RATIONAL AMUSEMENT

FOR

Winter Evenings;

OR,

A Collection of above

200 CURIOUS AND INTERESTING

PUZZLES AND PARADOXES

relating to

ARITHMETIC, GEOMETRY, GEOGRAPHY, &c.

WITH THEIR SOLUTIONS,

AND FOUR PLATES.

DESIGNED CHIEFLY FOR YOUNG PERSONS.

BY

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London:

SOLD BY LONGMAN, HURST, REES, ORME, AND BROWN;
G. AND W. B. WHITTAKER; AND
HARVEY AND DARTON.

AND BARRY AND SON, HIGH-STREET, BRISTOL.

1821.

PREFACE.

THE Author having occasionally met with *puzzles* relating to Numbers, Geometrical Figures, &c. interspersed in different books, apprehended that a collection of the best, with their *solutions*, might be made, sufficient to form a small volume.

After extensive enquiry, no work appearing to exist of the same description, he proceeded to collect from such materials as seemed most suitable for his purpose; and the result of his labours, in addition to a few *originals*, he now offers to the public.

It must, however, be noticed, that *Dr. Hutton's Mathematical Recreations*, in 4 vols. 8vo. contain, amongst many other curious and interesting subjects, several *puzzles* relating to numbers, &c. &c., particularly in the 1st vol.; but a work so large is rendered less accessible than a small volume at a moderate expence. The same may be said respecting *Hooper's Rational Recreations*, in 4 vols. 8vo.

From those works, particularly the former, the Author acknowledges having taken such *puzzles* as he considered the best. He has also taken a few from "The Circassian Puzzle," a recent performance, ingenious in its design, and calculated to afford much amusement for youth.

Although some of the puzzles relating to *numbers* admit of *regular calculation*, they are, nevertheless, of an amusing nature.

And, in reference to the puzzles in *Geometry*, although several of them may not be considered *strictly* entitled to that appellation, because they cannot be solved *mechanically*; yet they will be found equally curious and interesting in their nature, particularly those that are to be performed by means of the *compasses only*: in solving which, were a *ruler* to be used, it must be acknowledged that their difficulty as well as interest would vanish; for it must not be understood that any of these are to be performed merely by *repeated trials with the compasses;* and, here, the young students are advised not even to attempt such methods. Under these *limitations*, they will have to exercise their ingenuity; and, it is presumed, that they will become much more *familiar* with the relation between different lines in, and about a circle, &c., as well as with their values, than when a *ruler* is employed.

Such demonstrations as appeared either *simple*, or likely to satisfy the *investigating* mind, have been subjoined. In order to comprehend these, it will be necessary, in some instances, to refer to *Euclid;* without a knowledge of which indeed, we must not hope to experience (so far as respects Geometry) the truth of the motto taken from *Virgil*, and adopted by *Reynard* in his "Geometria legitima":

"*Felix* qui potuit rerum cognoscere *causas.*"*

In conclusion: it may be observed, that all the questions as well as their solutions, have been carefully revised; and both, in several instances, *new-modelled;* and it has been the Author's endeavour to render the *latter* as clear as the nature of the subject will admit.

☞ The young students are particularly recommended to give every question a *fair trial*, before they refer to the *key* for assistance. The same hint may be given to those of *riper* years, who also, it is hoped, will find some amusement in this little volume.

* "Happy the man, who, studying nature's laws,
Through known effects can trace the secret cause."
Dryden's Translation.

A

COLLECTION

OF

CURIOUS PUZZLES.

ARITHMETICAL PUZZLES.

1. To a thousand add one, twice fifty and ten,
Six-sevenths of a million's this sum I'll maintain.

2. It is required to express 100 by four 9's.

3. If from six ye take nine, and from nine ye take ten,
(Ye youths, now the mystery explain;)
And if fifty from forty be taken, there then
Shall just half a dozen remain.

4. It is required to place three 2's in such a manner as to form three numbers in Geometrical Progression.

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5. Express 12 by four figures each the same.

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6. A gentleman dying, left his executor a sum less than £2000, to be divided between his relations in such a manner, that his father and mother, son and grandson, brother and daughter, should each receive a sum not less than £666 13*s*. 3*d*. Required the relationship, and the sum to be divided?

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7. A person has a fox, a goose, and a peck of corn, to carry over a river; but on account of the smallness of the boat, he can only transport them one at a time: How can this be done, so that the fox may not be left with the goose, nor the goose with the corn?

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8. A schoolmaster to amuse his scholars, showed them a number, which he said was the sum of six rows, each consisting of four figures. He desired them to write down three rows of figures, to which he would add three more; and assured them that the sum of the whole should be equal to the number he shewed them. How must this be done?

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9. Supposing there are more persons in the

world than any of them has hairs upon his head, it then necessarily follows that some two of them at least must have exactly the same number of hairs on their heads to a hair. Required the proof?

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10. Place in a row, nine different figures, the sum of which shall be 45. Directly under the above, place another row, consisting of nine different figures, the sum of which shall also be 45. Subtract the lower figures from the upper, and the remainder shall still consist of nine different figures, their sum being 45 as before.

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11. It is required to distribute 21 bottles amongst three persons, 7 of them full of wine, 7 half full, and 7 empty, so that each may have the same quantity of wine, and the same number of bottles.

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12. In an Arabian manuscript was found this remarkable decision of a dispute. Two Arabians sat down to dinner: one had 5 loaves and the other 3. A stranger passing by, desired permission to eat with them; to which they agreed. The stranger dined, laid down 8 pieces of money and departed. The proprietor of the 5 loaves took up 5 pieces, and left 3 for the other, who objected, and insisted for one half. The cause came before Ali, the magistrate, who gave the following judgment: "Let the owner of the

five loaves have 7 pieces of money; and the owner of the three loaves 1. Quere, the justice of this sentence?

13. Three jealous husbands and their wives having to cross a river, find a boat without its owner, which can only carry two persons at a time: In what manner then can these six persons transport themselves over by pairs, so that none of the women shall be left in company with any of the men, except when her husband is present?

14. A man coming from Lochrin distillery with an 8-pint jar full of spirits, was met by a person going thither with a 3-pint jar, and one of 5 pints, both empty. Being pressed for time, he begged the person who was returning, to give him four pints out of his quantity. How are they to measure 4 pints exactly with these three jars only?

