THE

LADIES and GENTLEMENS

D I A R Y,

O R,

ROYAL ALMANACK;

For the Year of our LORD, 1777:

Being the First after Bissextile, or Leap Year.

CONTAINING,

Besides the CALENDAR, a great Variety of Ænigmas, Rebusses, Mathematical Solutions, &c. &c.

By REUBEN BURROW,
Late Assistant Astronomer at the Royal Observatory, and Teacher of the Mathematics.

LONDON:
Printed for T. CARNAN, in St. Paul's Church Yard, who dispossess-ed the Stationers Company of the exclusive Privilege of Printing Al-manacks, which they had monopolized 170 Years, to the discou-ragement of Genius and the great Prejudice of the Booksellers throughout the kingdom, in Consequence of a Patent obtained from King James I. which his most Sacred Majesty had no Right to Grant.

ECLIPSES in 1777.

This year there will be Five Eclipſes, Three of the Sun, and Two of the Moon, which will happen in the following Order: The Firſt Eclipſe of the Sun will happen on the 9th of *January*, at Forty-Nine Minutes after Three in the Afternoon, only Part viſible.---The Second will be an Eclipſe of the Moon, beginning *January* 23d, at Forty-Seven Minutes after Two in the Afternoon, Middle Eleven Minutes after Four, ends Thirty-Six Minutes after Five, Digits eclipſed 7°. 6'. Moon riſes at Twenty-Five Minutes after Four, conſequently only Part viſible.---The Third Eclipſe will be of the Sun, *July* 4th, at Twenty-One Minutes paſt Midnight, inviſible.---The Fourth will be an Eclipſe of the Moon, *July* 20th, at Forty-Two Miuutes paſt Noon.---The Fifth is an Eclipſe of the Sun, which happens on the 29th of *December*, at Ten at Night, inviſible.

COMMON NOTES, 1777.

Golden Number - -	10	Dominical Letter - -	E
Cycle of the Sun - -	22	Roman Indiction - -	19
Epact - - -	20	Number of Direction - -	9

The FOUR QUARTERS of the YEAR.

The Spring Quarter begins this Year the 20th of March, at 6 Hours 15 Minutes Morning, at which time the Sun enters *Equinoctial Sine Aries*, making equal Day and Night all the World over.

The Summer Quarter commences the 21ſt Day of June, at 4 Hours 33 Minutes, Morning, the Sun then entering into the *Sign Cancer*, making the longeſt Day to all the Northern, and the ſhorteſt to all the Southern Parts of the World.

The Autumnal Quarter begins the 22d Day of September, at 6 at Night, at which Time the Sun enters *Libra*, making again equal Day and Night to all Parts of the World.

The Winter Quarter begins the 21ſt of December, 10 Hours 20 Minutes, Morning, the Sun then entering into the *tropical Sign Capricorn*, making the ſhorteſt Day to the Northern, and longeſt to the Southern Inhabitants of the World.

WEIGHT and VALUE of the GOLD and SILVER COINS of England.

	WEIGHT.		VALUE.		
GOLD.	dwt.	grs.	l.	s.	d.
A Guinea	5	9,438	1	1	0
Half Guinea	2	16,719	0	10	6
Quarter Guinea	1	8,359	0	5	3
SILVER.					
A Crown	19	8,519	0	5	0
Half Crown	9	16,259	0	2	6
Shilling	3	20,903	0	1	0
Sixpence	1	22,451	0	0	6

Curre t Gold Coin muſt weigh as follows:

	dwt.	grs.
Guineas	5	8
Half Guineas	2	16
Quarter Guineas	1	8

Laſt Quarter	1 day	9 h. 9 m. evening	Sun enters Aquarius 19d. 2h. 54m. Apparent time.
New Moon	9 day	3 h. 39 m. afternoon	
Firſt Quarter	16 day	at noon	
Full Moon	23 day	4 h. 19 m. afternoon	
Laſt Quarter	31 day	6 h. 28 m. evening	

1	W	Circumciſion	8	4	3	56	22	58	☽ riſes		23
2	TH	Mars riſes 11 36	8	4	3	56	22	52	0 m	11	24
3	F		8	3	3	57	22	46	1	14	25
4	S		8	2	3	58	22	40	2	19	26
5	E	2 S. aft. Chriſt. *O.Chriſt.d.*	8	1	3	59	22	33	3	24	27
6	M	Epiphany *Twelfth Day*	8	0	4	0	22	26	4	35	28
7	TU		7	59	4	1	22	18	5	40	29
8	W	*Lucian*	7	58	4	2	22	10	6	45	30
9	TH	Sun eclipſed	7	57	4	3	22	1	☽ ſets		1
10	F		7	56	4	4	21	52	5 a	8	2
11	S		7	55	4	5	21	42	6	20	3
12	E	1 S. aft. Epiph. *O. N. Y. d.*	7	54	4	6	21	32	7	39	4
13	M	Camb. T. b. *Plow Mond.*	7	53	4	7	21	22	8	59	5
14	TU	Oxford Term begins	7	52	4	8	21	11	10	18	6
15	W		7	51	4	9	21	0	11	36	7
16	TH		7	50	4	10	20	49	Morn.		8
17	F	*Old Twelfth Day.*	7	49	4	11	20	37	0	56	9
18	S	Q. Ch. b. d. kept. Priſc.	7	48	4	12	20	24	2	14	10
19	E	2 S. aft. Epiph.	7	46	4	14	20	12	3	29	11
20	M	*Fabian* In 8 d. Hil. 1 Ret.	7	44	4	16	19	58	4	40	12
21	TU	*Agnes*	7	43	4	17	19	45	5	47	13
22	W	*Vincent*	7	42	4	18	19	31	6	43	14
23	TH	Hilary Term begins	7	41	4	19	19	17	☽ riſes		15
24	F		7	39	4	21	19	3	5 a	24	16
25	S	Converſion of St. Paul	7	37	4	23	18	48	6	29	17
26	E	Septuageſima Sunday	7	36	4	24	18	33	7	36	18
27	M	Pr. Aug. Fred. b. In 15	7	34	4	26	18	17	8	42	19
28	TU	[days of Hil. 2 Ret.	7	32	4	28	18	1	9	46	20
29	W		7	30	4	30	17	45	10	51	21
30	TH	King Charles beheaded	7	28	4	32	17	28	11	55	22
31	F		7	27	4	33	17	11	Morn.		23

Days	Leng. of Days.	Days increaſe.	Day breaks.	Sun Eaſt	Twilight ends.	Clock before Sun.	SevenStars South.
1	7 52	0 8	5 59	4 41	6 1	4 20	8 A 43
6	8 0	0 16	5 56	4 43	6 4	6 36	8 20
11	8 10	0 26	5 53	4 47	6 8	8 41	7 58
16	8 20	0 37	5 48	4 50	6 12	10 29	7 37
21	8 34	0 50	5 43	4 55	6 17	11 59	7 16
26	8 49	1 5	5 37	4 58	6 23	13 10	6 55

New Moon	8 day 4 h. 32 m. morning	Sun enters Pisces.
First Quarter	14 day 8 h. 18 m. night	17d. 17h. 46m.
Full Moon	22 day 9 h. 19 m. morning	Apparent time

D M	D W	Sundays, Holidays, &c.	☉ rises.	☉ sets.	☉'s declin.	☽ rises & sets.	☽'s age.
1	S		7 26	4 34	16 54S	1 m c	24
2	E	Candlemas d. Sexag. S.	7 24	4 36	16 37	2 7	25
3	M	*Blase* Mor. Purif. 3 Ret.	7 22	4 38	16 19	3 13	26
4	Tu	Mars rises 10 11	7 20	4 40	16 1	4 19	27
5	W	*Agatha*	7 18	4 42	15 43	5 19	28
6	Th		7 16	4 44	15 24	6 15	29
7	F	Venus sets 8 43	7 14	4 46	15 5	☽ sets	1
8	S		7 12	4 48	14 46	5 a 9	2
9	E	Shrove Sunday	7 11	4 49	14 27	6 32	3
10	M	In 8 days of Pur. 4 Ret.	7 10	4 50	14 7	7 54	4
11	Tu	Shrove Tuesday	7 8	4 52	13 48	9 19	5
12	W	Ash Wedn. Hil. T. ends	7 6	4 54	13 28	10 41	6
13	Th	Old Candlemas day	7 5	4 55	13 7	11 59	7
14	F	*Valentine*	7 3	4 57	12 47	morn.	8
15	S	Camb. Term divides	7 1	4 59	12 26	1 18	9
16	E	1 Sunday in Lent	6 59	5 1	12 6	2 29	10
17	M		6 57	5 3	11 44	3 37	11
18	Tu		6 55	5 5	11 23	4 36	12
19	W	Ember Week	6 53	5 7	11 2	5 26	13
20	Th		6 51	5 9	10 40	6 7	14
21	F		6 49	5 11	10 19	☽ rises	15
22	S	Procyon S. 9 1	6 47	5 13	9 57	5 a 23	16
23	E	2 Sunday in Lent	6 45	5 15	9 35	6 29	17
24	M	St. Matthias, Pr. Adol.	6 43	5 17	9 12	7 35	18
25	Tu	[Fr. born	6 41	5 19	8 50	8 39	19
26	W		6 39	5 21	8 28	9 44	20
27	Th		6 38	5 22	8 5	10 49	21
28	F		6 36	5 24	7 42	11 54	22

Days.	Leng. of Days.	Days increase.	Day breaks.	Sun East.	Twilight ends.	Clock before Sun	Seven Stars South.
1	9 9	1 25	5 28	5 5	6 32	14 8	6 A 31
6	9 27	1 43	5 21	5 10	6 39	14 34	6 11
11	9 45	2 1	5 12	5 16	6 48	14 41	5 51
16	10 3	2 19	5 4	5 21	6 56	14 27	5 31
21	10 23	2 39	4 55	5 27	7 5	13 56	5 12
26	10 43	2 59	4 46	5 33	7 14	13 9	4 54

Laſt Quarter 2 day 1 h. 42 m. afternoon. New Moon 9 day 3 h. 20 m. afternoon. Firſt Quarter 16 day 6 h. 11 m. morning. Full Moon 24 day 2 h. 54 m. morning.	S. enters Aries. 19d. 18h. 15m. Apparent time.

1	S	*David,*	6 34	5 26	7 20S	Morn.	23
2	E	3 Sunday in Lent. Chad.	6 32	5 28	6 57	1 2	24
3	M		6 30	5 30	6 34	2 6	25
4	Tu	Jupiter South 8 3	6 28	5 32	6 11	3 8	26
5	W		6 26	5 34	5 48	4 4	27
6	Th		6 24	5 36	5 24	4 53	28
7	F	*Perpetua*	6 22	5 38	5 1	5 36	29
8	S		6 20	5 40	4 38	6 10	30
9	E	Midlent Sunday	6 18	5 42	4 14	☽ ſets	1
10	M		6 16	5 44	3 51	6 a 56	2
11	Tu		6 14	5 46	3 27	8 22	3
12	W	*Gregory* M.	6 12	5 48	3 3	9 46	4
13	Th	Regulus South 10 20	6 10	5 50	2 40	11 7	5
14	F		6 8	5 52	2 16	Morn.	6
15	S		6 6	5 54	1 52	0 27	7
16	E	5 Sunday in Lent	6 4	5 56	1 29	1 38	8
17	M	*St. Patrick*	6 2	5 58	1 5	2 40	9
18	Tu	*Edward K. W. S.*	6 0	6 0	0 41	3 32	10
19	W	Mars riſes 7 7	5 58	6 2	0 18	4 15	11
20	Th	Equal Day and Night.	5 56	6 4	0 5N	4 48	12
21	F	Benedict. Camb. T. ends	5 54	6 6	0 29	5 19	13
22	S	Oxford Term ends	5 52	6 8	0 53	5 42	14
23	E	6 Sun. in Lent. Palm S.	5 50	6 10	1 16	☽ riſes	15
24	M		5 48	6 12	1 40	6 a 36	16
25	Tu	Lady-day	5 46	6 14	2 3	7 41	17
26	W		5 44	6 16	2 27	8 47	18
27	Th	*Maund. Thurſday*	5 43	6 17	2 50	9 52	19
28	F	Good Friday	5 41	6 19	3 14	10 59	20
29	S		5 39	6 21	3 37	Morn.	21
30	E	Eaſter Sunday	5 37	6 23	4 0	0 5	22
31	M	Eaſter Monday	5 35	6 25	4 23	1 7	23

Days	Leng. of Days	Days increaſe	Day breaks.	Sun Eaſt	Twilight ends.	Clock before Sun.	SevenStars South.
1	10 53	3 9	4 41	5 37	7 19	12 34	4 A42
6	11 13	3 29	4 31	5 43	7 29	11 26	4 24
11	11 33	3 49	4 20	5 50	7 40	10 9	4 5
16	11 53	4 9	4 9	5 56	7 51	8 43	3 47
21	12 13	4 29	3 58	6 1	8 2	7 12	3 29
26	12 33	4 49	3 46	6 7	8 14	5 30	3 11

Last Quarter 1 day 5 h. 31 m. morning.		Sun enters Taurus.
New Moon 7 day 0 h. 18 m. midnight.		19d. 7h. 7m.
First Quarter 14 day 6 h. 1 m. afternoon.		Apparent time.
Full Moon 22 day 7 h. 52 m. evening.		
Last Quarter 30 day 5 h. 18 m. afternoon.		

1	Tu	Easter Tuesday	5 33	6 27	4 47 N	2 m 4	24
2	W		5 31	6 29	5 10	2 54	25
3	Th	*R. Bishop Chichest.r*	5 29	6 31	5 33	3 37	26
4	F	*St. Ambrose*	5 27	6 33	5 55	4 15	27
5	S	*Old Lady-day*	5 25	6 35	6 18	4 46	28
6	E	1 S. after East. Low S.	5 23	6 37	6 41	5 12	29
7	M	Saturn rises 7 37	5 21	6 39	7 3	☽ sets	1
8	Tu		5 19	6 41	7 26	7 a 21	2
9	W	Oxf. and Camb. T. beg.	5 17	6 43	7 48	8 47	3
10	Th		5 15	6 45	8 10	10 13	4
11	F		5 13	6 47	8 32	11 30	5
12	S		5 11	6 49	8 54	Morn.	6
13	E	2 Sunday after Easter	5 10	6 50	9 16	0 40	7
14	M	From East. in 2 weeks 1	5 8	6 52	9 37	1 37	8
15	Tu	[Ret.	5 6	6 54	9 59	2 24	9
16	W	Easter Term begins	5 4	6 56	10 20	3 1	10
17	Th		5 2	6 58	10 41	3 32	11
18	F		5 0	7 0	11 2	3 55	12
19	S	*Alphege*	4 58	7 2	11 23	4 19	13
20	E	3 Sunday after Easter	4 56	7 4	11 43	4 35	14
21	M	From East. in 3 weeks 2	4 54	7 6	12 4	4 54	15
22	Tu	[Ret.	4 52	7 8	12 24	5 12	16
23	W	St. George	4 50	7 10	12 44	☽ rises	17
24	Th		4 49	7 11	13 3	8 a 54	18
25	F	St. Mark. Prs. M. born	4 47	7 13	13 23	10 5	19
26	S		4 45	7 15	13 42	11 8	20
27	E	4 Sunday after Easter	4 43	7 17	14 1	Morn.	21
28	M	From East. in 4 weeks	4 41	7 19	14 20	0 8	22
29	Tu	[3 Ret.	4 40	7 20	14 39	1 2	23
30	W		4 38	7 22	14 57	1 43	24

Days	Leng. of days.	Days increase.	Day breaks.	Sun Fast	Twilight ends.	Clock before Sun.	Seven Stars South.
1	12 55	5 11	3 31	6 15	8 29	3 47	2 A 50
6	13 15	5 31	3 18	6 21	8 42	2 18	2 32
11	13 35	5 51	3 5	6 27	8 55	0 54	2 13
16	13 53	6 9	2 51	6 33	9 9	0 A 23	1 55
21	14 13	6 29	2 36	6 39	9 24	1 31	1 37
26	14 31	6 47	2 20	6 44	9 40	2 28	1 18

New Moon 7 day 8 h. 8 m. morning.		
First Quarter 14 day 7 h. 46 m. morning.	Sun enters Gemini.	
Full Moon 22 day 11 h. 24 m. morning.	20d. 7h. 47m.	
Laſt Quarter 30 day 1 h. 18 m. morning.	Apparent time.	