15. This is a truth (tho' the number's even,)
That half of twelve 's exactly seven.

16. Fifteen Christians and fifteen Turks being in the same ship at sea, in a terrible storm, and it being thought indispensably necessary that half the number of passengers should be thrown overboard to prevent it from sinking, it was agreed on that they should be cast away by

lot. They were all to be placed in a ring, and every *ninth* man was to be cast away, till only *half* the number remained. How must they be placed, so that the lot may fall on every Turk, and thus every Christian be saved?

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17. Place four 5's so that their sum shall be $6\frac{1}{2}$.

---

18. Place the nine digits, so that the sum of the odd digits may be equal to the sum of the even ones.

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19. Place the nine digits in two different ways, so that in one case, their sum may be 17, and in the other, 31.

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20. Two-thirds of six, if multiply'd
By just one-fifth of seven,
Then Wilkes's number is descry'd
When product's fairly given.

---

21. Suppose three gentlemen and their three servants have to cross a river in a boat, capable of conveying only *two* persons at a time; and that they are told of the servants' design to murder them when thus separated. It is required to show, how these gentlemen may contrive to have their number, on both sides of the river, always equal or superior to the number of the servants, and thus be out of danger.

22. A gentleman having a castle situated on a square, and garrisoned by 48 soldiers, so ordered them, as that any two corners and the side between them should consist of 18 men: but supposing he had not a sufficient number, he hired 8 more, but still ordered them as before; afterwards 16 men were paid off, not being wanted, but still he kept up his 18 men. How must they be placed in each case?

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23. Come tell to me, what figures three,
When multiplied by four,
Make five exact, (it is the fact)
This unto me explore.

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24. One third of twelve, if you divide
By just one-fifth of seven,
The true result (it has been tried,)
Exactly is eleven.

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25. The sum of four figures in value will be,
Above seven thousand nine hundred and three;
But when they are halved, you'll find very fair
The sum will be nothing in truth I declare.

---

26. A walking out, finds, on turning round, that B is 400 yards behind him, desiring to overtake him; they each of them moved 200 yards with their faces towards each other in a direct

line, yet were still 400 yards asunder. How could this be?

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27. The sum of the digits a number will make,
From which, if just fifty you properly take,
One-third of that number is still left behind:
What it is, in whole numbers, endeavour to find.

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28. To give a person his choice out of three or four rows of figures which shall be written down for him; to let him multiply the row he makes choice of by any number; to let him suppress or rub out any of the figures; and even to alter the arrangement of the rest, to let him shew the figures that remain; and to tell him then what figure he has suppressed.

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29. In what manner may a number of nuns be disposed of in the eight external cells of a square convent, placed like those in a chess board, so that a blind abbess, who occupies a cell in the middle, shall always find whenever she visits them, 9 in each row, and yet some of them may have gone out, or a certain number of men may have been introduced so as to vary the number from 20 to 32?

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30. Six hundred and sixty so ordered may be,
That if you divide the whole number by three,

The quote will exactly in numbers express
The half of six hundred and sixty, not less.

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31. From half of five take one, then five shall still remain.

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32. A wine merchant having 32 casks of choice wine, ordered his clerk to arrange them in a square, so that each side should contain 9; which he did. But being roguishly disposed, he took *four* of them, and contrived to place the remainder so that each side still contained 9; having succeeded in concealing the theft, he ventured to take away other *four*, and placed the remainder as before. The rogue not yet contented, took away *four* more, and again contrived to place the remainder so that each side still contained 9. After this he could do no more. Required the arrangement of the casks in each case?

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33. If required to place four 2's in such a manner as to form four numbers in Geometrical Progression.

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34. On the day that I was born, my father laid by £5 for me, and on every succeeding birth-day he added £5, till my birth-day at 24 years of age, when I was married, and received my portion, which proved to be only £35. How is this accounted for?

35. Divide half of nine by half of five, the quotient shall be one.

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36. A blind abbess visiting her nuns, who were equally distributed in 8 cells, built at the four corners of a square, and in the middle of each side, finds an equal number of persons in each row or side containing three cells. At a second visit, she finds the same number of persons in each row, though their number was increased by the introduction of 4 men. And, coming a third time, she still finds the same number of persons in each row, though the 4 men were then gone, and had each carried away a nun with them. Required the distribution in each case?

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37. It is required to find, by extermination, all the prime numbers, except the number 2, from unity to 100?

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38. Place in a row nine figures, each different from the other; multiply them by 8, and the product shall still consist of nine different figures.

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39. A party of soldiers consisting of 40 men, having been quartered in a certain district, where they had rendered themselves odious by theft and outrageous conduct, the commanding officer was determined to decimate them, or to have every tenth man shot as a terror to the rest of the

army; but knowing from their general characters, which had been the ringleaders, desires to be informed how they may be placed, so that the lot may fall upon the most culpable.

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40. It is required to place three 3's in such a manner, as to form three numbers in Geometrical Progression, the common ratio of which shall be 3?

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41. Place in a row 9 figures, each different from the other; the sum of which shall be 45; and multiply them by any of the nine digits excepting 9, 6, and 3; and the several products shall still consist of nine figures, each different from the other, and their sum 45.

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42. It is required to place four 3's in such a manner, as to form six numbers in Geometrical Progression.

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43. Place five 3's in such a manner as to form six numbers in Geometrical Progression.

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44. It is required to express 78 by six figures each the same.

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45. There is a number consisting of nine digits, which being multiplied by five different digits, each of those products will contain exactly nine digits and a cipher.

46. To place 14 figures in such a manner as to exhibit not only the numbers to be multiplied, but the products beyond any common Multiplication Table.

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47. Two persons agree to take, alternately, numbers less than a given number; suppose less than 11, and to add them together till one of them has reached a certain sum; suppose 100. By what means can one of them infallibly attain to that number before the other?

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48. To find the difference between two numbers, the greatest of which is unknown.

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49. A number, consisting of the nine digits, being divided by 2 and 5 separately, the respective quotients, as well as the sum of the quotients, shall all consist of the same nine digits, though not in the same order.

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50. A person choosing any two out of several given numbers, and after adding them together, striking out one of the figures from the amount, to tell what that figure was.

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51. To distribute amongst three persons, 24 casks of wine, 8 of them full, 8 half full, and 8 empty, so that each shall have the same quantity of wine, and the same number of casks.

52. A gentleman has a bottle containing 12 pints of wine, half of which he is desirous of giving to a friend; but as he has nothing to measure it with except two other bottles, one of 7 pints and the other of 5. How must he manage to have the 6 pints in the bottle capable of containing 7 pints?

MAGIC SQUARES.