1	Th	St. Phil. and Ja.	4 36	7 24	15 15N	2 m 22	25
2	F		4 34	7 26	15 33	2 53	26
3	S	*Inv. of the Croſs*	4 33	7 27	15 51	3 20	27
4	E	Rogation Sunday	4 31	7 29	16 8	3 45	28
5	M	From Eaſt. in 5 weeks 4	4 30	7 30	16 25	4 8	29
6	Tu	*John à P. Lat.* [Ret.	4 28	7 32	16 42	☽ ſets	1
7	W		4 26	7 34	16 58	7 a 47	2
8	Th	Aſcenſ. day. Holy Thurſ.	4 25	7 35	17 15	9 9	3
9	F	On mor. of Aſcenſ. 5 Ret.	4 23	7 37	17 31	10 24	4
10	S		4 22	7 38	17 46	11 32	5
11	E	S. after Aſcenſion day	4 20	7 40	18 2	Morn.	6
12	M	Term ends. *Old May-day*	4 18	7 43	18 17	0 24	7
13	Tu		4 17	7 42	18 32	1 5	8
14	M		4 16	7 44	18 46	1 37	9
15	Th	Oxford Term ends	4 14	7 46	19 0	2 5	10
16	F		4 12	7 48	19 14	2 27	11
17	S	Arcturus South 10 25	4 11	7 49	19 27	2 46	12
18	E	Whit-Sunday	4 10	7 50	19 41	3 4	13
19	M	Q. Cha. b. 1744. *Dunſt.*	4 8	7 52	19 54	3 22	14
20	Tu	Whit-Tueſday	4 6	7 54	20 6	3 40	15
21	W	Ember Week	4 5	7 55	20 18	4 0	16
22	Th	Prs. Eliz. born	4 3	7 57	20 30	☽ riſes	17
23	F		4 2	7 58	20 42	9 a 1	18
24	S	*No N. but twil. till July* 21.	4 1	7 59	20 53	10 3	19
25	E	Trinity Sunday	4 0	8 0	21 3	10 59	20
26	M	*Auguſtin.* Mor. of Tr.	3 59	8 1	21 14	11 47	21
27	Tu	*Ven. Bede* [1 Ret	3 58	8 2	21 24	Morn.	22
28	W	Oxford Term begins	3 57	8 3	21 34	0 24	23
29	Th	K. Ch. II. Reſt. *C. Chriſti*	3 56	8 4	21 43	0 57	24
30	F	Trinity Term begins	3 55	8 5	21 52	1 24	25
31	S		3 54	8 6	22 0	1 47	26

Days	Leng. of Days.	Days increaſe.	Day breaks.	Sun Eaſt	Twilight ends.	Clock after Sun.	SevenStars South.
1	14 49	7 5	2 3	6 50	9 57	3 12	0 A59
6	15 5	7 21	1 46	6 55	10 14	3 43	0 40
11	15 21	7 37	1 24	7 0	10 36	3 58	0 20
16	15 36	7 52	1 0	7 4	11 0	4 0	0 0
21	15 50	8 6	0 18	7 8	11 42	3 48	11 M40
26	16 2	8 18	No Night	7 12	No Night	3 23	11 20

June hath XXX Days. 1777.

New Moon 5 day 3 h. 48 m. afternoon.	
First Quarter 12 day 11 h. 2 m. night.	S. enters Gemini.
Full Moon 21 day 1 h. 5 m. morning.	20d. 16h. 33m.
Last Quarter 28 day 6 h. 38 m. morning.	Apparent time.

1	E	1 S. aft. Trin. Nicom.	3 53	8 7	22 8 N	2 m 12	27
2	M	In 1 week aft. Tr. 2 Ret.	3 52	8 8	22 16	2 33	28
3	Tu		3 51	8 9	22 24	2 55	29
4	W	K. George III. born	3 51	8 10	22 31	3 23	30
5	Th	Pr. Er. Aug. b. *Boniface*	3 50	8 11	22 37	☽ sets	1
6	F	Lyra South 1 32	3 49	8 12	22 43	9 a 10	2
7	S		3 48	8 12	22 49	10 11	3
8	E	2 Sunday after Trinity	3 48	8 13	22 55	10 59	4
9	M	In 2 weeks aft. Tr. 3 Ret.	3 47	8 13	23 0	11 35	5
10	Tu	Princess Amelia born	3 47	8 14	23 4	Morn.	6
11	W	St. Barnabas	3 46	8 15	23 8	0 3	7
12	Th		3 45	8 15	23 12	0 28	8
13	F		3 45	8 16	23 16	0 49	9
14	S	Clock with the sun	3 44	8 16	23 18	1 8	10
15	E	3 Sunday after Trinity	3 44	8 16	23 21	1 24	11
16	M	In 3 weeks aft. Tr. 4 Ret.	3 44	8 17	23 23	1 43	12
17	Tu	*St. Alban*	3 43	8 17	23 25	2 1	13
18	W	Trinity Term ends	3 43	8 17	23 26	2 23	14
19	Th		3 43	8 17	23 27	2 49	15
20	F	*T. Edw. K. W. S.*	3 43	8 17	23 28	3 22	16
21	S	Long. day 16 34	3 43	8 17	23 28	☽ rises	17
22	E	4 Sunday after Trinity	3 43	8 17	23 27	9 a 41	18
23	M		3 43	8 17	23 26	10 23	19
24	Tu	St. John Bapt.	3 43	8 17	23 25	10 57	20
25	W		3 44	8 16	23 24	11 24	21
26	Th		3 44	8 16	23 22	11 48	22
27	F		3 44	8 16	23 20	Morn.	23
28	S		3 44	8 16	23 17	0 10	24
29	E	5 S. aft. Trin. St. Peter	3 45	8 15	23 13	0 33	25
30	M		3 45	8 15	23 10	0 54	26

Days	Leng. of Days.	Days increase.	Day breaks.	Sun East	Twilight ends.	Clock after Sun.	Seven Stars South.
1	16 14	8 32	No real night, but constant twilight.	7 16	No real night, but constant twilight.	2 37	10 M 56
6	16 22	8 38		7 18		1 47	10 35
11	16 28	8 44		7 19		0 49	10 15
16	16 32	8 48		7 20		bef. 13	9 54
21	16 34	8 50		7 20		1 18	9 33
26	16 32	dec. 2		7 20		2 21	9 12

July hath XXXI Days.

New Moon 4 day 0 h. 21 m. midnight.	
First Quarter 12 day 3 h. 34 m. afternoon.	Sun enters Leo.
Full Moon 20 day 0 h. 52 m. afternoon.	22d. 3h. 23m.
Last Quarter 27 day 10 h. 55 m. morning.	Apparent time.

1	Tu	Camb. Comm.	3 46	8 14	23 6N	1 m 18	27
2	W	*Visit. B. V. M.*	3 46	8 14	23 1	1 50	28
3	Th		3 47	8 13	22 57	2 27	29
4	F	Camb. T. ends. *Tran. St.*	3 48	8 12	22 51	☽ sets	1
5	S	*Old Midf. day* [*Mar.*	3 48	8 12	22 46	8 a 42	2
6	E	6 Sunday after Trinity	3 49	8 11	22 40	9 30	3
7	M	Oxford Act	3 50	8 10	22 33	10 0	4
8	Tu		3 50	8 10	22 26	10 25	5
9	W		3 51	8 9	22 19	10 48	6
10	Th		3 52	8 8	22 12	11 7	7
11	F		3 53	8 7	22 4	11 25	8
12	S		3 54	8 6	21 55	11 43	9
13	E	7 Sunday after Trinity	3 55	8 5	21 47	Midn.	10
14	M		3 56	8 4	21 38	Morn.	11
15	Tu	*Swithin*	3 57	8 3	21 28	0 21	12
16	W		3 58	8 2	21 18	0 44	13
17	Th		3 59	8 1	21 8	1 14	14
18	F		4 0	8 0	20 58	1 52	15
19	S	Oxford Term ends	4 1	7 59	20 47	2 38	16
20	E	8 S. aft. Trin.	4 3	7 57	20 35	☽ rises	17
21	M	Dog-days begin	4 4	7 56	20 24	8 a 53	18
22	Tu	*St. Mary Magdalen*	4 5	7 55	20 12	9 24	19
23	W		4 6	7 54	20 0	9 50	20
24	Th		4 8	7 52	19 47	10 13	21
25	F	St. James	4 10	7 50	19 34	10 36	22
26	S	*St. Anne, Mother of V. M.*	4 11	7 49	19 21	10 57	23
27	E	9 Sunday after Trinity	4 13	7 47	19 7	11 23	24
28	M		4 14	7 46	18 53	11 47	25
29	Tu		4 15	7 45	18 39	Morn.	26
30	W		4 17	7 43	18 24	0 21	27
31	Th		4 18	7 42	18 10	1 4	28

Days	Leng. of days.	Days increase.	Day breaks.	Sun East	Twilight ends.	Clock before Sun.	Seven Stars South.
1	16 28	0 6		7 19		3 21	8 M 52
6	16 22	0 12	No real	7 17	No real	4 16	8 31
11	16 14	0 20	Night.	7 15	Night.	5 2	8 11
16	16 4	0 30		7 12		5 35	7 50
21	15 52	0 42	0 24	7 8	11 36	5 55	7 30
26	15 38	0 56	1 1	7 4	10 59	6 1	7 10

New Moon 3 day 10 h. 45 m. morning.		Sun enters Virgo.
Firſt Quarter 11 day 9 h. 0 m. morning.		22d. 9h. 39m.
Full Moon 18 day 11 h. 8 m. night.		
Laſt Quarter 25 day 3 h. 54 m. afternoon.		

1	F	Lammas-day	4 19	7 41	17 54N	1m 54	29
2	S		4 21	7 39	17 39	☽ ſets	1
3	E	10 Sunday after Trinity	4 23	7 37	17 23	7 a 57	2
4	M		4 24	7 36	17 7	8 27	3
5	Tu		4 25	7 35	16 51	8 51	4
6	W	*Transfiguration*	4 27	7 33	16 35	9 10	5
7	Th	*Name of Jeſus*	4 28	7 32	16 18	9 30	6
8	F		4 30	7 30	16 1	9 47	7
9	S		4 31	7 29	15 43	10 6	8
10	E	11 S. aft. Trin. St. Law-	4 33	7 27	15 26	10 25	9
11	M	Prs. Brunſw. b. [rence	4 35	7 25	15 8	10 45	10
12	Tu	Pr. Wales b. Old Lam. d.	4 36	7 24	14 50	11 14	11
13	W		4 38	7 22	14 32	11 48	12
14	Th		4 40	7 20	14 13	Morn.	13
15	F	*Aſſumption Virgin Mary*	4 42	7 18	13 54	0 29	14
16	S	Prince Fred. born, 1763	4 44	7 16	13 35	1 22	15
17	E	12 Sunday after Trinity	4 45	7 15	13 16	2 26	16
18	M		4 47	7 13	12 57	3 37	17
19	Tu		4 49	7 11	12 37	☽ riſes	18
20	W		4 51	7 9	12 17	8 19	19
21	Th	Pr. Wm. Henry b. 1765	4 53	7 7	11 57	8 43	20
22	F		4 54	7 6	11 37	9 6	21
23	S		4 56	7 4	11 17	9 28	22
24	E	13 S. aft. Trin. St. Bar-	4 58	7 2	10 56	9 56	23
25	M	[tholomew	5 0	7 0	10 35	10 27	24
26	Tu		5 2	6 58	10 14	11 6	25
27	W		5 4	6 56	9 53	11 52	26
28	Th	*St. Auſtin*	5 6	6 54	9 32	Morn.	27
29	F	*Dec. St. John Baptiſt*	5 8	6 52	9 11	0 50	28
30	S		5 10	6 50	8 49	1 56	29
31	E	14 Sunday after Trinity	5 12	6 48	8 27	3 4	30

Days	Leng. of days.	Days decreaſe.	Day breaks.	Sun Eaſt	Twilight ends.	Clock before Sun.	SevenStars South.
1	15 20	1 14	1 29	6 59	10 31	5 50	6M47
6	15 5	1 29	1 49	6 54	10 11	5 25	6 28
11	14 49	1 45	2 7	6 49	9 53	4 45	6 9
16	14 31	2 3	2 25	6 44	9 35	3 50	5 50
21	14 13	2 21	2 39	6 38	9 21	2 42	5 32
26	13 55	2 39	2 52	6 33	9 8	1 23	5 13

New Moon 1 day 11 h. 32 m. night.	Sun enters Libra.
First Quarter 10 day 2 h. 41 m. morning.	22d. 6h. om.
Full Moon 17 day 8 h. 24 m. morning.	Apparent time.
Last Quarter 23 day 11 h. 3 m. night.	

1	M	*..ies* Dog-days end	5 14	6 46	8 6	☽ sets	1
2	Tu	London burnt, 1666	5 16	6 44	7 44	7 a 21	2
3	W		5 17	6 43	7 22	7 40	3
4	Th		5 19	6 41	6 59	7 59	4
5	F		5 21	6 39	6 37	8 17	5
6	S	Fomalhaut South 11 40	5 23	6 37	6 15	8 36	6
7	E	15 S. aft. Trin. *Eurchus*	5 25	6 35	5 52	8 56	7
8	M	*Nativity V. Mary*	5 27	6 33	5 29	9 22	8
9	Tu		5 29	6 31	5 7	9 52	9
10	W		5 31	6 29	4 44	10 30	10
11	Th		5 33	6 27	4 21	11 16	11
12	F		5 35	6 25	3 58	Morn.	12
13	S	Venus rises 1 35	5 37	6 23	3 35	0 12	13
14	E	16 S. aft. Trin. *Holy-*	5 39	6 21	3 12	1 22	14
15	M	[*cross-day*	5 41	6 19	2 49	2 36	15
16	Tu		5 43	6 17	2 26	3 58	16
17	W	Ember Week. *Lamb.*	5 45	6 15	2 2	5 21	17
18	Th		5 47	6 13	1 39	☽ rises	18
19	F		5 49	6 11	1 16	7 a 39	19
20	S		5 51	6 9	0 52	8 6	20
21	E	17S. aft. Tr. St. Matthew	5 53	6 7	0 29	8 36	21
22	M	K. George crowned 1761	5 55	6 5	0 6	9 13	22
23	Tu		5 57	6 3	0 17S	9 58	23
24	M		5 59	6 1	0 41	10 53	24
25	Th		6 1	5 59	1 4	11 55	25
26	F	*St. Cyprian*	6 3	5 57	1 28	Morn.	26
27	S		6 5	5 55	1 51	1 4	27
28	E	18 S. after Trinity	6 7	5 53	2 14	2 14	28
29	M	St. Mich. Prs. Ch. Aug	6 9	5 51	2 38	3 23	29
30	Tu	*St. Jerome* [bo. 1766	6 11	5 49	3 1	4 42	30

Days	Leng. of days.	Days de-crease.	Day breaks.	Sun East	Twilight ends.	Clock af-ter Sun.	Seven Stars South.
1	13 31	3 3	3 10	6 25	8 50	0 23	4 M 52
6	13 13	3 21	3 23	6 19	8 37	1 59	4 34
11	12 53	3 41	3 38	6 13	8 22	3 41	4 16
16	12 33	4 1	3 47	6 7	8 13	5 26	3 58
21	12 13	4 21	3 59	6 1	8 1	7 11	3 40
26	11 53	4 41	4 10	5 55	7 50	8 53	3 22

New Moon 1 day 2 h. 57 m. afternoon.
First Quarter 9 day 7 h. 34 m. evening.
Full Moon 16 day 5 h. 27 m. evening.
Last Quarter 23 day 9 h. 28 m. morning.
New Moon 31 day 8 h. 33 m. morning.

Sun enters Scorpio. 22d. 13h. 51m.

Apparent time.

1	W	*Remigius*	6 12	5 48	3 25	☽ sets	1
2	Th		6 14	5 46	3 48	6 a 31	2
3	F	Fomalhaut South 10 3	6 16	5 44	4 11	6 49	3
4	S		6 18	5 42	4 34	7 8	4
5	E	19 S. after Trin.	6 20	5 40	4 58	7 32	5
6	M	*Faith*	6 22	5 38	5 21	8 0	6
7	Tu	Jupiter rises 1 18	6 24	5 36	5 44	8 34	7
8	W		6 26	5 34	6 7	9 17	8
9	Th	*St. Denys*	6 28	5 32	6 30	10 7	9
10	F	Oxf. & Cam. Term. beg.	6 30	5 30	6 52	11 8	10
11	S	[*Old Mich. day*	6 32	5 28	7 15	Morn.	11
12	E	20 Sunday after Trin.	6 34	5 26	7 38	0 20	12
13	M	*Tr. K. Ed. Confessor*	6 36	5 24	8 0	1 36	13
14	Tu		6 38	5 22	8 22	2 58	14
15	W		6 40	5 20	8 45	4 23	15
16	Th		6 42	5 18	9 7	☽ rises	16
17	F	*Ethelred*	6 44	5 16	9 29	6 11	17
18	S	St. Luke	6 46	5 14	9 51	6 41	18
19	E	21 Sund. aft. Trin.	6 47	5 13	10 12	7 15	19
20	M		6 49	5 11	10 34	7 57	20
21	Tu		6 51	5 9	10 55	8 52	21
22	W		6 53	5 7	11 17	9 50	22
23	Th		6 55	5 5	11 38	10 59	23
24	F		6 57	5 3	11 59	Morn.	24
25	S	K. Geo. III. Ac. *Crispin*	6 59	5 1	12 19	0 11	25
26	E	22 S. aft. Tr. King Geo. [Proc.	7 1	4 59	12 40	1 20	26
27	M		7 3	4 57	13 0	2 28	27
28	Tu	St. Simon and Jude	7 5	4 55	13 20	3 37	28
29	W	Fomalhaut South 8 26	7 7	4 53	13 40	4 4[illegible]	29
30	Th		7 9	4 51	14 0	☽ sets	1
31	F		7 11	4 49	14 20	5 a 10	2

Days	Leng. of days.	Days decrease.	Day breaks.	Sun East	Twilight ends.	Clock after Sun.	Seven Stars South.
1	11 35	4 59	4 20	5 49	7 40	10 30	3 M 4
6	11 15	5 19	4 31	5 43	7 29	11 59	2 46
11	10 55	5 39	4 40	5 36	7 20	13 19	2 28
16	10 35	5 59	4 50	5 30	7 10	14 26	2 10
21	10 17	6 17	4 59	5 24	7 1	15 19	1 51
26	9 57	6 37	5 8	5 19	6 52	15 55	1 32

First Quarter	8 day	10 h.	31 m. morning	
Full Moon	15 day	3 h.	3 m. morning	S. ent. Sagittarius
Last Quarter	21 day	11 h.	32 m. night	21d. 9h. 59m.
New Moon	30 day	3 h.	24 m. morning	Apparent time.