If we take the common arithmetical progression of natural numbers, proceeding regularly from unity, and terminating in any *square number*, the terms of this progression may be so disposed within the cells of a geometrical square, that the sums of each row, taken either diagonally, laterally, or vertically, shall be the same; and such arithmetical squares are called MAGIC SQUARES. Nevertheless, numbers proceeding regularly or irregularly, whether from unity or otherwise, and whether they terminate in a square number or not; if disposed as above, are generally understood to form *Magic Squares* as well as the former.*

I. Questions in numbers forming *three* rows.

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1. Place the nine digits so as to form 15 every way.

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2. With the numbers 8, 9, 10, &c. to 16, to form 36 every way.

* Numbers in *geometrical* progression, may be placed as above, but then the *products* will be the same, and not the sums.

3. With every other number except 4 and 20, from 2 to 22, to form 36 every way.

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4. With the numbers 16, 17, 18, &c. to 24, to form 60 every way.

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5. With the nine first terms of the geometrical progression, 1, 2, 4, &c. to form a product of 4096 every way.

II. Questions in numbers forming *four* rows.

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6. With the numbers 1, 2, 3, &c. to 16, to form 34 every way.

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7. With the numbers 1, 2, 3, &c. to 19, excepting 5, 10, and 15, to form 40 every way.

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8. With the numbers 6, 7, 8, &c. to 24, excepting 10, 15, and 20, to form 60 every way.

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9. With the numbers 8, 9, 10, &c. to 23, to form 62 every way.

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10. With the numbers 3, 5, 7, 9, &c. to form ——— every way.

CURIOUS

ARITHMETICAL QUESTIONS

Requiring regular Calculation.

1. What part of 3d. is $\frac{1}{3}$ of 2d.?

2. If the half of five be seven,
What part of nine will be eleven?

3. What is the difference, in a commercial concern, between doubling an expence and halving a profit?

4. A and B at the same work can earn 40*s.* in 6 days; A and C can earn 54*s.* in 9 days; and B and C can earn 80*s.* in 15 days. What can each person earn alone per day?

5. A snail in getting up a maypole 20 feet high, ascended 8 feet every day, and came down 4 again every night. How long would he be in getting to the top of the pole?

6. Divide 1000 crowns between A, B, and C. Give A 129 more than B, and B 178 fewer than C.

7. Part 1500 acres of land: Give B 72 more than A, and C 112 more than B.

8. A Cheshire cheese being put into one of the scales of a false balance, was found to weigh 16lbs. and when put into the other only 9lbs.: What is the true weight?

9. A company at a tavern spent 6*s.* ¼*d.* and each of them had as many farthings to pay as there were persons in company. How many persons were there?

10. An Indian gardener being desirous of presenting a basket of oranges, of a peculiar quality, to the nawab, had seven gates to pass before he could reach the audience chamber; at the first of which he was obliged to give half the number he had to the porter; at the second, half what remained; and so on; when at length coming into the presence of the prince, he found he had only one orange left. How many had he at first?

11. A in five hours a sum can count,
Which B can in eleven;
How much more then is the amount
They both can count in seven?

12. When Gripus died, in sterling gold was found,
Left for his family, eight thousand pound,

To be bestowed, as his last will directed,
Which did provide that none should be
neglected;
For to each son (there being in number
five)
Three times each daughter's portion he
did give:
His daughters four, were each of them to
have
Double the sum he to the mother gave.
Now that his wish may justly be fulfill'd,
What must the widow have, and what
each child?

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13. A gentleman gave to the first of three poor persons that he met, half the number of shillings that he had about him, and one shilling more; to the second, half of what remained, and two shillings more; and to the third, half what now remained, and three shillings more; after which he found he had only 1 shilling left. How many shillings had he at first?

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14. The eldest of three sisters having 50 eggs to dispose of, the next 30, and the youngest 10, they so contrived it, that each of them sold their eggs at the same rate, and each got the same sum of money for them. How was this done?

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15. A woman carrying eggs to market, was asked how many she had. She replied, that

when she counted them by 2's, there was one left; when by 3's, there was one left; and when by 4's, there was one left; but when she counted them by 5's, there were none left. How many had she?

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16. A, B, and C, are to share 1000 acres of land in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, respectively: but C's part being lost by his death, it is required to divide the whole properly between the other two.

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17. A woman going to market, sells butter at a certain rate per lb.; the price she received for the whole was 2*s*. 2¼*d*. Required the different number of lbs. she could have, so that by selling each at a certain price per lb., she might always obtain the same money?

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18. A person went out with a certain number of guineas about him, in order to purchase necessaries at different shops: at the first he expended half the number he had and half-a-guinea more; at the second, half the remainder, and half-a-guinea more; and so on at a third and fourth shop; at the last of which, having paid for his articles, he found he had laid out all his money. How much had he at first?

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19. A poor woman, carrying a basket of apples, was met by three boys, the first of whom

bought half of what she had, and then gave her back 10; the second boy bought a third of what remained, and gave her back 2; and the third bought half of what she had now left, and returned her 1; after which she found she had 12 apples remaining. What number had she at first?

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20. What is the least number that can be divided by the nine digits without a remainder?

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21. A market-woman bought 120 apples at 2 a penny, and 120 more of another sort, at 3 a penny; but not liking her bargain, she mixed them together, and sold them out again at 5 for two pence, thinking she should get the same sum; but on counting her money, she found to her surprise, that she had lost, 4d. How did this happen?

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22. The three Graces carrying each an equal number of oranges, were met by the nine Muses, who asked for some of them; and each Grace having given to each Muse the same number, it was then found that they had all equal shares. How many had the Graces at first?

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23. A country woman carrying eggs to a garrison where she had three guards to pass, sold at the first, half the number she had and half an egg more; at the second, half of what remained

and half an egg more; and at the third, the half of the remainder and half an egg more: when she arrived at the market place, she had three dozen still to sell. How was this possible without breaking any of the eggs?

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24. To find the weights with which any number of lbs. from 1 to a given number, can be weighed in the simplest manner.

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25. A gentleman employed a bricklayer to sink a well to the depth of 20 yards, and agreed to give him £20 for the whole; but the bricklayer falling sick when he had finished the 8th yard, was unable to go on with the work. How much was then due to him, on a supposition that the labour increases in arithmetical proportion as the depth?

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26. A labourer received £2 8*s*. for thrashing 60 quarters of grain, viz. wheat and barley; for the wheat he received 12*d*. per quarter, and for the barley 6*d*. How many quarters did he thrash of each?

To be solved independently of the Rule of Position.