1	S	All Saints	7	12	4	48	14	39	5 a 39	3
2	E	23 S. aft. Tr. Pr. Ed. bo.	allSo		4	46	14	58	6 0	4
3	M	Mor. of all Souls 1 Ret.	7	16	4	44	15	17	6 30	5
4	Tu		7	18	4	42	15	35	7 15	6
5	W	Powder Plot, 1605	7	20	4	40	15	53	8 1	7
6	Th	*Leonard.*	7	21	4	39	16	11	8 58	8
7	F	D. of Cum. b. 1745	7	23	4	37	16	29	10 4	9
8	S	Prs. Auguf. So. bo. 1768	7	24	4	36	16	46	11 15	10
9	E	24 S. aft. Trin. Ld. May-[day	7	26	4	34	17	4	morn.	11
10	M		7	28	4	32	17	20	0 33	12
11	Tu	*St. Martin*	7	30	4	30	17	37	1 54	13
12	W	Mor. of St. Mart. 2 Ret	7	31	4	29	17	53	3 18	14
13	Th	*Britius*	7	32	4	28	18	9	4 41	15
14	F		7	34	4	26	18	25	☽ rifes	16
15	S	*Machutus*	7	36	4	24	18	40	5 a 6	17
16	E	25 S. after Trinity	7	37	4	23	18	55	5 46	18
17	M	*Hugh B. Lincoln*	7	38	4	22	19	10	6 35	19
18	Tu	In 8 days of St. Mar. 3 Ret.	7	40	4	20	19	24	7 34	20
19	W		7	42	4	18	19	38	8 40	21
20	T	*Edw. K. and Mart.*	7	44	4	16	19	52	9 53	22
21	F		7	45	4	15	20	5	11 4	23
22	S	*Old Mart. Day. Cecilia*	7	46	4	14	20	18	morn.	24
23	E	26 S. after Trin. *Clement*	7	48	4	12	20	30	0 15	25
24	M		7	49	4	11	20	42	1 25	26
25	Tu	D. of Glouceft. b. 4 Ret.	7	50	4	10	20	54	2 30	27
26	W		7	51	4	9	21	5	3 36	28
27	Th		7	52	4	8	21	16	4 41	29
28	F	Michaelm. Term ends	7	53	4	7	21	27	5 48	30
29	S		7	54	4	6	21	37	☽ fets	1
30	E	Advent S. St. Andrew	7	55	4	5	21	46	4 a 33	2

Days.	Leng. of Days.		Day decreafe.		Day breaks.		Sun Eaft.		Twilight ends.		Clock after Sun.		Seven Stars fouth.
1	9	35	6	59	5	17	5	17	6	43	16	13	1 M 8
6	9	17	7	17	5	26	5	6	6	34	16	5	0 48
11	9	0	7	34	5	32	5	1	6	28	15	38	0 28
16	8	45	7	49	5	38	4	56	6	22	14	50	0 8
21	8	30	8	4	5	44	4	52	6	16	13	40	11 A 43
26	8	18	8	16	5	49	4	48	6	11	12	11	11 20

Firſt Quarter 7 day 10 h. 48 m. night		
Full Moon 14 day 1 h. 39 m. aftern.	S. ent. Capricorn	
Laſt Quarter 21 day 5 h. 5 m. aftern.	20d. 22h. 20m.	
New Moon 29 day 10 h. 0 m. night	Apparent time.	

1	M		7	57	4	3	21	56	5	a 7	3
2	Tu		7	58	4	2	22	5	5	51	4
3	W		7	59	4	1	22	13	6	44	5
4	Th	Aldebaran South 11 35	8	0	4	0	22	21	7	47	6
5	F		8	0	4	0	22	29	8	55	7
6	S	*Nicholas*	8	1	3	59	22	36	10	10	8
7	E	2 Sunday in Advent	8	2	3	58	22	42	11	26	9
8	M	*Conception Virgin Mary*	8	3	3	57	22	49	morn.		10
9	Tu		8	4	3	56	22	54	0	43	11
10	W		8	5	3	55	23	0	2	3	12
11	Th		8	5	3	55	23	5	3	25	13
12	F		8	6	3	54	23	9	4	52	14
13	S	*Lucy*	8	6	3	54	23	13	6	17	15
14	E	3 Sunday in Advent	8	6	3	54	23	16	☽ riſes		16
15	M		8	7	3	53	23	19	5	a 2	17
16	Tu	O Sap. Camb Term ends	8	7	3	53	23	22	6	8	18
17	W	Emb. Week. Oxf. Term	8	7	3	53	23	24	7	20	19
18	Th	[ends	8	8	3	52	23	26	8	35	20
19	F		8	8	3	52	23	27	9	45	21
20	S	Aldebaran South 10 25	8	8	3	52	23	27	10	57	22
21	E	4 Sun. Adv. St. Thomas	8	8	3	52	23	28	morn.		23
22	M	[Shorteſt day	8	8	3	52	23	27	0	6	24
23	Tu		8	8	3	52	23	27	1	13	25
24	W		8	8	3	52	23	26	2	18	26
25	Th	Chriſtmas Day	8	7	3	53	23	24	3	24	27
26	F	St. Stephen	8	7	3	53	23	22	4	39	28
27	S	St. John	8	7	3	53	23	19	5	36	29
28	E	Sun. aft. Chriſtmas. In-	8	7	3	53	23	16	6	39	30
29	M	[nocents	8	6	3	54	23	13	☽ ſets		1
30	Tu		8	6	3	54	23	9	4	a 26	2
31	W	*Silveſter*	8	5	3	55	23	4	5	25	3

Days.	Leng. of Day.		Day de-creaſe.		Day breaks.		Sun Eaſt.		Twilight ends.		Clock af-ter Sun.		Seven Stars South.	
1	8	6	8	28	5	54	4	45	6	6	10	22	11	A 1
6	7	58	8	36	5	57	4	43	6	3	8	19	10	39
11	7	50	8	44	5	59	4	41	6	1	6	3	10	17
16	7	46	8	48	6	0	4	40	6	0	3	39	9	55
21	7	44	8	50	6	1	4	40	5	59	1	10	9	33
26	7	46	Incr.	2	6	0	4	40	6	0	1	20	9	10

ANSWERS *to the* QUERIES, REBUSSES, *&c. in Laſt Year's* DIARY

Query I. *Anſwered by* Caput Mortuum.

THIS difference is one of thoſe operations of Nature which, doubtleſs, will never be accounted for; though it is probably effected by attraction and repulſion; but in what manner?--- We obſerve that the *Sun-Flower* generally keeps turning its bloſſom towards the ſun; we behold with admiration, the phænomena of the *ſenſitive plant*, and *Venus fly-trap*, but when we would enquire the *cauſe* our reaſon is at a ſtand, and we are left to lament the circumſcribed ſtate of human knowledge.

Query III. *Anſwered by Mr.* I. Dalby.

Theſe ſeem to be the ſpecies of worms called by Linnæus, *Gordius aquaticus pallidus*, with black extremities; though I have ſeen ſome thouſands of them entirely black; but as he ſays they are bred in clay, it is probable that they change to a pale colour ſoon after coming into the water. Merrett, in his *Pinax Britannicarum*, calls them *ſeta aquaticus*, and mentions the ſame thing of their being vulgarly taken for animated horſe-hairs: his words are, "Vulgo creditur oriri, ex ſeta caudæ equinæ aquis immerſâ." He has not taken notice of their colour.

Query IV. *Anſwered by Mr.* French Johnſon.

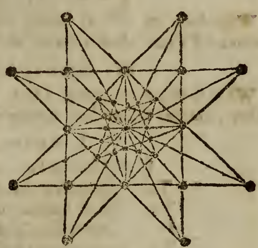
Sound the *s* in unlooſe ſoftly (as in looſe morals) and the myſtery will vaniſh; ſo then unlooſe morals will be good morals, and unlooſe will ſignify to be tyed.

Queries II. *and* V.

Are obliged to be deferred till next year, as no ſatisfactory anſwers have been received.

Anſwer to Mr. Dalby's *Paradox, with a new one propoſed, by Mr.* J Wales.

The ſcheme in the margin the
muckle mon ſhows,
To plant three and thirty in twenty-
four rows;
Now four in each row, and in rows
that are even,
The number of plants, I would plant
twenty-ſeven.



Anſwers to the three Rebuſſes, by Mr. John Clarke, *of* Lincoln.

I called laſt night on *Dalby*---he was gone
With *Rachel Rogers* up to *Iſlington*;
What could the errand be they went upon?

Anſwers to the Enigmas in laſt Year's Diary.

I. A Candle	V. A piece of Muſic
II. A Woman	VI. Coffee
III. A Bed	VII. A Picture
IV. A Louſe	

An

An answer to all the Enigmas, by Mr. William Francis, *of* Reading.

Man! what is he? a reptile on the earth II.
A scene of misery from his very birth;
His prime once past, how subject to decay!
Prone to the grave, and downward lies his way:
What various ills his every state attend,
Each coming day but hastens on his end;
To *bed* by sicknefs, pain, and grief, confin'd, III.
All out of *tune* in body and in mind; V.
Vermin sometimes, concomitants of age, IV.
Sadden the *picture*, and his death presage; VII.
Teas, soups, and cordials, can't prolong his stay, VI.
But like a *candle* snuff he dies away. I.

Answer to the Prize, by Philomathes.

Your portraiture, ingenious *Clarke*,
Does elegant appear;
Pray write some more, but not too dark,
Against another year.

Ingenious Answers were also given by Mess. Rogers, Clarke, Dalby, G. Little, Johnson, Moody, Wales, and several others; but the prize of ten Diaries fell to the lot of Mr. John Clarke.

New QUERIES, REBUSSES, *&c. to be answered next Year.*

I. QUERY, *by Mr.* Robert Moody.

What is the reason that dead bodies sooner rot in a dry than a moist church yard?

II. QUERY, *by Miss* Polly Tayrt.

Are not children naturally ambidextrous?

III. QUERY, *by Mr.* Isaac Dalby.

Why does an object, when viewed with a magnifying lens, seem farther off than when viewed with the naked eye?

IV. QUERY, *by Mr.* John Burrow.

What is the reason that a body moving forward upon rollers, moves twice as fast as the rollers themselves?

I. REBUS, *by* Regulus.

If the fairest fair you'd know,
Take the initials here below:
The highest station and command,
In this great, free, and happy land;
The greatest beauty or disgrace
Upon a pretty female's face;
The point within the azure skies,
From whence the sun is seen to rise;
The city which ten years employ'd
The bravest Greeks, before destroy'd.

II. REBUS, *by Mr.* Isaac Dalby.

One third of the pleasure of each toping blade,
When joined to a beast which the Lord never made,

Will

Will tell you what brought an unfortunate bard,
To ample repentance in Lazarus' ward.

III. Rebus, *by Mr.* Iſaac Dalby.

A large purſe, and four ſevenths of a miſer,
With juſt the two-thirds of a ſheep;
Twice a letter of capital ſize, Sir,
Join'd to the beginning of ſleep;

Theſe name you a Sunday retreat,
Near London for cit and for ſtranger,
Where Venus and Mercury meet,
And your carcaſe and purſe are in danger.

New Enigmas *to be anſwered in the next Year's* Diary.

I. Enigma, *by Mr.* William Francis, *of* Reading.

I Was born in a ſcuffle 'twixt father and mother,
And quickly convey'd to be nurs'd by another;
Tho' a black naſty jade, yet to tell you the truth,
She her duty perform'd, and befriended my youth:
A ſly beggar's brat thence ſtole me away,
And ſo altered my dreſs that I ſhine bright and gay;
I'm lively and briſk when I've food at command,
And chiefly ſubſiſt on the fat of the land;
On animal food tho' I moſtly do thrive,
I frequently feaſt on the ſpoils of the hive;
I'm always aſpiring, which haſtens my fate,
And my ruin compleats---a tale for the great:
Ye Enigmatiſts, who in dark myſteries delight,
In next Ladies Diary bring me to light.

II. Enigma, *by Mr.* T. Fiſhbourne.

Ye peaceful bards a-while attend,
And hearken to a faithful friend;
A friend you'll ſay, I make no doubt,
When once my name you have found out.
My downy wings around me ſpread,
My healing balm propitious ſhed,
Exert my kind relieving art,
And heal the ſorrow wounded heart;
I am a kind conſoling gueſt,
And calm the tumults of your breaſt;
I gently ſooth your ſoul to peace,
And make each jarring paſſion ceaſe;
From me your chiefeſt bleſſings flow,
A cordial I'm for every woe;
I chear your gloom, to joys invite.
And make your cares and burdens light;
From envy, pride, and diſcord free,
Are every one poſſeſſed of me,

All ſeek me in a different way,
Then what's my name, ye witty ſay.

III. Enigma.

Who's he that's no bodys friend,
Whoſe levees yet great men attend;
Who in retirement loves to ſneak,
Yet for domeſticks, oft does ſeek?
Folly and innocence him dread,
He's hated, yet he's follow'd,
And is interr'd before he's dead.
His retinue's kept at others coſt,
And when he's curſt he proſpers moſt.

IV. Enigma.

I ſtand but on one leg, yet do ſuſtain
Much weight, beſide a noted rogue in grain,
And 'twere an ill wind which blew him no gain.
He gives me clothes when faſt he'd have me run,
But ſtrips me naked when his work I've done;
Then I, with arms acroſs, expos'd do ſtand,
Forc'd to ſubmit to every turn of hand,
And to inconſtant unſeen powers command.
I once encounter'd was by hardy fool,
Who'ad got my nameſake lodg'd within his ſcull;
He me attack'd in wild and frantic mood,
And I my ground, tho' in ſwift motion, ſtood;
He from my arms receiv'd a ſtunning blow,
Yet what I was the coxcomb did not know;
And you're more wiſe, If you gueſs what I'm now.

V. Enigma.

Cloſe to my owner I adher'd,
'Till bloody hands me from him tear'd;
In warmth and quietneſs we liv'd,
And, while together, well we thriv'd;
But naked now men me expoſe,
And I excite them too to blows.
Dumb was I born, ſtill have no voice,
Yet courts and camps I fill with noiſe.
I liv'd in peace, now ſerve in wars,
Was innocent, but now at bars
Am try'd, where I move endleſs jars.
Great rogues trade in me by whole-ſale,
In parcels too they me retail;
But when their greater uſe I fail,
Small louſy thieves do in me deal,
And ſerve their ends of me piece-meal.

Prize Enigma *(of 10 Diaries) by Mr.* Iſaac Dalby.

Ye meddlers, who are always rude,
And unpolitely will intrude

Like

Like Marplot, and cannot forbear
To thruſt your noſes every where,
Be circumſpect---I'm one in keeping,
That pays impertinence for peeping----
Not care I, tho' perhaps in huff,
You take at once diſguſt and ſnuff.
There's ne'er a Slakenbergius-ſnout,
Nor Proclus' like, ſo large about,
That poets* ſing, he could not wipe it,
His hand b'ing much too ſmall to gripe it;
Nor pimpl'd knob, nor that with ſcars,
Curtail'd of half in Venus' wars,
That I reſpect,---for great and ſmall,
I play St. Dunſtan with 'em all.---
And this is done, Sirs, in a trice,
Tho' I'm not ſhap'd like tongs or vice;
But rather ſeem, (except in colour)
Like Mynheer Van Dunk's Kevenhuller;
With mouth extenſive, deep and round,
Deſcending to a depth profound---
Yet like a hag, long paſt her prime,
Whoſe teeth are drawn by quacks and time,
I am, tho' odd is the relation,
Incapable of maſtication;
But each fair belle by kindneſs led,
Prepares my food before I'm fed,
Then after, which you'll think is aukward,
They take great pains to feed me backward---
Laborious taſk! which brings to view
Things ſeldom ſeen, or ſeen by few;
But this alas! diſturbs my reſt,
And ſtorms invade my peaceful breaſt---
Loud thunders roll, and winds long pent,
In caverns deep now find a vent;
Rocks burſt, and with impetuous ſweep,
Are hurl'd into the briny deep.
From yon black cloud which ſeems to rend
In twain, the rattling ſtreams deſcend,
Waves upon waves now ſeem to ride
And iſlands float along the tide;
While dreadful as a cataract roars,
The ſurges 'gainſt the neighb'ring ſhores---
But ſtraight there ruſhes from behind,
Some poet damn'd, to me conſign'd,
Who gently on the ſurface glides,
And then the raging ſtorm ſubſides.
  Now ladies, after this diſgrace,
Dare you to look me in the face?---
No:---and tho' daily I befriend ye,
'Tis ten to one but I offend ye.
  Not fam'd at all for much diſcerning,
I cannot boaſt of taſte or learning;

Yet

Yet of what's form'd by nature's hand,
The fundamentals underftand ;
My aid fubfervient to her laws,
Is fought when fhe'd her paths difclofe ;
Behold an Æfculapian big,
With cane and large important wig,
And pair of fupplemental eyes,
(The certain marks of being wife)
Explore my bowels for the ftate,
Of health and fearch for hidden fate--
In vain,-- no fecrets with me reft,
Tho' daily lodg'd within my breaft.
Know I'm compared to a punk,
But never was detected drunk ;
Yet in North Britain, as 'tis faid,
I puke upon each ftranger's head,
A moft uncivil falutation,
Tho' not peculiar to that nation,
For the Athenian Sage of old,
The fame experienc'd from a fcold.
Now fhould you ever me affail,
I'll make your worfhip turn your tail,
And tho' you'd ftop me you will find,
That fearlefs I am clofe behind.