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27. X, Y, and Z in company make one joint stock of £4262. X's money was in 4 months, Y's 6 months, and Z's 9. They gained £420, which was to be divided as follows, viz. $\frac{1}{2}$ of X's gain

was to be $\frac{1}{3}$ of Y's, and $\frac{1}{8}$ of Y's to be $\frac{1}{4}$ of Z's. Quere each man's gain and stock?

This question appears in some works on Arithmetic, but the answers given are incorrect.

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28. A factor delivers 6 French crowns and 4 dollars for 53*s*. 6*d*.; and at another time, 4 French crowns and 6 dollars for 49*s*. 10*d*. What is the value of each?

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29. Suppose a dog, a wolf, and a lion were to devour a sheep, and that the dog could eat up the sheep in one hour, the wolf in $\frac{3}{4}$ of an hour, and the lion in $\frac{1}{2}$ an hour: now if the lion begins to eat $\frac{1}{8}$ of an hour before the other two, and afterwards all three eat together; in what time will the sheep be devoured?

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30. There were 25 cobblers, 20 taylors, 18 weavers, and 12 combers spent 133*s*. at a meeting; to which reckoning 5 cobblers paid as much as 4 taylors, 12 taylors paid as much as 9 weavers, and 6 weavers as much as 8 combers. How much did each company pay?

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31. How many changes may be made in the order of words of the following Hexameter verse, so that *mala* may always be the last word but two?

Mars, mors, sors, lis, vis, styx, pus, nox, fex,
mala, crux, fraus.

32. Suppose a cow to have a she-calf at the age of two years, and then to continue yearly to have the like; and every one of her young to have a she-calf at the age of two years, and afterwards every year likewise. How many would spring from the old cow and her brood in 20 years?

GEOMETRICAL PUZZLES.

1. Three-fourths of a cross, and a circle complete,
   Two semicircles a perpendicular meet;
   Then a triangle set upon two feet,
   Two semicircles, and then a circle complete.

The whole, when properly joined, will exhibit a plant possessing extraordinary virtues.

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2. How can the mouth of a square well be filled up with three square stones?

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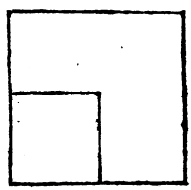
3. A gentleman purchases a piece of land, in the form of a square, and incloses one fourth part, as is represented in the annexed figure, to build a house and other conveniences upon.



He desires to have the remainder of the land divided into four equal and similar parts, to be appropriated to such uses as he may hereafter think suitable. The plan is required.

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4. There is a square piece of land containing 25 acres, designed for the reception of 24 poor men and their governor; each man is to have a

house situated within his own ground; the governor's in the centre. How many people's land must the governor pass through, before he can get to the outside of the whole?

5. A general had a small army of men,
Which form'd a long square of just twelve times ten,
But still, without having his number increas'd,
In twelve equal rows he would have them placed;
And just eleven men he would have in each row,
Himself equidistant from each row also;
Now how he must place them I gladly would know.

6. Mathematicians affirm that of all bodies contained under the same superficies, a sphere is the most capacious: But they have never considered the amazing capaciousness of a body, the name of which is now required, of which it may be truly affirmed, that supposing its greatest length 9 inches, greatest breadth 4 inches, and greatest depth 3 inches, yet under these dimensions it contains a solid foot?

7. How must a board that is 16 inches long and 9 inches broad, be cut into two such parts, as, when joined together, shall form a square?

8. A master joiner gives to one of his men a plank, 10 feet long and 2 feet wide, to be made into a square table; but the plank not to be cut into more than five pieces. The man, unskilled in Geometry, wishes to know how this must be done.

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9. A person wants to have two oval stools, of equal size, sawn out of a circular board, and to have holes made in them to take them up by. It is required to shew how this must be done, without any waste in the wood except what is occasioned by the saw.

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10. To form *one* square into *five* equal squares.

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11. Divide a circle into 4 equal parts by three lines of equal length.

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12. A ship at sea is found to leak on account of a square hole in the bottom, a side of which measures 6 inches; and the carpenter having only a rectangular board 9 inches long, and 4 inches wide, would be glad to be informed how he must cut it into two parts, so that, when properly joined, they may exactly fill up the hole.

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13. It is required to cut each of two equal squares into two such parts, that when the four parts are properly joined together, they shall make a square.

14. A carpet measures 20 feet by 10 feet, and is to cover a floor (of the same area) measuring 15 feet by 13 feet 4 inches. How must this be done, by cutting the carpet only into two pieces?

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15. Cut two squares, one of which is double the size of the other, into four such pieces, as, when properly joined, shall form three-fourths of a square.

Note. To have one square double the size of another, the side of the latter must be equal to the diagonal of the former.

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16. In a solid piece of wood let there be made three holes, one of them square, another circular, and the third triangular,* and let the side of the first, the diameter of the second, and the base and perpendicular (measured from the vertex to the middle of the base) of the last,† each measure the same. It is required to describe the form of *one* piece of wood, &c, that will exactly fill each of the *three* holes.

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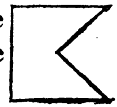
17. A lady has a square dressing table, each side of which is 30 inches, but wishes to have it enlarged by the application of 6 square feet of

* To state merely the forms of the three holes is not sufficient, their relative dimensions must likewise be given; as it will be found that with no other dimensions could the problem be possible. † The triangle must also be equal-legged.

plank to it, so that each side may be a foot longer. It is required to shew how this must be done.

18. Out of six equal squares, make *three-fourths* of a square similar to the annexed figure.



19. To draw a line that shall continually approach a given straight line; and if both should be ever so far produced (still continuing to approach) their nearest distance from each other shall become less than any assignable quantity whatever, but yet they shall manifestly never meet; which it is here required to shew.

20. * Cut out 15 pieces of pasteboard, &c. of the form of the annexed figures.

*Though the pieces in the key correspond in size with the above, yet it is not necessary for that to be the case: if they are considered too *small* to be cut out for real amusement, *larger* pieces may be made, observing that:

1. With regard to the triangles, they are all to be equal-legged, right-angled, and the legs of each equal to the hypothenuse of next last preceding.

2. The length of the rectangle must be equal to the hyp. of triangle 3, and the breadth equal the hyp. of triangle 1.

3. Fig. 6, breadth *ab* or *de* = leg of triangle 1; *af* = twice that leg, and *fe* = thrice ditto. It may also be ob-

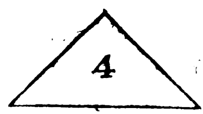
Fig.

1 marked 1 4 triangles of equal size

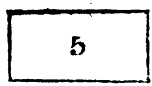
2 marked 2 1 triangle twice as large as fig. 1

3 marked 3 4 triangles each twice as large as fig. 2

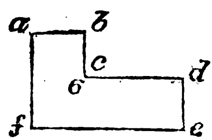
4 marked 4 1 triangle as large as fig. 3



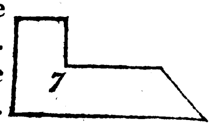
5 marked 5 1 rectangle as large as fig. 4



6 marked 6 2 hook-like pieces, each as large as fig. 5



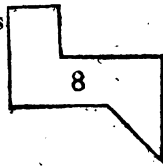
7 marked 7 1 hook-like piece as large as fig. 6, with the addition of fig. 1. at the side.



served, that all the pieces together consist of 72 triangles each of the size of triangle 1.

Fig.

8 marked 8 1 hook-like piece as large as fig. 7, with the addition of fig. 1 at the side.



By employing the whole of the above pieces, it is required to form a square.

---

21. With all the above pieces to form a right-angled triangle.

---

22. With all the above pieces to form a parallelogram.

---

23. With all the above pieces to form a rhomboid.

---

24. With all the above pieces to form a trapezoid.

---

25. With all the above pieces to form three right-angled triangles, one of which shall be equal in surface and form to the remaining two.

---

26. With all the above pieces to form three squares.

---

27. With all the above pieces to form a rectangle and a rhomboid, of the same base and altitude.

28. With one sweep of the compasses, and without altering the opening, or changing the centre, to describe an oval.

---

29. To make the same body pass through a square hole, a round hole, and an oval hole.

---

30 A has a field 500 poles in circumference, which is square; B has one of the same circumference, which is an oblong; and proposes to A an exchange. Ought the latter to accept the offer?

---

31. A green-grocer purchased, for a certain sum, as many heads of asparagus as could be contained in a string a foot in length; being desirous to purchase double that quantity, he returned next day to the market, with a string of twice the length, and offered double the price of the former quantity, for as many as it would contain. Was his offer reasonable?

---

32. To describe a circle, or any determinate arc of a circle, without knowing the centre, and without the compasses.

---

33. To find by a simple, accurate, and direct method, with the compasses *only*, the centre of a circle which shall pass through three given points.

34. It is required to divide a circle into any proposed number of parts, that shall be in any ratio to each other.

---

35. Whatever angle any two right lines can possibly form that meet with each other; a third line may nevertheless be drawn in such a manner as to be perpendicular to them both.

---

36. To make a triangle that shall have three right angles.

---

37. To find the linear expressions for the sq. r. of 2, 3, &c. to 10.

---

38. To inscribe a square in a given circle, by means of the compasses *only*, supposing the centre to be known.

---

39. To inscribe a dodecagon in a given circle, with the compasses *only*, supposing the centre to be known.

---

40. To find a *right line* nearly equal to the circumference of a given circle.

---

41. To find a *right line* nearly equal to $\frac{1}{4}$ of the circumference of a given circle, without *first* finding the *whole* circumference.

42. Find the centre of a given square by means of the compasses *only*.

---

43. Bisect a given straight line by means of the compasses *only*.

---

44. Inscribe a regular pentagon in a given circle, by means of the compasses *only*.

TREES PLANTED IN ROWS.

1. Your aid I want, nine trees to plant
In rows just half a score;
And let there be in each row three.
Solve this: I ask no more.

2. Fain would I plant a grove in rows,
But how must I its form compose
With three trees in each row;
To have as many rows as trees;
Now tell me, artists, if you please;
'Tis all I want to know.

3. Ingenious artists, if you please
To plant a grove, pray show,
In twenty-three rows with fifteen trees,
And three in every row.

4. It is required to plant 17 trees in 24 rows, and to have 3 trees in every row.

5. Ingenious artists, pray dispose
Twenty-four trees in twenty-four rows.
Three trees I'd have in every row;
A pond in midst I'd have also.
A plan thereof I fain would have,
And therefore your assistance crave.

6. Fam'd arborists, display your power,
And show how I may plant a bower
With verdant fir and yew:
Twelve trees of each I would dispose,
In only eight-and-twenty rows;
Four trees in each to view.

---

7. Plant 27 trees in 15 rows, 5 in a row.

---

8. Ingenious artists, if you please,
Now plant me five-and-twenty trees,
In twenty-eight rows, nor less, nor more;
In some rows five, some three, some four.

---

9. It is required to plant 90 trees in 10 rows, with 10 trees in each row; each tree equidistant from the other, also each row equidistant from a pond in the centre.

---

10. A gentleman has a quadrangular irregular piece of ground, in which he is desirous of planting a quincunx, in such a manner, that all the rows of trees, whether transversal or diagonal, shall be right lines. How must this be done?

Note. A real quincunx is a plantation of trees disposed in a square, consisting of five trees, one at each corner, and the fifth in the middle; but in the present case, the trees are to be disposed in a quadrangle, one at each corner (as in the square,) and the fifth at the point of intersection of the two diagonals.

GEOGRAPHICAL PARADOXES.

1. There are two remarkable places on the globe of the earth, in which there is only one day and one night throughout the whole year.

2. There are some places on the earth, in which it is neither day nor night, at a certain time of the year, for the space of 24 hours.

3. There is a certain place on the globe, of a considerably southern latitude, that has both the greatest and least degree of longitude.

4. There are three remarkable places on the globe, that differ in latitude, as well as in longitude; and yet, all of them lie under the same meridian.

5. There are three remarkable places on the continent of Europe, that lie under three different meridians; and yet, all agree in both latitude and longitude.

6. There are two remarkable places belonging to Asia, that lie under the same meridian, at a small distance from each other; and yet, their

respective inhabitants, in reckoning their time, differ an entire natural day every week.

---

7. There is a particular place on the earth, where the winds (though frequently veering round the compass) always blow from the north point.

---

8. There is a certain island in the Ægean sea, upon which, if two children were born at the same instant of time, and living together for several years, should both die on the same day, or even at the same hour and minute of the day; yet, the life of the one would surpass that of the other several months.

---

9. There is a certain hill in the south of Bohemia, on the top of which, if an equinoctial sun-dial be duly erected, a man that is completely blind, may know the hour of the day by the same, if the sun shines.

---

10. There are divers places on the continent of Africa, and the islands of Sumatra, and Borneo, where a certain kind of sun-dial being duly fixed, the gnomon will cast no shadow at certain periods during the year: and yet, the exact time of the day may be known thereby.