Anfwers to the Mathematical Queftions *propofed in laft Year's* Diary.

I. Question, *anfwered by Mr.* Robert Moody.

IT is evident that if B advances his goods $13\frac{1}{2}$ per cent. and allows $7\frac{1}{2}$ per cent. advance on A's fugar for paying $\frac{1}{4}$ of the amount in ready money, that the whole of A's advance muft be 21 per cent. then $121 \times 6,25 \div 100 =$ the price of 1 lb. and 24480 pence, the price of 36 pieces of B's goods, divided by the price of 1 lb. is $3237\frac{3}{121}$ the number of pounds of fugar ; and 2*l*. 16*s*. 8*d*. $\times 12 =$ 34*l*. the ready cafh which A gives B for his fugar.

II. Question *anfwered by Lieutenant* Wheldale.

Analyfis. Let AB the bafe, ACB a fegment of a circle containing the given vertical angle, and ACB the required triangle, draw FZ ⊥ to FK and the perpendicular CZ upon it, then by a known property $AK + KB : AC + CB :: \sqrt{KF} : \sqrt{CZ}$, therefore $AC + CB = 2AK\sqrt{CZ} \div \sqrt{KF}$, wherefore S or $AC + CB + CD = 2AK\sqrt{CZ} \div \sqrt{KF} + CD = S$, let $4AK^2 \div KF = R$, then $S - CD = \sqrt{R \times CZ} = \sqrt{R \times CD + R \times DZ}$, confequently $S^2 - R \times FE = \overline{R + 2S} \times DC - DC^2$, whence this conftruction. Take $EQ = R + 2S$ and cut it in n fo that $Qn \times nE = S^2$

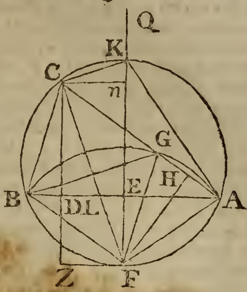
$= S^2 - R \times FE$ and draw $n\,C \parallel$ to AB, cutting the circle in C, the vertex of the required triangle.

Note. This is prob. 5 of Newton's Univerſal Arithmetick.

The ſame anſwered by Mr. Jeremiah Ainſworth.

CONSTRUCTION.

Having drawn the circle, &c. as before, take $EQ =$ the ſum of the ſides and perpendicular, draw alſo AK and to twice AK let a line be added ſo that the rectangle of the part added, and the whole be $= FQ \times FK$, then apply the chord FC equal to the additional part, and join A, C, and B, which will be the triangle required.



For from F with the diſtance FA or FB deſcribe a circle, let fall the perpendiculars CD and FH, and join the lines as in the figure, then $CF \times FL = FE \times FK$ and $CD \times KF = CL \times CF$ by the known properties of the circle, but $CL \times CF = CF^2 - CFL = CF^2 - KFE$, alſo from the ſimilar triangles CFH and KFA, $CH \times KF = CF \times KA$, whence it follows that $2\,CH \times KF + CD \times KF$, or $\overline{2\,CH + CD} \times KF = 2\,KA \times CF + CF^2 - KFE$; and conſequently $\overline{2\,CH + CD + FE} \times KF = \overline{2\,KA + CF} \times CF$, which is, by conſtruction, equal to $FQ \times FK$, wherefore $2\,CH + CD + FE = FQ$ and $2\,CH + CD = EQ$; but $2\,CH = CB + CA$ by *prop.* 9. *article* XII. wherefore $2\,CH + CD = EQ = CB + CA + CD$ - - - Q. E. D.

Limitation. EQ muſt not be greater than $2\,AK + KE$.

III. QUESTION.

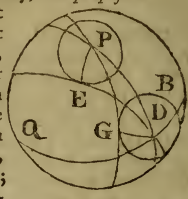
A ſmall omiſſion was made in copying this queſtion for the preſs; however, as that which the propoſer intended, may be eaſily reſolved by Prob. III. Art. 9, in laſt year's Diary, as well as moſt other problems of the ſame kind, wherein the limits of the ſum or difference of the ſides are concerned, thoſe queſtions ſeem to require no other notice than a reference to the aforeſaid article.

IV. QUESTION, *anſwered by Mr.* J. Ainſworth.

By prop. 22, Simpſon's Trig. as cot. of half the obliquity of the ecliptic is to its tangent, ſo is the ſine of the ſum of the ſun's longitude and right aſcenſion, to the ſine of their difference; hence when the difference is a maximum the ſine of the ſum will be ſo too, and conſequently equal to radius, and the ſum itſelf $=90$ degrees, whence the difference will be $2^\circ.\ 28'$. and the longitude $= 46^\circ.\ 14'$. which anſwers to May 7th; and it is evident that the common increaſe of longitude and right aſcenſion during the interval, muſt be 180 degrees, whence the time will appear to be November 8, and the interval 185 days,

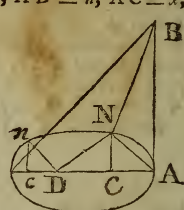
days, confequently the principal will be 118*l.* 7*s.* 6*d* $\frac{3}{4}$. *Solution were alfo given by Meff. Afpland, Barker, Boucher, Fininley, Hardy, Lynn, and Moody.*

V. QUESTION, *anfwered by Mr.* Ifaac Dalby, *the propofer.*



Let P be the given point, and QB the great circle. Through P draw a great circle P D at right angles to QB, then about the points P, D, as poles, defcribe two leffer circles, fo that their diftances are each equal to the given leg, through P, D draw great circles P G, D E, to touch the leffer circles refpectively, then having drawn the great circles P E, D G, the triangles D E P, P G D, will anfwer the conditions of the problem; that is, the fide D E is a *min.* and its complement to a femicirc. a *max.* when the given fide P E is drawn from the given point P; but if the given fide D G (P E) falls into the given great circle QB, then P G is a *min.* and its complement to a femicirc. a *max.*---For P D being the fhorteft portion of a great circle that can be drawn from P to meet QB, and the hypothenufe common to both the triangles D E P, P G D, therefore D E, P G are each a *min.* and their complements to femicircles, forming two other right-angled triangles, muft be each a *max.*'

VI. QUESTION, *anfwered by Mr.* Vidgen, *of the* Tower, London.



Let A B $= b$, length of the ftring B A D $= m$, A D $= n$, A C $= x$, then D C $= n \backsim x$ and let C N $= y$, B N^2 $=$ A B^2 $+$ A C^2 $+$ C N^2 $=$ A B^2 $+$ A C^2 $+$ N D^2 $-$ D C^2 $= \overline{m - \text{ND}}|^2 = m^2 - 2m \times$ N D $+$ N D^2, whence $m^2 - 2m \times$ N D $=$ A B^2 $+$ A C^2 $-$ C D^2, that is, $m^2 - 2m \sqrt{n^2 - 2nx + x^2 + y^2} = b^2 - n^2 + 2nx$. But from this equation to determine the nature of the curve, let $zn = m$ then will $b + n = zn$ and $b = \overline{z - 1} \times n$, and we fhall have $z^2 n^2 - \overline{z^2 + 2z - 1} \times n^2 + n^2 - 2nx = 2zn \times \sqrt{n^2 - 2nx + x^2 + y^2}$, $\overline{zn - x}|^2 = z^2 \times \overline{n^2 - 2nx + x^2 + y^2}$, $\frac{x^2}{z^2} - \frac{2nx}{z} = x^2 - 2nx + y^2$ and $\frac{z^2 - 1}{z^2} \times x^2 - \frac{2z - 2}{z} \times nx + y^2 = 0$. Let the tranfverfe diameter E A $= a$ then B E $= \sqrt{a^2 + \overline{nz - n}|^2}$, and B E $+$ E D $= \sqrt{a^2 + \overline{nz - n}|^2} + a - n = zn$,

whence by reduction $n = \frac{\overline{z + 1} \times a}{2z}$, which being fubftituted we have

$y^2 = \frac{z^2 - 1}{z^2} \times \overline{ax - x^2}$ the equation for an ellipfis.

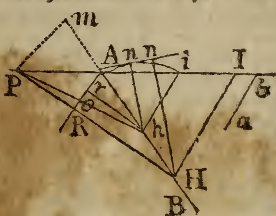
A very neat and general folution was alfo given by Mr. Brown, the propofer.

The

The same answered by Mr. Jeremiah Ainsworth *of* Manchester.

It is evident that if a plane be supposed to pass through the given points B and D, a conic section will be described thereon by a point keeping the cord tight, whether the sum of the parts of the cord be given (as in the question) or their difference, and it will be an ellipse in the first case, an hyperbola in the second, and a parabola when one of the points is supposed to be removed to an infinite distance; now if this plane, with the figure thereon, be revolved about the line B D, a solid will be generated by the curve, the intersection of which by any plane whatever, it is known will be a conic section; and, therefore, whatever angle the planes in the question are supposed to make, the curve will be an ellipse, except when one of them is perpendicular to the line B D, in which case it will be a circle. *In a manner very little different the solution was given by Mr. Isaac Dalby and Mr. John Burrow.*

VII. Question, *answered by Mr.* Isaac Dalby.

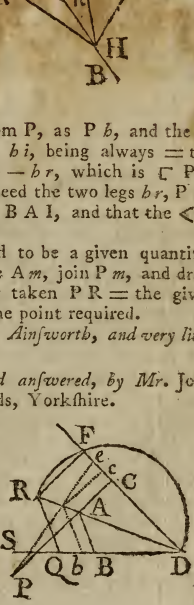


Construction. From any point in A H as b, draw a line $bi \parallel ba$ (the line given in position) with which as radius describe an arc, in, from A draw a tang. thereto, and make the $\angle$ R A B $=$ $\angle$ B A n, from P draw P H $\perp$ A R, and H I $\parallel$ ba, and the thing is done. For drawing H n, bn $\perp$ A n, we have by sim. $\triangle$, s, H n $=$ H R $=$ H I, therefore P H $-$ H I $=$ P R, which is a minimum, because if any other line be drawn from P, as P b, and the $\perp$ br let fall upon A R, then the lines H I, bi, being always $=$ the perp. H R, br; therefore P b $-$ bi $=$ P b $-$ br, which is $\sqsupset$ P R by what the two hypothenuses bo, P o exceed the two legs br, P R. Here it is necessary that the $\angle$ A ba $\sqsupset$ $\angle$ B A I, and that the $\angle$ P A R, B A I be less than right ones.

If instead of a *min.* the diff. was required to be a given quantity, produce B A till bi : b A : : the given diff. : A m, join P m, and draw A R parallel thereto, then from P having taken P R $=$ the given diff. and produced it to meet A B, it gives the point required.

A solution equally elegant was given by Mr. Ainsworth, and very little different from the following one.

The same problem rendered more general, and answered, by Mr. John Burrow, of Rounday, *near* Leeds, Yorkshire.

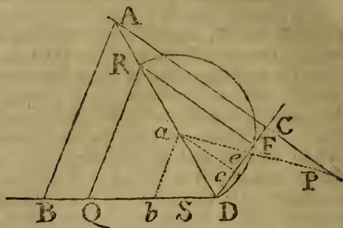


Let D R, D S be two lines given in position, P a given point, R Q a line given in position; it is required to draw P A to cut D R in A, so that drawing A B parallel to R Q cutting D S in O, the sum or difference of P A and A B may be the least possible.

On any line D R describe a semicircle, in which let R F be inscribed equal to R Q; join D F, and draw P C $\perp$ to D F

cutting

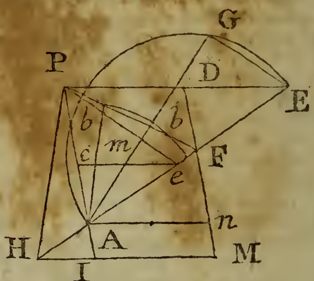
cutting D R in A, then if A B be drawn parallel to R Q, P A and A B are the lines required. For draw any other line P a and $a\,b$ || to A B, alfo draw $a\,c$ || to A C; then becaufe $\left\{\begin{matrix} Pa + ac \\ Pa - ac \end{matrix}\right\}$ is $\left\{\begin{matrix} \sqsubset \\ \sqsupset \end{matrix}\right\}$ than $\left\{\begin{matrix} PA + AC \\ PA - AC \end{matrix}\right\}$ and



P $a \pm ac =$ P $a \pm a\,b$, therefore P $a \pm a\,b$ is $\left\{\begin{matrix} \sqsubset \\ \sqsupset \end{matrix}\right\}$ than P A $\pm$ A B, or P A $\pm$ A C; for A C $=$ A B becaufe R Q $=$ R F, confequently P A $\pm$ A B is the greateft or leaft poffible. Q. E. D.

The fame anfwered by Mr. Thomas Mofs, *the propofer.*

Let P E be the line given in pofition, in which conceive P D to be a given difference inftead of a minimum, and draw D F || to A P meeting B A in F and draw F P, then from A with the diftance P D defcribe a circle cutting F P in the point (or points) b, draw A b and P H || thereto meeting A B in H, then H M drawn || to E P is the line required.



For produce P A to meet H M in I, and draw A n || to to P E, then A F : F H : : A n : H M : : A b : H P and becaufe A $n =$ A b, H M $=$ H P and ∵ H P $-$ H I $=$ I M $=$ P D.

Scholium. Hence it appears that the problem is impoffible when the difference of the fides is fuch that a circle defcribed therewith from the center A will neither cut nor touch P F, and that when it touches P F the difference will be the leaft, for it may be eafily proved that D F M will be the neareft line that can be drawn || to A P meeting A E, when a circle defcribed from A (as above) does not cut but touch the line drawn from P to the interfection in A E.

The problem then becomes this---From P to draw a line P e meeting A E in e, fo that $e\,c$ being drawn || to E P and A m perpend. to P e, A m may be $= e\,c$, and the conftruction is as follows:

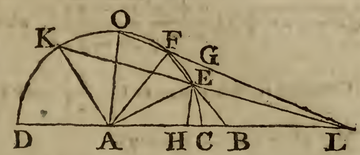
Upon A E defcribe a circle, in which apply A G $=$ E P; draw E G, and || thereto draw P e, and the thing is done.

Demonftration. Becaufe the triangles A $c\,e$, A P E, A $m\,e$ and A G E are fimilar, therefore A e : A E : : $e\,c$: E P : : A m : A G, but E P $=$ A G, confequently $e\,c =$ A m.

VIII. Question

VIII. QUESTION, *anſwered by the Rev. Mr.* Lawſon, *the propoſer.*

We muſt firſt take notice that the firſt member of this queſtion was wrong printed. Inſtead of the *ratio of the angle* A O L *to* A K L, it ſhould have been *the ratio of the arc* F C *to* E C.



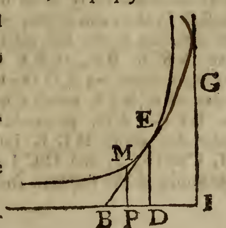
1. Now the ratio of the arc F C to E C is thus ſhewn to be greater than the ratio of the angle F L C to E L C. From the center A draw A F, A E. draw the chord F E, which produced may meet the diameter in B. and will L center and radius L E deſcribe the arc H G. Sector A F E : △ A E E B is greater than △ A F E : △ A E B. But Sect. A F E : Sector A E C is greater ſtill. ∴ Sect. A F E : A E D, i. e. arc F E : E C is greater than △ A F E : △ A E B, i. e. than line F E : E B. and by inverſion arc E C : arc F E is leſs than line E B : line F E. Juſt in the ſame manner we may ſhew that Sect. H L E : Sect. E L G, i. e. arc H E : arc E G is greater than △ B L E : △ E L F, i. e. than line B E : line E F. ∴ B E : E F is leſs than arc H E : arc E G. Since then arc E C : arc F E is leſs than line E B : line F E, and line E B : line F E is leſs than arc H E : arc E G, ∴ arc E C : arc F F is leſs than arc H E : arc E G, i. e. than angle A L E : angle E L F. ∴ by perm. and comp. arc F C : to arc E C is greater than angle F L C : angle E L C. Q. E. D.

2. Let A O, A K be joined. Then by the 1ſt part and permutation, angle F A L : ang. F L A is greater than ang. E A L : ang. E L A, and by comp. F A L + F L A or A F O or A O F : F L A is greater than E A L + E L A or A E K or A K E : E L A, that is, A O L : O L A is greater than A K L : K L A, or by perm. A O L : A K L is greater than O L A : K L A. Q. E. D.