---

11. There is a certain island in the vast Atlantic ocean, which being descried by a ship at sea, and

bearing due east off the said ship, at twelve leagues distance by estimation; the true course for touching upon the said island, is to steer six leagues due east, and as many due west.

---

12. There are divers remarkable places upon the Terraqueous Globe, the sensible horizon of which is commonly fair and serene: and yet, it is impossible to distinguish properly in it, any of the intermediate points of the compass, or even so much as two of the four cardinals themselves.

---

13. There is a certain island in the Baltic Sea, to the inhabitants of which, the sun is visible in the morning before he rises, and in the evening after he sets.

---

14. There is a certain village in the kingdom of Naples, situated in a very low valley; and yet the sun is nearer to the inhabitants thereof, every noon by 3000 miles and upwards, than when he either rises or sets to those of the said village.

---

15. There is a certain village in the south of Great Britain, to the inhabitants of which, the sun is less visible, about the winter Solstice, than to those who reside upon the island of Iceland.

---

16. There is a remarkable place on the earth, of a considerably southern latitude, from the

meridian of which, the sun does not remove for several days at a certain time of the year.

---

17. There is a certain place on the earth, of a considerably northern latitude, where, though the days and nights (even when shortest) consist of several hours, yet, in that place it is mid-day or noon every quarter of an hour.

---

18. There is a very remarkable place upon the Terraqueous Globe, where all the planets, notwithstanding their different motions and various aspects, always bear upon one and the same point of the compass.

---

19. There is a certain noted part of the earth, where the sun and moon, although at the time of full moon, may both happen to rise at the same moment of time, and upon the same point of the compass.

---

20. There is a remarkable river on the continent of Europe, over which there is a bridge, of such a breadth, that more than 3000 men may pass abreast, along the same, without any inconvenience.

---

21. There is a certain city in the southern part of China, the inhabitants of which observe the same manner of walking as the Europeans;

and yet, they frequently appear to strangers as if they walked on their heads.

---

22. There are three places on the continent of Europe, each distant from the others 1000 miles, and yet, there is a fourth place so situated, in respect of the other three, that a man may travel on foot, from it to any one of the former places, in one Artificial Day, at a certain time of the year, and that without any hurry or fatigue.

---

23. There are three places on the continent of Europe, that lie under the same meridian, at such a distance, that the latitude of the third exceeds that of the second, as much as the latitude of the second exceeds that of the first; and yet, the true distance of the first and third, from the second or intermediate place, may not be the same by many miles.

---

24. There are two places on the continent of Europe, so situated, that although the former lies east of the latter, yet the latter is not west of the former.

---

25. There is a certain place in Great Britain, where the stars are always visible at any time of the day, provided the sky be not overcast with clouds.

---

26. It may be shown by the Terrestrial Globe,

that a person may sail in 24 hours from the river Thames in England, to the city of Messina in Sicily, at a certain time of the year; provided there be a brisk north wind, a light frigate, and an azimuth compass.

---

27. There are two places on the earth which bear exactly north and south of each other, and their distance is 100 miles, but the true course to one of these places from the other, is to sail 50 miles due north, and 50 miles due south.

---

28. There are two places situated in the Torrid Zone, that are not more than 60 miles asunder; but if a ship sail from one to the other, on one particular point of the compass, the difference of time between these places will be found to be more than 23 hours.

---

29. There are three remarkable places on the Terraqueous Globe, to the inhabitants of which all the stars are visible, on three certain nights of the year.

---

30. It may be demonstrated by the globe, that the Sun, Moon, and several of the Planets, do not move exactly 15 degrees hourly, from the meridian of several places on the continent of America, but come later to their meridians, than the preceding day.

31. There are certain places in north latitude, where the longest Artificial Day, is longer by some hours, than that, in the same degree of south latitude.

---

32. There is a certain island in Europe, for which, in four or five particular months of the year, if the ablest astronomers would calculate the Moon's rising, they would not only differ in minutes and seconds, but whole hours.

---

33. Two travellers arriving at a certain place on the globe, found, that, although the Sun was shining, their shadows disappeared; but, proceeding some miles farther, observed that their shadows circuited all points of the compass.

---

34. There are certain places on the earth, to which, if you bring a Mariner's Compass, ever so well touched and rectified, the Needle becomes useless, for it will turn indifferently to any point of the compass, though neither iron nor loadstone be near.

---

35. There is a certain place where the Planets may be seen constantly moving forward, in the same regular and uniform manner; though, to most places of the earth, they appear at the same time to be stationary, retrograde, or to move very unequally.

36. There are certain planets said to be in conjunction with the Sun, not only when they appear in the same degree of their orbit with the Sun, but when they are in that degree of their orbit, diametrically opposite to the former.

---

37. There are sundry places, which when coldest, are much hotter than any part of the Torrid Zone, and yet, not a burning mountain; and, there are other places, which, when hottest, are much colder than the Frigid Zone.

---

38. There is a certain wall in the city of Dublin, the situation of which is due east and west; yet, the Sun will shine on the north side thereof before six in the evening, all the summer, and on the south side, before six in the morning.

---

39. There are three distinct places on the earth, all differing both in longitude and latitude, and distant from each other 2000 miles; and yet, they all bear upon one and the same point of the compass.

---

40. To several parts of this globe, there are certain planets, which are so far from coming to an opposition, that they form neither Trine, Square, nor Sextile Aspect with the Sun.

---

41. There is a certain place in Great Britain, where, when the tide is in, the sheep may be

seen feeding on a certain neighbouring island; yet, when the tide is out, and the water at the lowest, not one can be seen, though they be feeding there at the same instant.

---

42. There are ten places on the earth, distant from one another 300 miles, and upwards; and yet, none of them have either latitude or longitude.

---

43. There is a certain island, situated between England and France; and yet, that island is farther from France than England is.

---

44. There are ten places, all under the same meridian, exactly ten miles from each other; and yet, their situation is such, that it is impossible for them all to be equidistant.

---

45 There are several places on the globe, where the Sun is frequently upon the same point of the compass, both in the forenoon and afternoon.

---

46. There is a certain place upon the globe of the earth, where, if 72 men were placed together in a small circle, with their faces outward, (the centre of which circle must necessarily be behind every one of them,) and, if every man was to travel straight forward, they would all go due north; and, if they were to continue

travelling, on that point of the compass, they would, after having gone about 5400 miles*, be placed in a certain circle sideways, 300 miles* distant from one another; the centre of which circle would be directly under the feet of every man; and, if they went about 5400 miles* more, they would all meet again in another circle, with their faces inward, the centre of which circle would then be before them; and this, without turning their bodies round about.