3. Since by part 2d and permutation A O L : O L A is greater than A K L : K L A, by comp. A O L + O L A or D A O : O L A is greater than A K L + K L A or D A K : K L A, and by perm. D A O : D A K : : arc O D : K D is greater than ang. O L A : ang. K L A.

This queſtion was alſo anſwered by Mr. Iſaac Dalby and Mr. John Burrow. N. B. The method of reſolving the 9th queſtion is ſelf-evident from the 7th prop. of art. ix. in laſt year's Diary.

X. QUESTION, *anſwered by Mr.* Todd, *the propoſer.*



If $a = 400$, $n = PM$, $PD = x$, and $DE = y$, then $\dot{y} : \dot{x} : : y : \frac{y\dot{x}}{\dot{y}} = DB$; alſo the tangent $BE = \frac{y}{\dot{y}} \sqrt{\dot{x}^2 + \dot{y}^2}$ and $y + \frac{y}{\dot{y}} \sqrt{\dot{x}^2 + \dot{y}^2} = a$ by the queſtion, whence $\dot{x} = \frac{\dot{y}\sqrt{a^2 - 2ay}}{y}$, and thence the equa-

 tion

tion of the fluents is $x = 2\sqrt{a^2 - 2ay} - a\,|\,\frac{a + \sqrt{a^2 - 2ay}}{a - \sqrt{a^2 - 2ay}} -$ $2\sqrt{a^2 - 2an} + a\,|\,\frac{a + \sqrt{a^2 - 2an}}{a - \sqrt{a^2 - 2an}}$, where $x = 0$ when $y = n$.

Corollary 1. When $y = GI = \frac{a}{2}$, $x = PI = a\,|\,\frac{a + \sqrt{a^2 - 2an}}{a - \sqrt{a^2 - 2an}}$ $- 2\sqrt{a^2 - 2an} = 39,44492$.

To find the curve $ME = z$; because $\dot{z} = \sqrt{\dot{x}^2 + \dot{y}^2} = \frac{a\dot{y}}{y} - \dot{y}$, we have $z = a\,|\,y - y - a\,|\,n + n = a\,|\,\frac{y}{n} - y + n = ME$.

Corollary 2. When $y = GI$, $z = MEG = a\,|\,\frac{a}{2n} - \frac{1}{2}a + n =$ $65,0728$.

Lastly, to find the area of $PMED = A$. Because $\dot{A} = y\dot{x} = \dot{y}\sqrt{a^2 - 2ay}$, $A = -\frac{\overline{a^2 - 2ay}|^{\frac{3}{2}}}{3a} + \frac{\overline{a^2 - 2an}|^{\frac{3}{2}}}{3a}$ where $A = 0$ when $y = n$.

Corollary 3. When $y = GI = \frac{1}{2}$, then $A = \frac{\overline{a^2 - 2an}|^{\frac{3}{2}}}{3a} = 6666\frac{2}{3}$ the area of PMGI.

Scholium. When $y = 0$, x will be infinite, or an asymptote to the curve, and the greatest ordinate GI is equal to the tangent at the point G.

This question was also answered by Mr. J. Aspland.

XI. QUESTION, *answered by the Rev. Mr.* Crakelt, *the proposer.*

Construction. Upon any assumed line, A B, as diameter, describe a circle; and, having formed the angle B A C equal to half the given difference of the angles above the base, joined the points B, C, and drawn C D perpendicularly to A B, make 2 B C to B E in the ratio of the given difference of the sides to the line bisecting the base; and A B to B E, as B E to B F; then having determined A G the less of two reciprocals to A C^2, whose sum may be equal to B F + 2 A D, perpendicularly to A B draw the chord H G I, join the points H, C and C, I, and H C I will be a triangle similar to the required one.

Demonstra-

Demonſtration. Draw C G, A H, and A K perpendicularly to H C. Then, ſince by *con.* $AG \times BF + AG \times 2AD - AG^2 = AC^2 = CG^2 + AG^2 + AG \times 2DG$ (*Euc.* ii. 12.) $= CG^2 + AG^2 + AG \times \overline{2AD - 2AG} = CG^2 + AG \times 2AD - AG^2$, therefore will $CG^2 = AG \times BF$. But, by ſimilar triangles $HK^2 : BC^2 :: AH^2 = AB \times AG$ (*Euc.* vi. 8. cor.) $: AB^2 :: AG : AB$ (*Euc.* vi. 1.) $:: AG \times BF = CG^2 : AB \times BF = BE^2$ by conſtruction; conſequently, by permutation, $HK^2 : CG^2 :: BC^2 : BE^2$, or $HK : CG :: BC : BE$. Now it is well known that H K is equal to half the difference betwixt H G and L I, wherefore by doubling the antecedents of the laſt proportion, we ſhall have, $2HK$ or $HC - CI : CG :: 2BC : BE$. And that the difference betwixt the angles C I H and C H I is equal to 2 B A C is manifeſt; becauſe the difference betwixt the arches H C and I C is equal to twice the arch B C.

Scholium. If with the other data, the ſum inſtead of the difference of the ſides had been given, make 2 A C to B E in the ratio of the ſum of the ſides to the biſecting line, and CG^2 equal to $BG \times BF$, that is, B G the leſs of two reciprocals to BC^2 whoſe ſum is $BF + 2BD$, and in the demonſtration uſe C K, A C, and B H inſtead of H K, B C, and A H, and every thing elſe will follow.

Very elegant ſolutions were alſo given by Mr. George Sanderſon, and Mr. Iſaac Dalby; Mr. Ainſworth alſo gave excellent ſolutions both to the queſtion itſelf, and that mentioned in the above ſcholium, with ſeveral others, ſome of which will be inſerted in future.

XII. Question, *anſwered by* Archimedes.

Suppoſe A, B, C, and D to be the four players, A being the dealer, then by prop. 6. corollary 2, of Simpſon's Chances, the probability that any one of the players B, C, D, has of holding not more than four trumps will be expreſſed by $\frac{39 \cdot 38 \cdot 37}{51 \cdot 50 \cdot 49}(12) \times \Big[\frac{27}{39} + \frac{1}{39} \times 13\ 12 + \frac{1}{28 \cdot 39} \times 12 \cdot 13\ \frac{12}{1}\ \frac{11}{2} + \frac{1}{29 \cdot 28 \cdot 39} \times 11 \cdot 12 \cdot 13\ \frac{12}{1}\ \frac{11}{2}\ \frac{10}{3} + \frac{1}{30 \cdot 29 \cdot 28 \cdot 39} \times 10 \cdot 11 \cdot 12 \cdot 13\ \frac{12}{1}\ \frac{11}{2}\ \frac{10}{3}\ \frac{9}{4}\Big]$ which reduced is $= \frac{432385952}{466921735}$, which taken from unity there will remain $\frac{64535783}{466921735}$ for the probability that each of the players B, C, D, has of holding 5 or more trumps, and from the ſame problem it is evident that the probability of the dealer's not holding more than four trumps will be expreſſed by $\frac{39 \cdot 38 \cdot 37}{51 \cdot 50 \cdot 49}(12) \times \Big[1 + \frac{1}{28} \times 12\ \frac{12}{1} \times \frac{1}{29 \cdot 28} \times 11 \cdot 12\ \frac{12}{1}\ \frac{11}{2} + \frac{1}{30 \cdot 29 \cdot 28} \times 10 \cdot 11 \cdot 12\ \frac{12}{1}\ \frac{11}{2}\ \frac{10}{3}\Big]$ which reduced will be $=$

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$\frac{331188221}{466921735}$, and therefore $\frac{135733514}{466921735}$ will be the probability that the dealer A holds 5 or more trumps; consequently $3 \times \frac{64535783}{466921735} + \frac{135733514}{466921735}$ or $\frac{329340863}{466921735}$ will at laſt expreſs the probability that ſome one of the four players holds 5 or more trumps; and therefore the required odds that ſome one of the players holds 5 or more trumps are as 329340863 to 137580172, being nearly as 12 to 5, or ſtill nearer as 67 to 28.

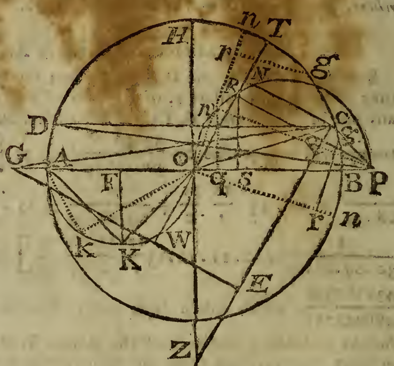
Note, the probability of ſome two of the players each holding 5 or more trumps, being inconſiderable, is neglected.

Nearly in the ſame manner this queſtion was alſo anſwered by Mr. Robert Moody, and Mr. Ainſworth, &c.

PRIZE QUESTION, *anſwered by Mr.* Iſaac Dalby.

Lemma. If upon a given hypothenuſe A O a right angled triangle A K O be conſtructed, the rectangle of the legs A K × O K will be a maximum when they are equal.----For letting fall the ⊥ F K from the cent. F, and let the ſemicirc. A K O be deſcribed, then will H K = O K, and by ſim. △*s* we have A K × O K = A O × F K which is a max. becauſe A O is conſtant, and F K the greateſt ⊥ to A O that can be drawn within the ſemicirc.

Conſtruction. Let A B be the diam. O the center, and P the given point. Upon O P, A O, let ſemicircles be deſcribed, take O K, A K equal to each other and in the ſemicircle O P and ⊥ O P make R S a fourth proportional to P O, O K, A K, through R draw O T, in which take O N = O K (A K) and draw N C ⊥ N O meeting the circumf. in C, then draw the chord C D ∥ A B and the thing is done.



Demonſtration. Join D P, P C, P R, O C, produce B A till A G = B P and draw C G, alſo let O H be ⊥ A B, and C Z be drawn ∥ N O meeting H O produced in Z, alſo draw G E ⊥ C Z, and produce N O to W.

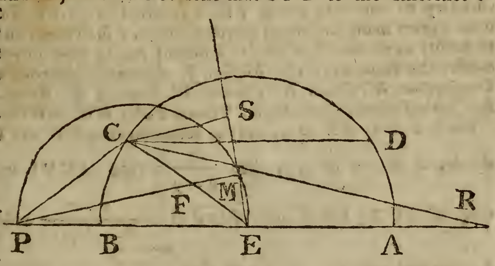
Since

Since by conftruction O N (O K, A K) is a mean proportional between O P, R S, and the $\angle$ O R P a right one, it is alfo a mean proportional between O R, R P, that is, O R : O N : : C N : R P (becaufe O A $=$ O C, and O N $=$ O K $=$ A K, C N is $=$ O N) whence by compofition and divifion O N $+$ O R : O N $-$ O R : : R P $+$ C N : R P $-$ C N ; but becaufe G E, P R, C N are $\parallel$ to each other and $\perp$ C Z, N W, we have O N $+$ O R $=$ C E (O W being $=$ O R) O N $-$ O R $=$ C Q, R P $+$ C N $=$ G E and R P $-$ C N $=$ P Q, hence the laft proportion becomes C E : C Q : : G E : P Q, therefore the $\triangle$ s G C E, P C Q are fim. and fo C Z bifects the $\angle$ G C P, hence if a circle is conceived to pafs through the points P, C, D, G, it will alfo pafs thro' Z, and the $\angle$ C Z H will be $=\frac{1}{2}$ the $\angle$ C P D, but the $\angle$ C Z H $=$ T O H $=$ R P O ; now the $\angle$ R P O is evidently a max. when R S is a max. or when P O $\times$ R S is a max. but P O $\times$ R S $=$ N O $\times$ C N (by conftruction) which (becaufe N O $=$ C N) is a max. by the foregoing lemma.

If it is required that the $\angle$ C P D fhall be of a given mag. inftead of a maximum, the conftruc. will be thus.——Draw O n, O n making the $\angle$ s, H O n, B O n, each $=$ half the propofed $\angle$, draw P w $\perp$ O n and let fall the perp. $w\,q$, then in the femicircle A O having taken O k, A k, fo that their rectang. may be $=$ O P $\times$ $w\,q$, make O r, O r each $=$ O k, and draw the perpendiculars $r\,g$, $r\,g$, then if chords be drawn from the points g, $g \parallel$ A B, either will anfwer the conditions of the prob.—The demonftration is evident from that already given.

The fame anfwered by Reuben Burrow, *the propofer.*

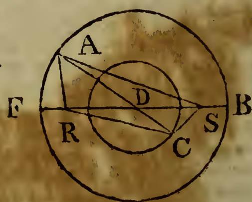
Analysis. Suppofe the thing done, and let C be the required point, P the point in the diameter produced, and C D the chord required ; alfo let E R $=$ E P, (E being the center) and join the points D, C, P, and R ; then it is evident that C P D is the difference of the angles C P R and C R P, but C P D being a maximum, this difference is a maximum alfo ; Now when the difference of C P R and C R P is conftant, it is well known that the vertex C is an hyperbola, paffing thro' P ; therefore when this difference is the greateft, it is evident the hyperbola will touch the circle in the point C ; and if E S be fuppofed one of the affymptotes paffing thro' the center E, and C S, P M perpendicular thereto, C S $\times$ S E will be equal to P M $\times$ M E ; but becaufe every other point of the hyperbola, except C, falls without the circle



circle, it is evident that C S $\times$ S E muft be a maximum; but C E being given, and C S E a right angled triangle, C S $\times$ S E will be greateft when C S = S E, wherefore the triangle C S E or its equal P M E is given; whence this *Conftruction.*

On P E defcribe a femicircle, infcribe therein the triangle P M E whofe area is half the fquare of the radius of the given circle; take M F = M E and E F being joined will cut the circle in C the point required.

A different folution may be deduced from the 59th problem of Simpfon's Algebra; but the problem will be confidered in a more general manner fome future opportunity.



Corollary. Hence if B A F be a given circle, and the points R, S in B F equidiftant from the center D, the point A may be found, where the difference of the angles R A D, D A S is the greateft; for take D C a third proportional to D F and D R, and with that diftance and the center D, defcribe a circle, then find the point C by the foregoing problem where the difference of R C S and C R S is the greateft, and C D produced to cut the other circle gives the point A required. For A D $\times$ D C = D R^2 = D R $\times$ D S ∴ A, R, S and C are points in a circle, confequently S A D = S R C and R A D = R S C, therefore R A D — D A S is the greateft poffible.

Scholium. The problem in the laft corollary has been thought worthy of the attention of feveral learned men, particularly the famous *P. Frifi*, who in the *Atta del' Siena* has beftowed feveral pages thereon; the conclufion there given is exceeding fimple, but the procefs is in effect fluxional; *Cramer* has alfo given a fluxional folution in his "*Analyfe des Lignes Courbes*," but as this problem is nothing more than a corollary to the laft, and as I have received anfwers by fluxions to it from a great many ingenious correfpondents, I fhall infert one of them, efpecially as no lefs than twelve different people have folved it almoft exactly the fame way, viz. *Meffrs. J. Afpland, Edward Boucher, D. Cunningham, W. Dixon, W. Fininley, W. Francis, W. Hardy, J. Hartley, James Pringle, John Roper, Thomas Todd, William Wilkin.*

Let B E = a, P E = B, and the perpendiculars C N and D W = x, alfo let E N = E W = y, then $\frac{x}{b-y}$ = tang. of C P E and $\frac{x}{b+y}$ = tang. of D P E; hence tang. of C P D = $\frac{2yx}{b^2-y^2+x^2}$ which put into fluxions and reduced, gives $x = \frac{a}{b}\sqrt{\frac{b^2-a^2}{2}}$ the diftance of the chord required from A B.

A geometrical folution was alfo given by Mr. Ainfworth, who is entitled to the prize of twelve Diaries, the filver medal was adjudged to

M

Mr. Ifaac Dalby, as his folution was the only geometrical one that came in the limited time.

ARTICLE XI.

A Supplement to a former Article, concerning the Equation of Payments. by Reuben Burrow.

AS there is fcarcely any fubject that has caufed more difputing and wrangling among arithmetical writers, than the equation of payments; and as the lateft writers on arithmetic have only given us the miftakes of former authors intermixt with peremptory affertions and invidious remarks of their own, I thought it might be a means of putting a ftop to fuch reflections by confidering the fubject in a more general manner than it has hitherto been. In order to this, let us fuppofe that one perfon owes another the fums of money M, N, P, and Q, &c. payable at different times, and that the creditor is willing to receive the whole fum at one fingle payment, at a time when it will be of equal advantage to him whether he receives it thus, or receives the payments in their proper order; let us, in the firft place, fuppofe the creditor to receive his debts as they become due, then it is evident that at the time of the laft payment he will have received the fum of the fingle payments, together with the intereft arifing from each, from the time of becoming due to the time of the laft payment; and it is alfo evident that if the debtor had paid the creditor the whole fum at once, at a time when being put out to intereft it might have amounted at the end to the fame fum as that arifing from the fingle fucceffive payments and their intereft; the creditor would then have received exactly the fame advantage by the one method as by the other; and confequently the fubject is reduced, both in fimple and compound intereft, *to find in what time the whole fum of the fingle payments would produce the fame amount as that which arifes from the aggregate of each payment, together with the intereft of each from its time of becoming due to the time of the laft payment.*

This principle I fhall now apply both to fimple and compound intereft; in order to which, let M, N, P, Q, R, &c. be the payments in fucceffion; t, t', t'', t''', &c. the intervals of time between the firft and laft, fecond and laft, third and laft payments, &c. and $r =$ the rate of intereft, alfo let x be the interval between the required or equated time, and that of the laft payment: then becaufe $s\,(tr + 1)$ is the general expreffion for the amount of the fum s in the time t, the fum of the amounts aforefaid will be $= M\,(tr + 1) + N\,(t'r + 1) + P\,(t''r + 1) + Q\,(t'''r + 1) +$, &c. which by the aforefaid principle muft be $= (M + N + P +$, &c.$) \times (xr + 1)$ which equation being multiplied and $M + N + P + Q +$, &c. taken from both fides, there remains $Mrt + Nrt' + Prt'' + Qrt''' +$, &c. $=$ $(M + N + P + Q +$, &c.$)\,xr$, and dividing the whole by r we have

have $x = \frac{M t + N t' + P t'' + Q t''' +, \&c.}{M + N + P + Q +, \&c.}$ which gives exactly the old rule, viz. " *Multiply each payment by its time of continuance, and divide the ſum of the products by the whole debt.*"

The ſame principle may be applied to any number of payments at compound intereſt, for $s\ r^t$ expreſſes the amount of any ſum s, in the time t, wherefore $r^t M + r^{t'} N + r^{t''} P + r^{t'''} Q +, \&c. + R =$ $(M + N + P + Q +, \&c. + R)\ r^x$, conſequently $r^x =$ $\frac{r^t M + r^{t'} N + r^{t''} P + r^{t'''} Q +, \&c. + R}{M + N + P + Q + \ldots\ldots + R} = a$; and hence we find $x = \log. a \div \log. r = \log. \frac{a}{r}$, which is nothing more than finding the amount of all the payments from the times they become due to the time of the laſt; then with this *amount*, and the ſum of all the payments as a *principal*, finding the *time* of continuance, according to the common rules of intereſt; and this method, with reſpect to compound intereſt, agrees exactly with Kerſey's rule.

But as " *Mr. Profeſſor Hutton, F. R. S.*" has thought proper to condemn Kerſey's rule as *falſe*, and to give the preference to a rule of *Mr. Malcolm's*, which he ſays is " *the only true one*," it will not be improper here to ſhew that Malcolm's and Kerſeys are in effect the ſame, and that both agree with the foregoing rule, when compound intereſt is allowed.

The principle on which Malcolm has founded his calculation, is the equality between the intereſt and diſcount at the equated time; but as there is apparently ſome difficulty in determining which debts are to bear intereſt, and which are to be diſcounted, he has been obliged to introduce the tedious and incorrect method of finding the time for two payments, and then making uſe of a third, and ſo on; however, this is a method which there is not the leaſt occaſion for, ſince whatever interval is aſſumed for the equated time to happen in, the inveſtigation will be exactly the ſame; and that aſſumption will have no other effect than to render the proceſs more methodical; thus if the time be ſuppoſed to fall in the interval between P and Q, and the letters to ſignify the ſame as before; then the intereſt of M for the time $t - x$; of N for the time $t' - x$, and of P for the time $t'' - x$, will be equal to the diſcount of Q for the time $x - t'''$, &c. and R for the time x, R being the laſt payment. Now $(r^x - 1)\ s$ expreſſes the intereſt of any ſum s for the time x; alſo $s\ r^{-x}$ is the principal which would amount to s in th time x; conſequently the diſcount is $s - s\ r^{-x}$ or $(1 - r^{-x})\ s$: but as all the diſcounts are to be *ſubſtracted* from the ſum of the intereſts, in order to make the equation vaniſh, it is the ſame thing as *adding* them with a *contrary* ſign; but $(1 - r^{-x})\ s$ when

when its ſign is changed, does not differ from the expreſſion for the intereſt, except in the ſign of its index; wherefore, *if the intereſt be found with a contrary index, it will be equivalent to the diſcount with its ſign changed.*

Now the intereſt of M for the time $t - x$ is $= (r^{t-x} - 1)$ M, that of N for the time $t' - x$ is $= (r^{t'-x} - 1)$ N, and that of P for the time $t'' - x$ is $= (r^{t''-x} - 1)$ P; alſo the diſcount of Q with its ſign changed in the time $x - t''' = (r^{t'''-x} - 1)$ Q, &c. and the diſcount of the laſt payment is $(r^{-x} - 1)$ R: theſe terms being added together, and the whole made equal to nothing, alſo the equation multiplied by r^x and divided by the ſum of the payments, gives

$$r^x = \frac{r^t \mathrm{M} + r^{t'} \mathrm{N} + r^{t''} \mathrm{P} + r^{t'''} \mathrm{Q} + \text{\&c.} + \mathrm{R}}{\mathrm{M} + \mathrm{N} + \mathrm{P} + \mathrm{Q} + \ldots\ldots \mathrm{R}};$$

which equation is exactly the ſame as the foregoing, and the ſame concluſion would have followed had the equated time been ſuppoſed in any other of the intervals.

I cannot conclude this ſubject without obſerving, that having mentioned the above to Mr. Dalby, he ſhewed me a paper wherein he had not only deduced the very ſame concluſions, but alſo confirmed the principle on which they are founded by many ſubſtantial arguments. Hence it appears, that the common method of computing the equated time at ſimple intereſt is true, and that Kerſey's rule is true alſo in compound intereſt; As to *Profeſſor Hutton*'s aſſertions to the contrary, they have juſt as much validity as *Dr. Horſley*'s confirmation of *Stewart*'s theory of the *Sun*'s diſtance; and the ſame anſwer which *Mr. Landen* gave the *Doctor* is equally applicable to the *Profeſſor.*

Some Miscellaneous Problems, *with their* Solutions. *By* Reuben Burrow.

ARTICLE XII.

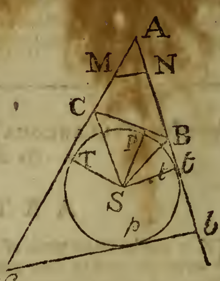
IN a poſthumous work of Dr. Simſon's (printed at Lord Stanhope's expence, and by that nobleman preſented to men of ſcience) which I have lately ſeen, there is an appendix inſerted by the editor, containing geometrical ſolutions to ſeveral problems, ſome of which are taken from Newton's Univerſal Arithmetic, and others elſewhere; but as the ſolutions there given are very long, and I had anſwers to the ſame problems by me, I flatter myſelf that to inſert them here will not be unacceptable, both on account of their ſimplicity and the impoſſibility of procuring the book aforeſaid; I was farther induced, by ſome remarks at the end of a book compiled by the *Rev. Dr Horſley, Secr. R. S.* entitled, *Apollonii Pergæi inclinationem, &c.* wherein that gentleman has been pleaſed to beſtow his cenſures very liberally on the *im-*

mane equations, and the *odious ambages* and modes of folution, which he fays the modern *plebians* have *fweat* about; and after having condefcended himfelf to give a folution of Newton's 7th problem as a fpecimen, and to refer to two propofitions of Euclid, by which he fays the reft might be effected, modeftly concludes that thofe geometers aforefaid, know nothing of Euclid's Data.

Whether we ought to include *Caftillioneus* among thofe geometers that are ignorant of the data, the doctor has not informed us; however this is certain, that he had actually folved Newton's Problems by thofe very propofitions referred to, ten years before the doctor pointed out *the fame method*; and fince the doctor in his propofals for printing a new edition of Newton's works, has, in a very particular manner, informed us of his intention to give geometrical folutions to all thofe problems, I had an additional motive in the clumfinefs of his method, to infert what follows; to which if fome (not *immane*) folutions be added, which are given in the London Magazine for 1775, by *Mr. George Sanderfon*, taylor, in *Doctors-Commons*, particularly a geometrical one to the 7th problem aforefaid, which this induftrious compiler did not folve without algebra, there will not remain in the *Arithmetica Univerfalis* a fingle queftion, relating to triangles, of any difficulty; this I point out in order to fave the Rev. Doctor fome trouble in his new edition; and though it has been his method hitherto, in all his *Notes, Remarks, and Compilations*, to be very fparing of the names of thofe authors whofe works he has made free with, yet I hope, at the fame time, that he will not forget to do Mr. Sanderfon the juftice to which his merit fo defervedly entitles him.

Proposition I. Theorem.

If A B, A C be two lines drawn from a given point to touch a circle in T and *t*, and C B be any line touching the circle and intercepted between A C and A B, then will A C + C B + B A be conftant when C B is on that part of the circle next A; and A *c* + A *b* — *b c* will be conftant when *c b* is drawn on the contrary part For C P = C T, B P = B *t*, *c p* = *c* T. *b p* = *b t*, and therefore A T = A C + C P, and A *t* = A T = A B + B P = A *c* — *c p* = A *c* — *c* T = A *b* — *b p*, confequently 2 A T = AC + C B + B A = *c* A + A *b* — *b c*.



Corollary 1. If M, N be two given points, and M N be on the fame fide of C B the perimeter of all the trapeziums M C B N will be invariable, or the difference between the three fides and a fourth, &c.

Corollary 2. Hence the fum of the two oppofite fides of any quadrilateral figure, circumfcribing a circle is equal to the fum of the other two fides. Moreover, if there be any number of circles whatever, touching A C in the point T the perimeters of all triangles, &c. defcribed in the fame manner on each circle as that above, will all be equal.

Corollary.

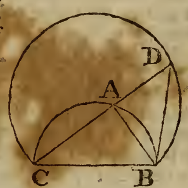
Corollary 3. Hence if the perimeter, two angles and the included ſide of a trapezium be given, together with one of the oppoſite or adjacent ſides or angles, or the area, &c. the figure may be conſtructed by this and the following propoſitions.

Corollary 4. Hence Newton's 4th problem, which is the firſt in Dr. Simſon's appendix, may be generally ſolved by making C A B = the given vertical angle, A T = A t = half the given perimeter, and drawing the circle to touch AT and A t in T and t; then having deſcribed a circle from the center A, with a diſtance equal to the given perpendicular, draw a line C B to touch both circles, cutting the lines containing the given angle in C and B, then C A B will be the triangle, and the truth of the propoſition is ſelf-evident.

Proposition II. Problem.

If T P t be a circle given in magnitude and poſition and A T, A t tangents drawn to it from a given point; it is required to draw a line C B to touch the circle ſo that the part C B intercepted between A T and A t may be of a given length. See *fig.* 1.

Analyſis. Becauſe A C + C B + B A is given = 2 A T and C B alſo given, A C + A B is conſequently given, and the angle A; hence this conſtruction. Deſcribe on the given line C B a ſegment of a circle containing an angle equal to that made by the lines A T, A t, and another on the ſame line containing half that angle, in which let C D be inſcribed equal to the given ſum of A C and A B, cutting C A B in A; then if C A, A B be taken in the firſt figure equal to the ſame lines in that annexed, the poſition of the tangent will be determined.



Corollary 1. Hence the third and eighth problems of Newton's Univ. Arith. may be generally ſolved; for the vertical angle perimeter and area being given, T A, A t (ſee fig. 1.) and the angle A, are given, alſo S T A t S is given, and becauſe C A B is given, by ſuppoſition, T C B t S is alſo given; but this laſt quantity is equal to C B × S P, and S P being known C B is alſo given, and conſequently the triangle may be conſtructed by this problem.

Corollary 2. The above problem alſo includes the ſolution of Newton's tenth.

Proposition III. Problem.

The ſame things being given as in the laſt, it is required to draw the tangent C B ſo that its parts C P, B P may obtain a given ratio.

Analyſis. Becauſe A T S, A t S (ſee fig. 1.) are right angles, therefore A and T S t taken together are equal to two right angles; alſo T S C = C S P and P S B = B S t therefore C S B = half T S t = half the ſupplement of the angle A, whence the following conſtruction.

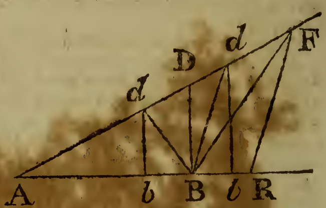
Take any given line C B and divide it according to the given ratio in P, and draw P S perpen. to C B, then on C B deſcribe a ſegment of a circle containing an angle equal to half the ſupplement of A, interſecting P S in S, and make S B t = S B C and S C T = S C P, then

 t B

t B, T C when produced to meet in A, determine a triangle A C R fimilar to that required.

Scholium. By the foregoing problems a great number of queftions relating to the perimeters of triangles and trapeziums may be readily refolved, and it is worth remarking, that whatever queftions are folveable thereby with refpect to perimeters, when the tangent is drawn next the vertical point; fimilar ones may be found the fame way when the tangent is drawn on the part fartheft diftant; and the difference between the fum of the fides and bafe will then be concerned in like manner as the perimeter was in thofe foregoing.

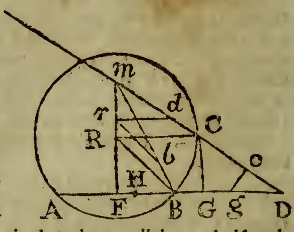
Proposition IV. Theorem.



If A B, A D be two lines given in pofition meeting at A, and B D be drawn ⊥ to A B cutting A D in D, then will the ratio of A D to D B be the greateft poffible, and of all lines A d and d B the ratio of thofe which interfect neareft D is greater than that of thofe interfecting farther off.

For draw d b parallel to D B and join d, B; then A D : D B : : A d : d b, but d B is greater than d b, therefore the ratio of A d to d b or A D to D B is greater than that of A d to d B. Again draw B F farther diftant from D than d, join B F and draw F R ∥ to B d; then A d : d B : : A F : F R; but F B is greater than F R, confequently the ratio of A d to d B is greater than that of A F to F B.

Proposition V. Problem.



A and B are two given points, and D C a line given in pofition; it is required to find a point G in A B, fo that G C being drawn to cut D C in a given angle, the rectangle of A G and G B may be equal to the fquare of G C.

Cafe 1. Bifect A B by the normal F m cutting D C, and join B m, take any point r in F m and draw r d meeting D m, fo that it may be ⊥ to a line to which G C is required to be parallel; and alfo take $r b = r d$, then draw B R parallel to b r and from the center R with the diftance R B defcribe a circle cutting D C in C; draw C G making D C G = the given angle and G is the point required.

For $r d = r b$ ∴ R C = R B and C G is ⊥ to R C, becaufe it is ⊥ to r d by conftruction, confequently C G is a tangent, and therefore the rectangle A G B = G C².

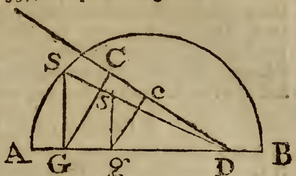
Cafe 2. Defcribe a circle on A B, and from any point g draw g c making the given angle with D C and g s ⊥ to A B and = g c, continue

D s

D s to cut the circle in S and the normal S G will divide A B in G the point required. For $gc = gs$ ∵ G C = G S, whence A G B = $GS^2 = GC^2$.

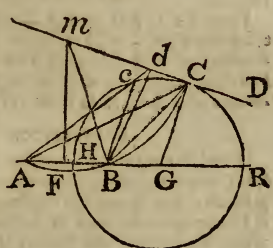
Corollary 1. If G C be required to be ⊥ to D C the center of the circle will fall in m.

Corollary 2. If from the center G with the diſtance G C a circle be deſcribed, cutting A B in H, then will all lines drawn from A and B to its circumference have the ſame ratio which A H has to H B, as is evident from the Lemma, page 337, Simpſon's Algebra.



Corollary 3. If A D be a given line and B a point given, another point G may be found where A G × G B may have a given ratio to GD^2, by taking any line gD, deſcribing a ſemicircle thereon, and in it taking gc^2 to gD^2 in the given ratio, then drawing F m ⊥ to and biſecting A B, and cutting D C in m, then taking m C = m B and drawing C G parallel to cg, and G will be found; for $GC^2 : GD^2 :: gc^2 : gD^2$ and m being the center C G is a tangent to the circle, and conſequently its ſquare = A G × G B; wherefore $AGB : GD^2 :: gc^2 : gD^2$, viz. in the given ratio. Alſo in the ſecond figure $AGB = GS^2 = GC^2$ and GC^2 is to GD^2 in the given ratio, therefore A G × G B is to GD^2 in the ſame ratio; and in a ſimilar manner may be the reſt of Apollonius's problems on Determinate Section be reſolved, as will be evident to any perſon that takes the trouble of obſerving the method which Mr. *Wales* took in collecting his book thereon from the ſolutions that had been given before by Mr. *Simpſon* and *Snellius*.

Proposition VI. Problem.



C D is a line given in poſition and A, B two given points: it is required to find a point C in the line C D, where the ratio of A C to C B may be the greateſt poſſible.

Biſect A B by the perpendicular F m, meeting D C in m, and take m C = m B, then C will be the point required.