---

47 Christians the week's *first* day for sabbath hold,
The Jews the *seventh*, as they did of old,
The Turks the *sixth*, as we have oft been told.
Hew can these three, in the same place, and day,
Have each his own true sabbath; tell, I pray.

---

48. I have twelve times seen Bissextile;
Pray tell how that can be,
Since twelve times four make forty-eight,
And I'm but forty-three.

---

49. My wife and I did disagree,
Resolv'd at last to part were we:
She set off east, and I steer'd west,
(Believe me this is not a jest,)

* Geographical Miles.

When each had gone miles fifty-three,
She was not fifty yards from me;
And though it was such stormy weather,
We travell'd all that time together.
How can this be, pray clear the doubt,
And let us know the diff'rent rout.

---

50. I thrash'd ten quarters of fine wheat,
I ate just thirty pounds of meat;
Besides a calf that weigh'd eight stone,
I ate the whole, pick'd every bone:
Yet more—myself to satisfy,
Ate three roast pigs, which made me dry;
Drank sixteen pints of cherry brandy,
Then ate five pounds of sugar candy.
All this I did—all in one day:
'Tis true, I tell you, what I say.

---

51. One day, I saw the sun arise,
I'm sure I saw him set likewise:
But, wonderful—that day
I vouch again he rose; and then
Beneath th' horizon went: explain
How this could be, I pray.

---

52. There is a large and spacious plain, in a certain country in Asia, capable of containing 600,000 men, drawn up in battle array; but, if they were brought thither, and then drawn up, it would be impossible for more than one man to stand upright.

53. There is a certain European city, the houses of which are built with firm stone, and mostly high and strong; and yet, the walls are neither parallel to each other, nor perpendicular to the plane on which they stand.

54. It is a matter of fact, that three certain travellers went on a journey, in which their heads travelled full twelve yards more than their feet; and yet, they all returned alive with their heads on.

55. There is a certain famous church, in England, which is situated in such a remarkable manner, that when a man has travelled 80, 90, or 100 miles regularly, he shall be neither nearer to it, nor farther from it, than when he first set out.

56. There are several tracts of land, very near London, each of which is not above 20 feet over, measured any way; and yet, a horse may travel on either of them, very regularly, 30 miles per day. But their situation is such, that, whilst the right, or off legs of the horse, pass over 30 miles, his left, or near legs (next to which the driver generally goes,) will unavoidably pass over nearly 31 miles, the horse being all the time sound, and in just proportion.

57. There are thirteen places on the continent of Europe, the nearest distance of which being

just one mile, they are all equidistant, as you pass regularly from one to another; and yet, not more than two of them are in a right line.

---

58. A certain mount in Devonshire doth stand,
Whose lofty head o'erlooks the neighbouring land;
And such is the known property of th' hill,
That if a vessel you with liquor fill
At the hill top, or vertex of the place,
It will hold less than if filled at the base.
Ingenious youths! be pleased to tell to me,
What is the cause of this deficiency?

---

59. Two fav'rite fields near to my dwelling lie,
Their soil alike in depth and quality.
The furthest distant, twenty acres measures,
The nearest ten; but fraught with latent treasures:
For, till'd alike, this yields me as much grain
As does the first, though just as large again.

---

60. There is a certain fire, which affords the more heat the further it is from us, and the less heat the nearer it is to us.

A KEY

TO

THE COLLECTION OF

CURIOUS PUZZLES.

A KEY

TO

THE COLLECTION OF

CURIOUS PUZZLES.

A KEY TO THE

ARITHMETICAL PUZZLES.

1. MILLIO.

2. $99\frac{2}{9}$.

3. From $\left.\begin{matrix}\text{SIX}\\ \text{IX}\\ \text{XL}\end{matrix}\right\}$ take $\left\{\begin{matrix}\text{IX}\\ \text{X}\\ \text{L}\end{matrix}\right\}$ then $\left\{\begin{matrix}\text{S}\\ \text{I}\\ \text{X}\end{matrix}\right\}$ will remain

4. $\frac{2}{2\times 2}$, $2-\frac{2}{2}$, $\frac{2\times 2}{2}$, which denote $\frac{1}{2}$, 1, and 2 respectively, the common ratio being 2.

5. $11\frac{1}{1}$.

6. Suppose two widows, A and B, not akin, to be left each with a son, and that A's son marries B, and B's son marries A; and that A's son has a son by B. Where B's son is the person that leaves the money.

Then to his

				£	s	d.
Father,	who is the same as his	son,	is left	666	13	3
Mother,		daughter,		666	13	3
Grandson,		brother,		666	13	3
				£1999	19	9

7. The man must first take over the goose; then return, and take over the fox, and bring back the goose; then, leaving the goose, he must carry over the corn; after which he must return and take the goose over.

8. The number shown by the master is 29997, which is 9999×3. Now, if every figure, in each row of the master's, be made a complement to 9 of the scholars' row, it is evident that the sum will be equal to the number proposed.

Thus: suppose the scholars write down three rows, as follows:

	7285	The scholars' rows
	5829	
	3456	
Then we shall have	2714	The master's rows
	4170	
	6543	
Total	29997	The number proposed.

9. The greatest variety that can be in the number of hairs, is equal to the greatest number that any person has: viz. one person having but one, another two, another three, and so on to the greatest number; but as, by the supposition, there are still more persons, whatever number they may have, some one of the preceding must have the same. Hence the proposition is manifest.

10.

```
 987654321
 123456789
 ---------
 864197532
```

---

11. As there are in all $10\frac{1}{2}$ bottles of wine, and 21 bottles, each person must have $3\frac{1}{2}$ bottles of wine (i. e. 7 bottles half full) and 7 bottles.

		Bottles full.	Half full.	Empty.
Therefore, the	1st must have	3	1	3
	2nd	3	1	3
	3rd	1	5	1
Or thus, the	1st may have	2	3	2
	2nd	2	3	2
	3rd	3	1	3

---

12. Suppose each of the 8 loaves to be divided into three equal parts, making 24 (parts) and that each person eats an equal share or 8 parts. Then the stranger must eat 7 parts of the man's who contributed 5 loaves or 15 parts, and only

F 2

1 part of his who contributed 3 loaves or 9 parts. From whence it appears that the sentence was just.