For draw C G ⊥ to D C meeting A B in G, and with the center G and diſtance G C deſcribe a circle, which of courſe touches D C in C, alſo join A C, C B and drew any other lines A d, d B cutting the circle in c, and D E in d; then becauſe m is the circle's center and C G ⊥ to m C, C G is a tangent, and $AG \times GB = GC^2 = GH^2$, therefore A G : G H :: G H : G B, and conſequently H B : B R :: H A : A R; wherefore A H : H B :: A C : C B :: A c : c B; but the ratio of A c to c B is greater than that of A d to d B by prop. 4, and therefore

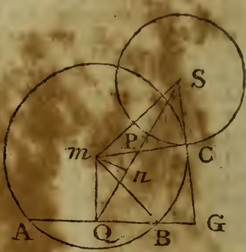
therefore the ratio of AC to CB is alſo greater than that of Ad to dB, conſequently is the greateſt poſſible.

Corollary 1. The other interſection of the circle gives another point, but the method is the ſame for all caſes.

Corollary 2. Hence, if there be an indefinite number of right lines parallel to CD, the locus of all the points C will be an hyperbola; for Fm is given in poſition, and the diſtances mC are ſet off in a direction making a conſtant angle with Fm and equal to mB.

Scholium. The above problem is Dr. Simſon's 5th, the ſolution there given takes up ſeven quarto pages: as to the 4th it has been already done the ſame way by Mr. Simpſon; the 2d. and 3d. are the ſame as that propoſed in laſt year's Diary by Mr. Sanderſon, different ſolutions of which may be ſeen in the anſwers for this year; and the firſt is ſolved in the 4th corollary of the firſt propoſition.

Proposition VII. Lemma.

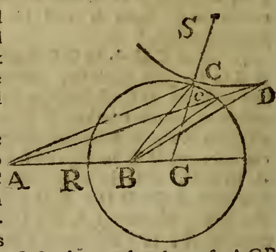


A and B are two given points, and SC a given circle: it is required to find the point G in AB, ſo that GS being drawn to the center and meeting the circumference in C, the ſquare of CG may be equal to the rectangle AG and BG.

Draw $Qm \perp$ to and biſecting AB join SQ which biſect in P, and take Pn ſo that $2Pn \times SQ = QB^2 + SC^2$, then nm drawn $\perp$ to QS gives m the center of a circle, which being deſcribed with the diſtance mB cuts the circle SC in C, then CS being drawn, cuts AB in G the point required.

For $2Pn \times SQ = Sn^2 - nQ^2 = Sm^2 - mQ^2 = QB^2 + SC^2$, therefore $Sm^2 - SC^2 = QB^2 + Qm^2 = mB^2$; but mB is by conſtruction $= mC$, therefore $Sm^2 - SC^2 = mC^2$, conſequently mCS is a right angle, and CG a tangent to the circle ABC, whence it follows that $AG \times GB = GC^2$.

Proposition VIII. Problem.

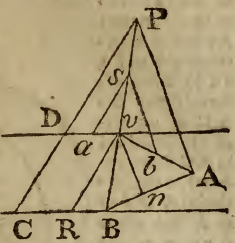


A and B are two given points, and SDC a circle given in magnitude and poſition: it is required to find a point C in the circumference of the circle where the ratio of the lines AC and CB may be the greateſt poſſible.

Through the center S draw the line SCG by the laſt propoſition ſo that $AG \times GB = GC^2$, then C will be the point required. For with the center G and diſtance GC deſcribe a circle, and draw any other lines AD, DB, the firſt cutting the circle GCc in c; then becauſe AGB $= G$

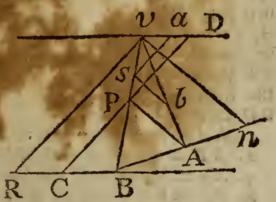
$= GC^2 = GR^2$, A C : C B : : A c : c B; but the ratio of A c to c B is greater than that of A D to D B, confequently the ratio of A C to C B is greater than that of A D to D B, and therefore the ratio of A C to C B is the greateft poffible.

PROPOSITION VIII. PROBLEM.



B v and B C are two lines given in pofition and A a given point: it is required to find the point P in the line B v fo that A P being joined, and P C drawn parallel to a line given in pofition, the ratio, fum, or difference of A P and P C may be given.

1. Draw R v parallel to the line given in pofition, and at fuch a diftance that v R may be equal to the given fum or difference, join A v and A B, and take any point a from which draw $a\,s$ parallel to R v, and take $s\,b = s\,a$ cutting A v in b, and draw A P parallel to $b\,s$, meeting B v in P, then will the fum or difference of A P and P C be equal to v R. For $v\,s$: v P : : $a\,s$: P D : : $s\,v$: A P, therefore A P = P D and C P + P A = C D = R v.



2. For the ratio; take $v\,n$ to v R in the given ratio which AP is required to have to P C, and parallel to $v\,n$ draw A P, and the thing is done; for B v : B P : : $v\,n$: P A : : v R : P C, therefore $v\,n$: v R : : P A : P C.

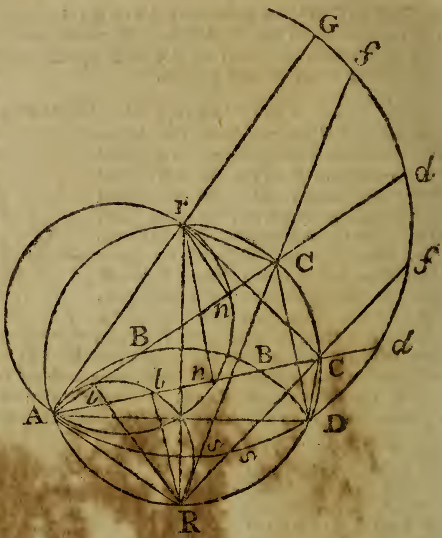
Corollary 1. Hence the 48th problem of Newton's Algebra may be folved geometrically; by continuing A B (fee Newton's figure) and finding the point E in B C fo that F E being drawn perpen. to the horizon meeting A B produced in P, the ratio of A E to E P may be as the weight D to the weight E: for if P E reprefent the weight E, B E will reprefent its force down the plane, B C, and as A E reprefents the force of D, E B reprefents its force in the direction E B, and confequently the weights are in equilibrio.

Corollary 2. If the ratio of C P to P A be required to be the greateft poffible, let A P be perpen. to A B: the reafon is evident from the 4th propofition.

Scholium. The application of this problem is very extenfive, particularly in mechanics, wherein lines are often required to be drawn parallel to the direction of gravity, &c. the problems of gunnery (abftracting the air's refiftance) may alfo be conftructed by it, in a much fimpler manner than any publifhed hitherto, as I fhall fhew hereafter.

PROPOSITION IX. THEOREM.

If A C D R be a circle, A D a chord, R r a diameter ⊥ to A D, and A C D, A C D triangles in the fegment A C D; then if from the



the centers r and R with the distances r A and R A circles A s $d f$ G and A B D be drawn; also on the diameters A r and A R circles r n A and A b R be described; then if the side A C of any triangle inscribed in A C B be produced to cut the circle A D d G in d, the circle A n r in n, the circle A B D in B and the circle A b R in b; also if R b, D B, r B, $r n$ and R B be drawn, then will the parts of the triangles A C D be as follows, viz.

1. A d = sum of the sides = A C + C D
2. A B = difference of the sides
3. A n = half sum of the sides = C b
4. A b = half difference of the sides = n C
5. C s^2 = rectangle of the sides.
6. b R A = half difference of the angles at the base = r A C

1. For the angle A r D = 2 A G D because r is the center, therefore A C D = 2 A d D = A d D + C D d, consequently C d = C D and A C + C D = A d.

2. In order to avoid the multiplicity of lines, suppose A R drawn through the point bisecting the arch A B D, and D R joined, then the proof that C D = C B, and consequently that A B = A C — C B will be thus; A B = B D and R B = R D, therefore A R B = B R D and R B D = R D B; but C B D + A B D = 2 right angles = C B D + 2 R D B, therefore C B D + 2 R D B = C D B + 2 R D B, and consequently C B = C D in this case; but the angles A B D and A C B are constant, and consequently C m is = C D in every case.

3. Because A n r is a right angle and r the center of A D d G ∴ A n = n d = half A C + C B, also A b = b B and B C = C d ∴ b A = half A d = A n = n d.

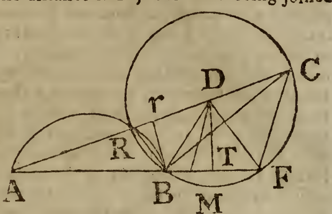
4. A b = half A B = half A C — C D.

5. A C × C d = s C × C f = C s^2, because s C = C f by Euclid, prop. 3.

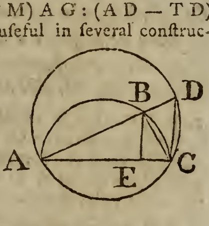
6. A R

6. A R b = A D B = C D A − C D B = C D B + B D A (C B D) B D A − C A D = C D B − C A D ∴ 2 A D B = C D A − C A D, consequently A R b equal half the difference of the angles at the base equal r A C, &c.

In the same manner if A D F be any triangle, and from the center D a circle be described with the distance D F; the lines being joined as in the figure, and D M parallel to C F; then it is evident that A B is the difference of the segments of the base, and A R that of the sides, and because A C F = half A D F, D M bisects A D F, and therefore M D T = half the difference of the angles at the base; but if B r be perpen. to A D, A B r =



A D T and A B R = A C F = A D M, therefore R B r = M D T, consequently A R B = R B r + R r B = 90° + half the diff. of the angles at the base, and therefore when the difference of the segments of the base is given, and the difference of the angles; the locus of all the points R will be a circular segment described on A B, containing an angle equal to 90 degrees + half the difference of the angles at the base; hence if S − s, A − a, and m − n be given, the triangle may be constructed, by taking A B = m − n, and on it describing the segment containing the angle 90 + $\frac{1}{2}$ (A − a) then taking A R = S − s and making R B D = B R D and D will be the vertex: hence also the locus of the vertices of all triangles which have the same difference of the angles at the base, and difference of the segments of the base, will be a circle; for as A R B is constant, and B R r also, and R B C a right angle, B C R is also constant as well as its double B D R, and R B D; wherefore B R is in a constant ratio to B D and B C, and consequently the points D and C are in circles; and hence a great number of cases of triangles may be constructed; for instance, if m − n, A − a, and either S, P, or S : s or S × s be given, and many others; and similar methods may be used when A R is constant and A B and the other parts variable. Hence also if M G = M F, A G : A R : : A R : A B for A F : A D + D F : : A R : A B : : A M : A D : : F M : F D, therefore A R : A B : : (A M − F M) A G : (A D − T D) A R. The following problem will also be useful in several constructions, viz. if A B C, A D C, be two given circles intersecting in A and C, and it be required to draw A B D so that the rectangle of A B and B D may be given: because the angles B and D are constant; the ratio of B C to B D is given, and consequently the ratio of A B × B C to A B × B D, but A B × B C = B E × by the diameter of the circle, and this being given, B E is also given; in the same manner may A B be drawn to have a given ratio to B D.



Some parts of this problem are not new, but were here brought together into one view for the sake of making references in order to shorten the solutions of problems.

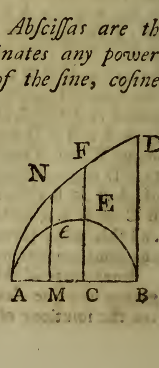
E Scho-

Scholium. In the foregoing propofitions I have feldom given folutions to more than one cafe; there are fome that admit of more cafes, but the method laid down will be applicable to the reft with fo little alteration, that I did not think it neceffary to be more particular. I do not doubt but fuch a proceedure will be looked upon as deviating from geometrical ftrictnefs by fuch as have formed their ideas of the method of the ancients from the fpecimen given by the *Reftorer* (as he is called) of *Apollonius de Inclinationibus*; however I cannot fee the ufe of multiplying cafes without neceffity, nor what end it can anfwer to repeat the fame thing, for each trifling alteration, when a fingle example would ferve: In a propofition there are certain things given, and thofe things are fufceptible of various fituations; now either the method of folution varies according as thofe fituations vary, or not; if it doth then it is neceffary to increafe the number of cafes till there be a folution for each fituation; if the method do *not* vary, what end can it anfwer to repeat the fame thing over and over for no other purpofe but to exhibit the various difpofitions of the data; when the fame end may be fully accomplifhed by only increafing the number of diagrams? Nay I do not even fee any neceffity for this laft; *Euclid* does not ufe it, and if by " the inclination of two ftreight lines which meet together," we underftand *either* of the angles made at the point of interfection, (a fenfe in which there is great reafon to believe that *Euclid* intended to be underftood) there will not then be the leaft occafion for feveral additions which *Dr. Simfon* and others have made to the elements; for inftance, propofition A, in book 6, will be included in prop. 3 preceeding it; and the additional theorem inferted in the data by *Lord Stanhope*, will fcarcely amount even to a fecond cafe of prop. 97. &c.

I know this method is contrary to the practice of feveral that arrogate to themfelves all knowledge in ancient geometry; but if it be agreeable to common fenfe, and give the fame degree of evidence and inftruction in a lefs compafs, it certainly cannot be without its ufe, and may, for that reafon, at leaft be tolerated.

ARTICLE XIII.

Of finding the Areas of Curves whofe Abfciffas are the fame as thofe in a Circle, and their Ordinates any powers of the correfponding Arc or Multiples of the fine, cofine, &c. By Mr. William Wilkin.



1. LET AEB be a femicircle whofe diameter is AB and center C, and from B, C, and any point M erect the perpendiculars BD, CF, and MN; and let MN be equal to any power of the arc Ae; to find the quadrature of the fpace ANM or the fluent of $z^m \dot{x}$, (putting AB $=2a$ M$e=v$, AM, $=x$, MN $=y$, A$e=z$, and the index of the power $=m$.)

Assume the fluent $= z^m x + q$, then will $z^m \dot{x} + m z^{m-1} \dot{z} \; x \times \dot{q} = z^m \dot{x}$, and therefore $\dot{q} = -m z^{m-1} \dot{z} . x = -\frac{a m z^{m-1} x \dot{x}}{\sqrt{2ax - x^2}} = -a m z^{m-1} \times (\dot{z} - \dot{v})$ and by taking the fluents $q = -a z^m$ + fluent $a m z^{m-1} \dot{v}$. Again assume the fluent of $a m z^{m-1} \dot{v} = a m z^{m-1} v + r$, then will $a m z^{m-1} \dot{v} + a m . \overline{m-1} . z^{m-2} \dot{z} v + \dot{r} = a m z^{m-1} \dot{v}$, therefore $\dot{r} = -a m . \overline{m-1} . z^{m-2} \dot{z} v = -a^2 m . (m-1) . z^{m-2} \dot{x}$, then again assume the fluent $r = -a^2 m . \overline{m-1} . z^{m-2} . x + s$, and by proceeding as before $\dot{s}$ will be $= a^2 m (m-1).(m-2). z^{m-3} \dot{z} . x = \frac{a^3 m . (m-1).(m-2). z^{m-3} x \dot{x}}{\sqrt{2ax - x^2}}$
$= a^3 m . (m-1) . (m-2) \times z^{m-3} \times (\dot{z} - \dot{v})$ and therefore $s = a^3 m . (m-1) . z^{m-2}$ — fluent $a^3 m . (m-1) . (m-2) . z^{m-3} \dot{v}$. Whence again assume $-a^3 m . (m-1) . (m-2) . z^{m-3} v + t$ for the fluent so will $\dot{t} = a^3 m . (m-1) . (m-2) . m-3 . z^{m-4} \dot{z} . v = a^4 m . (m-1) . (m-2) . (m-3) . z^{m-4} \dot{x}$ and $t = a^4 m . (m-1) . (m-2) . (m-3) . z^{m-4} x + v$: consequently (the law of continuation being manifest) the fluent of the given expression will be $= z^m x - a z^m + a m z^{m-1} v - a^2 m . (m-1) . z^{m-2} . x + a^3 m . (m-1) . z^{m-2} - a^3 m . (m-1) . (m-2) . z^{m-3} v + a^4 m . (m-1) \ldots\ldots (m-3) . z^{m-4} x - a^5 m \ldots . (m-3) . z^{m-4} +$, &c.

Corollary 1. If w be put equal $a - x$ (the cosine of the arc z) in the above expression, it will become $= -w z^m + a m z^{m-1} v + a^2 m . (m-1) . z^{m-2} w - a^3 m . (m-1) , (m-2) . z^{m-3} v - a^4 m . (m-1) \ldots (m-3) . z^{m-4} w +$, &c, the same as found at page 590 of Mr. Simson's Flux. 2d edit.

Corollary 2. If $m = 1$. then shall the fluent of $z \dot{x}$ or the area of the curve space A N M (whose ordinate M N is = circular arc A e,) $= z x - a z + a v = z x - a z + a \sqrt{2ax - x^2}$; whence $x = a$, or the ordinate passes through the center C, the area A F C = A C^2, and when $x = 2a$. the area of the whole curve A D B A = A C $\times$ circumference A E B = the circle whose diameter is A B.

Corollary 3. If $m = 2$, the area A N M $= z^2 x - a z^2 + 2 a z v - 2 a^2 x =$ (when $x = 2a$) $a z^2 - 4 a^3 =$ A C $\times$ $(\overline{\text{A EB}}|^2 - \overline{\text{AB}}|^2$ = the excess of the circle whose diameter is A B above twice the square of the rad, A C, for the whole space A D B.

Corollary 4. If $m = 3$ the area $= z^3 x - a z^3 + 3 a z^2 v - 6 a^2 z x + 6 a^3 z - 6 a^3 v =$ (when $x = 2a$) $a z^3 - 6 a^3 z$.

Corollary 5. If $m = 4$ the area $= z^4 x - a z^4 + 4 a z^3 v - 12 a^2 z^2 x + 12 a^3 z^2 - 24 a^3 z v + 24 a^4 x =$ (when $x = 2a$) $a z^4 - 12 a^3 z^2 + 48 a^5$, &c. &c.

2. Suppose the ordinate of the curve M N to be always equal to a rectangle of any power of the arc and the versed sine $= z^m x^n$ to find the curve of the space A M N or the fluent of $z^m x^n \dot{x}$. Assume the fluent $= \frac{z^m x^{n+1}}{n+1} + r$, then in fluxions $\dot{r} = -\frac{m x^{n+1} z^{m-1} \dot{z}}{n+1}$

$$= -\frac{m x^{n+1} z^{m-1}}{n+1} \times \frac{a \dot{x}}{\sqrt{2ax - x^2}} = -\frac{a m z^{m-1}}{n+1} \times \frac{x^{n+1} \dot{x}}{\sqrt{2ax - x^2}}$$

$= -\frac{a m z^{m-1}}{n+1} \times \frac{x^{n+\frac{1}{2}} \dot{x}}{\sqrt{2a - x}}$. Again assume the fluent $r = -\frac{a m z^{m-1} A}{n+1} + s$ (putting $A =$ fluent $\frac{x^{n+\frac{1}{2}} \dot{x}}{\sqrt{2a - x}}$,) then will $\dot{s} = \frac{a m . \overline{m-1} . z^{m-2} \dot{z} A}{n+1}$. Now by finding the value of A or the flu. of $\frac{x^{n+\frac{1}{2}} \dot{x}}{\sqrt{2a - x}}$ (which may be easily had from Emer. Table, Form 11. when n is any affirmative whole number, and thence s, or the fluent of $\frac{a m . \overline{m-1} . z^{m-2} \dot{z} A}{n+1}$ and assuming other variable determinate values as before the required fluent of $z^m x^n \dot{x}$ will therefore be evidently $\frac{1}{n+1} \times : z^m x^{n+1} - a m z^{m-1} A + a m . (m-1) . z^{m-2} B - a m . \overline{m-1} . \overline{m-2} . z^{m-3} C$, &c. A being as above, $B =$ fluent of $A \dot{z}$, $C =$ fluent of $B \dot{z}$. &c.

But as this cannot be pursued in a general manner by this method it will be necessary to shew how to proceed in particular cases, when m and n are given in numbers.

Thus. 1. If m and n each equal 1, then will $\dot{A} = \frac{x^{\frac{3}{2}} \dot{x}}{\sqrt{2a - x}}$, and by taking the fluents $A = \frac{3}{2} a z - \frac{3a + ax}{2} \times \sqrt{2ax - x^2}$ therefore the area will be $= \frac{z x^2}{2} - \frac{3 a z}{4} + \frac{3a^2 + ax}{4} \sqrt{ax - x^2}$ which when x is $= 2a$ becomes $= \frac{5 a^2 z}{4}$ the whole area of the curve. Or the fluent of the expression $z x \dot{x}$.

2. If $m = 1$ and $n = 2$ then will $\dot{A} = \frac{x^{\frac{5}{2}} \dot{x}}{\sqrt{2a - x}}$ and consequently

A

$A = \frac{5}{2}a^2 z - \overline{\frac{1}{3}x^2 + \frac{5}{6}ax + \frac{5}{2}a^2} \times \sqrt{2ax - x^2}$ and the area $= \frac{z^m x^{n+1}}{n+1} - \frac{amz^{m-1}A}{n+1}) \frac{zx^3}{3} - \frac{5a^3z}{6} + \frac{ax^2}{9} + \frac{5a^2x}{18}$ $+ \frac{5a^3}{6}\sqrt{2ax - x^2}$ — (when $x = 2a$) $\frac{11}{6}a^3 z$ for the whole space A D B or the fluent of $z x^2 \dot{x}$.

3. If $m = 2$ and $n = 1$, then $\dot{A} = \frac{x^{\frac{3}{2}}\dot{x}}{\sqrt{2a - x^2}}$ and $\dot{s} = \frac{3a^2 z\dot{z}}{2} - \frac{3a^2 + ax}{2}\sqrt{2ax - x^2} \times (\dot{z}) \frac{a\dot{x}}{\sqrt{2ax - x^2}}$, hence by taking the fluents $s = \frac{3a^2 z^2}{4} - \frac{3a^3 x}{2} - \frac{a^2 x^2}{4}$, therefore the area $\frac{z^m x^{n+1}}{n+1} - \frac{amz^{m-1}A}{n+1} + s) = \frac{z^2 x^2}{2} - \frac{3}{4}a^2 z^2 + \frac{3a^2 z + axz}{2}$ $\sqrt{2ax - x^2} - \frac{3a^3 x}{2} - \frac{a^2 x^2}{4} =$ (when $x = 2a$) $\frac{5}{4}a^2 z^2 - 2a^4$.

4. If m and n each equal 2, then $\dot{A} = \frac{x^{\frac{5}{2}}\dot{x}}{\sqrt{2a - x}}$ and $\dot{s}$ $= \left(\frac{am.\overline{m-1}.z^{m-2}A}{n+1} \times \dot{z}\right) = \frac{5}{3}a^3 z\dot{z} - \frac{2}{9}a^2 x^2 \dot{x} - \frac{5}{9}a^3 x\dot{x}$ $- \frac{5}{3}a^4 \dot{x}$, whence $s = \frac{5}{6}a^3 z^2 - \frac{2}{27}a^2 x^3 - \frac{5}{18}a^3 x^2 - \frac{5}{3}a^4 x$ and the area $= \frac{z^2 x^3}{3} - \frac{5}{6}a^3 z^2 + \frac{2}{9}azx^2 + \frac{5}{9}a^2 zx + \frac{5}{3}a^3$ $z\sqrt{2ax - x^2} - \frac{2}{27}a^2 x^3 - \frac{5}{18}a^3 x^2 - \frac{5}{3}a^4 x =$ (when $x = 2a$) $\frac{11}{6}a^3 z^2 - \frac{136}{27}a^5$.

5. If $m = 3$ and $n = 2$, $\dot{A} = \frac{x^{\frac{5}{2}}\dot{x}}{\sqrt{2a - x}}$ as before, ($\dot{s} = 5a^3 z^2 \dot{z}$ $- \frac{2}{3}a^2 x^2 \dot{x} - \frac{5}{3}a^3 x\dot{x} - \frac{5}{2}a^4 \dot{x}$ and $\dot{t} = -\frac{5}{2}a^3 z^2 \dot{z} - \frac{2}{9}a^3 x^3$ $\dot{x} - \frac{5}{6}a^4 x^2 \dot{x} - 5a^5 x\dot{x}$), or $B = \frac{5}{4}a^2 z^2 - \frac{1}{9}ax^3 - \frac{5}{12}a^2 x^2$ $- \frac{5}{2}a^3 x$ and $C = \frac{5}{12}a^2 z^3 - \frac{1}{36}a^2 x^4 - \frac{5}{36}a^3 x^3 - \frac{5}{4}a^4 x^2$, whence by substituting these values in the above general expression, the area becomes $= \frac{z^3 x^3}{3} - az^2 . A + 2az B - 2a C = \frac{z^3 x^3}{3}$ $\frac{5}{6}a^2 x^3 + \frac{1}{3}ax + \frac{5}{6}a^2 x + \frac{5}{2}a^3 \times z^2 y(-\frac{2}{9}ax^3 - \frac{5}{6}a^3 x^2 - 5$ $a^4 x) \times z + \frac{1}{18}a^3 x^4 + \frac{5}{18}a^4 x^3 + \frac{5}{2}a^5 x^2$.

6. If m and n each equal 3; then $\dot{A} = \frac{x^{\frac{7}{2}}\dot{x}}{\sqrt{2a - x}}$ or $A =$ $\frac{35}{8}a^4 z - \frac{1}{4}x^3 y - \frac{7}{12}ax^2 y - \frac{35}{24}a^2 xy - \frac{35}{8}a^3 y$, $B = \frac{35}{16}a^4 x^2$

$-\frac{1}{16}ax^4-\frac{7}{36}a^2x^3-\frac{35}{48}a^3x^2-\frac{35}{8}a^4x$, $C=\frac{35}{48}a^4z^3-\frac{1}{80}a^2x^5-\frac{7}{144}a^3x^4-\frac{35}{144}a^4x^3-\frac{35}{16}a^5x^2$, and $D=\frac{35}{172}a^4x^4-\frac{1}{480}a^3x^6-\frac{7}{720}a^4x^5-\frac{35}{456}a^5x^4-\frac{35}{48}a^6x^3$ substitute these values in the theorem and it will give $\frac{z^3x^4}{4}-\frac{35}{32}a^5z^3+\frac{3}{16}ax^3+\frac{7}{16}a^2x^2+\frac{35}{32}a^3x+\frac{105}{32}a^4\times z^2\dot{y}-\frac{3}{32}a^2x^4-\frac{7}{24}a^3x^3(-\frac{55}{32}a^4x^2-\frac{105}{16}a^5x)\times z+\frac{3}{160}a^3x^5+\frac{7}{96}a^4x^4+\frac{35}{96}a^5x^3+\frac{105}{32}a^6z^2$ for the area, &c. &c.

Suppose the ordinate of the curve MN to be always equal to the rectangle of any power of the arc, versed sine and sine $=z^mx^nv^r$ to find the area of the space AMN or the fluent of $z^mx^nv^r\dot{x}$.

Assume $\frac{z^m v^r x^{n+1}}{n+1}+s$ for the fluent, then $\dot{s}=-\frac{1}{n+1}\times:mx^{n+1}v^rz^{m-1}\dot{z}+rx^{n+1}z^mv^{r-1}\dot{v}=-\frac{1}{n+1}\times:mz^{m-1}x^{n+1}v^r\times\frac{a\dot{x}}{v}+rz^mx^{n+1}v^{r-1}\dot{v}=-\frac{1}{n+1}\times:rz^mx^{n+1}v^{r-1}\dot{v}+amz^{m-1}x^{n+1}v^{r-1}\dot{x}$. Again assume $s=-\frac{1}{n+1}\times amz^{m-1}A+rz^mB+t$ (putting A = the fluent of $x^{n+1}v^{r-1}\dot{v}$ and B = fluent of $\frac{\dot{x}}{v}$ A) then will $\dot{t}=\frac{1}{n+1}\times:am.\overline{m-1}z^{m-2}A\dot{z}+rmz^{m-1}B\dot{z}$. Assume now this fluent $=\frac{1}{n+1}\times:am.\overline{m-1}z^{m-2}C+rmz^{m-1}D+u$ (putting C = fluent of $\dot{z}$ A and D = that of $\dot{z}$ B) by proceeding as before, and assuming another value for u, the law of continuation will be evident, putting again in this value E = flu. $\dot{z}$ C and E = that of $\dot{z}$ D; The fluent will therefore be generally expressed thus $\frac{z^mv^rx^{n+1}}{n+1}-\frac{1}{1+n}(rz^mA+amz^{m-1}B+rmx^{m-1}C+am.\overline{m-1}.z^{m-2}D)-(rm.\overline{m-1}.z^{m-2}E+am.\overline{m-1}.\overline{m-2}.z^{m-3}F+$ &c.)

To exemplify this theorem, take the 1 Exam. given by Mr. Simpson to his solution of the same problem at page 393 of his fluxions. Then will $m=1$, $n=0$, and $r=1$; whence $zvx-zA+$ fluent $\dot{z}A=zvx-\frac{1}{2}x(xv-az+av)-\frac{1}{2}ax^2+$ fluent of $\frac{1}{2}:(vz-az+av)\times\frac{av}{x}=zvx-\frac{1}{2}\times(zvx+az^2-avz-ax^2)$

$+\frac{1}{4}$

$+\frac{1}{4}\times(ax^2-az^2)+\frac{1}{2}a^2x=\frac{1}{2}zvx+\frac{1}{4}az^2-\frac{1}{2}avx-\frac{1}{4}ax^2+\frac{1}{2}a^2x=\frac{1}{4}az^2-\frac{1}{2}avz+\frac{1}{2}zvz+\frac{1}{4}av^2$.

N. B. In the above quoted ſolution x is the coſine, and in this x is the verſed ſine, if therefore in the concluſion $a-x$ be ſubſtituted for the verſed ſine it will appear to correſpond with the above.

Scholium. From theſe may be had the ſolutions of ſeveral queſtions that have been propoſed in the annual publications relating to circular areas and cycloidal ſpaces. The method may be purſued much farther, and extended to different enquiries of a ſimilar nature, where arcs of any kind, hyp. logs. &c. are involved with the fluxions of their contemporaneous parts, though perhaps not in ſo conciſe a manner as by the method of aſſuming a ſeries with unknown coefficients, &c. yet to beginners it will appear much more plain and intelligible, for whoſe uſe and improvement the application of the theorems is intended.

New MATHEMATICAL QUESTIONS *to be anſwered in next Year's* DIARY.

[1] XIV. QUESTION, *by Mr.* Edward Boucher.

GIVEN $\left.\begin{array}{l} x^3y+y^3x=a \\ x^6y^2+y^6x^2=b \end{array}\right\}$ to find x and y.

[2] XV. QUESTION, *by Mr.* Fininley.

GIVEN the difference of the ſegments of the baſe, the difference of the angles at the baſe, and the rectangle made by one of the ſides, and a line to which the other ſide hath a given ratio: to find the triangle.

[3] XVI. QUESTION, *by Mr.* John Lynn, *of* Sunderland

LET the given line A B be perpendicular to the the indefinite line A Q, and drawing any right line B E from the fixt point B, to cut A Q in E, and taking E C thereon in a given ratio to E A; it is required to find the nature of the curve, &c.

[4] XVII. QUESTION, *by Mr.* George Sanderſon.

IN a triangle A C B, the baſe A B is given, and the difference of the ſides A C and C B: it is required to conſtruct the triangle geometrically when the difference of A D and D C is the leaſt poſſible; C D being drawn from the vertex of the triangle to meet the baſe in a given angle.

[5] XVIII. QUESTION, *by Mr.* Iſaac Dalby.

IF A B and A C be tangents to a given circle meeting in the given point A, and from this point with a given diſtance a circle be deſcribed; it is required to draw a tangent to the firſt circle, cutting the laſt in Q, and the tangents in N and R, ſo that N Q may be equal to Q R.

[6] XIX. QUESTION, *by Mr.* William Wilkin.

IF a caſk is formed by the revolution of the quadratrix of Dinoſtratus, about the diameter of the generating ſemicircle; it is required to determine the number of ale gallons it will contain when the bung and head diameter are 40 and 30 inches reſpectively.

[7] XX. QUESTION, *by Mr.* D. Cunningham.

REQUIRED the ſum of any number of terms of the infinite ſeries $\frac{2\cdot4\cdot6}{3}+\frac{4\cdot6\cdot8}{3\cdot3}+\frac{6\cdot8\cdot10}{3\cdot3\cdot3}+$, &c.

[8] XXI. QUESTION, *by Mr.* Thomas Mofs.

IF from the extremities S and V of the bafe of a triangle S T V two lines be drawn through a given point N meeting T V and S T in C and A; and the line T N be joined meeting A C in B; alfo if from A and C parallel lines be drawn meeting the bafe in M and P; then will A B be to B C as A M to C P: required the demonftration?

[9] XXII. QUESTION, *by Mr.* Thomas Todd.

A Has 1000 l. due from B one year hence, befides D pounds feven years hence; to determine D pounds, with the equated time, as given by Malcolm's method, fo that A may gain 20 l. more by this equatement than if he had received his money as it came due, 5 per cent. per annum fimple intereft being allowed to both A and B.

[10] XXIII. QUESTION, *by Mr.* Jeremiah Ainfworth.

HAVING a circle given in magnitude and pofition, the center of which is fituated in a line bifecting an angle made by two lines given in pofition: it is required to draw a tangent to the circle, fo that the fegment intercepted between thefe two lines may be of a given length.

[11] XXIV. QUESTION, *by the Rev. Mr.* Crakelt.

GIVEN the triangle A B C, and the pofition of the point P in the fide B C; it is required to draw the line D E through the faid point, meeting A C in D, and A B produced in E, in fuch fort, that the fum of tho areas of the two triangles P C D and P B E may be equal to the area of the trapezium A D P B.

N. B. This is Queftion 337 of the Gent. Diary.

[12] XXV. PRIZE-QUESTION, *by* Reuben Burrow.

IF A B, A C and A N be lines given in pofition, meeting in the point A, and P a given point; it is required to draw the line P E, meeting A B in D and A C in E, in fuch a manner, that E F being drawn parallel to N A to meet A B in F, the perimeter of the triangle D E F may be the greateft poffible; without algebra.

Whoever gives the beft Solution to this Queftion, before Candlemas-day, *fhall receive Six* Diaries *and a Silver Medal*; *and the Perfon who gives the next beft Solution fhall be intitled to the Prize of Twelve* Diaries.

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