---

13. First, two women must pass over; then one of them must bring back the boat, and repass with the third woman; then one of the three women must bring back the boat, and stay with her husband, whilst the other two men pass over to their wives; then one of these men with his wife must bring back the boat, and, she remaining, he must take over the other man. Lastly, the woman who is found with the three men, must return with the boat, and at twice take over the other two women.

---

14. Out of the 8 pint jar fill the 5, and out of the 5 fill the 3; then 3 pints are left in the 8, and 2 in the 5. Empty the 3 pint jar into the 8, and it will have 6 in it; then empty the 5 (which had two in it) into the 3. Lastly, fill the 5 out of the 8 (which had 6 in it) and 1 will be left in the 8, then out of the 5 fill the 3 (which had 2 in it) and the 5 will have 4 in it, as was required.

---

15. The half of twelve will seven be,
Cut through the middle as you see. XII.

---

16. Place the first 30 numbers, viz. 1, 2, 3, &c. (in their natural order) in a ring or row; and,

beginning to count at 1, make a mark, by way of distinction, over every *9th* number, (passing over those already marked) till there be 15 marks, which will denote the places where the Turks are to stand; then it will be found that the

1st	is to be over the	9th man		
2nd	9+9 =	18th		
3rd	18+9	27		
*4th	(27+9)—30	6		
5th	6+9+1	16,	passing over 1,	viz. the 9th
6th	16+9+1	26,	1,	ditto 18th
7th	(26+9+2)—30	7,	2,	27th, 6th,
8th	7+9+3	19,	2	&c. &c.
9th	19+9+2	30,	2	
10th	9+3	12,	3	
11th	12+9+3	24,	3	
12th	(24+9+5)—30	8,	5	
13th	8+9+5	22,	5	
14th	(22+9+4)—30	5,	4	
15th	5+9+9	23,	9	

that is, in the natural order of numbers, the 5th, 6th, 7th, 8th, 9th, 12th, 16th, 18th, 19th, 22nd, 23rd, 24th, 26th, 27th, 30th must be Turks.

From which it appears, that, if C denote the Christians, and T the Turks, they ought to be placed as under:

CCCCTTTTTCCTCCCTCTTCCTTTCTTCCT.

* It must be evident that the excess above 30 will denote the place of the 4th man, as likewise of the 7th, 12th, and 14th; also, that as many as have been previously reckoned, so many more than 9 must be reckoned the next time.

☞ Suppose the five vowels, a, e, i, o, and u, to denote the numbers 1, 2, 3, 4, and 5, respectively, then it will be found that the vowels in either of the following verses will denote the number of each that must be placed together, beginning with the Christians.

1. From num-bers, aid, and art,
   Ne-ver will fame de-part.

2. In the Latin Hexameter:
Po-pu-le-am vir-gam ma-ter re-gi-na fe-re-bat.

---

17. $5{\cdot}5 + \frac{5}{5}$.

---

18. $2 + 4 + 6 + {\cdot}8 = 12{\cdot}8$.
$1 + 3 + 7 + \frac{9}{5} = 12{\cdot}8$.

---

19. $\frac{2}{8} + \frac{1}{4} + {\cdot}5 + {\cdot}3 + {\cdot}7 + 6 + 9 = 17$
$\frac{4}{8} + \frac{1}{2} + 3 + 5 + 6 + 7 + 9 = 31$.

---

20. Two-thirds of SIX = IX or 9,
One-fifth of SEVEN = V or 5.
and $9 \times 5 = 45$, the number required.

---

21. One gentleman must pass over the river with his servant, and the former must return with the boat; then two servants must pass over, and one of them return with the boat. Next, the two gentlemen must pass over, and one of them must return with a servant. Lastly, the two gentlemen must pass over, and the servant who first crossed must return with the boat, and, at twice, take over the other two servants.

22. First Arrangement.

6	6	6
6		6
6	6	6

Second Arrangement.

4	10	4
10		10
4	10	4

Third Arrangement.

8	2	8
2		2
8	2	8

---

23. $1 \cdot 25 \times 4 = 5.$

---

24. One-third of TWELVE is LV or 55,
One-fifth of SEVEN is V or 5;
and 55, divided by 5, gives 11.

---

25. The four figures are 8888, which being divided by a line drawn through the middle, become $\frac{0\,0\,0\,0}{0\,0\,0\,0}$, the sum of which is eight 0's, or nothing.

---

26. A moved 200 yards backward with his face towards B's, and B moved 200 yards forward with his face towards A's.

---

27. Invert the figure 6, and it will become 9, and write the figure 3 in the ten's place, and it will become 30; to which add the remaining 7 digits, viz, 1, 2, 4, 5, 7, 8 and 9, and the sum will be 75; from which take 50, and there remains 25, which is $\frac{1}{3}$ of 75.

28. Suppose the following sets of figures are given to choose from: viz,

364851
234765
823644

which are all divisible by 9 without a remainder, though the person must not be told so; and suppose he chooses the third line, and multiplies it by 6, the product will likewise be divisible by 9 without a remainder. Now supposing he suppresses the 6, then the sum of the remaining digits is 30, which contains three 9's and 3 over; which 3, wants 6 to make up 9, and therefore 6 was the figure suppressed.

When the sum of the digits, remaining after the suppression of a figure, is divisible by 9 without a remainder, then the figure suppressed must be either 9 or 0.

---

29. First Arrangement. Second Arrangement.

	4	1	4		3	3	3
20 Persons	1		1	24 Persons	3		3
	4	1	4		3	3	3

Third Arrangement. Fourth Arrangement.

	2	5	2		1	7	1
28 Persons	5		5	32 Persons	7		7
	2	5	2		1	7	1

---

30. If the given figures be turned upside down, they will become 990, the third part of which, viz: 330, is half of 660.

31. One-half of FIVE is IV; take away I, and V or 5 remain.

---

32. First Arrangement.

1	7	1
7		7
1	7	1

Second Arrangement.

2	5	2
5		5
2	5	2

Third Arrangement.

3	3	3
3		3
3	3	3

Fourth Arrangement.

4	1	4
1		1
4	1	4

---

33. $\frac{2}{2^{2+2}}$, $\frac{2-\frac{2}{2}}{2}$, $\frac{2}{2-\frac{2}{2}}$, $\frac{2^{2+2}}{2}$,

denoting $\frac{1}{8}$, $\frac{1}{2}$, 2 and 8 respectively, the common ratio being 4.

---

34. The person was born in a Bissextile year, on the 29th of February.

---

35. Half of IX, when cut through the middle, is IV; and half of FIVE is also IV; and the quotient resulting from dividing the former by the latter is evidently 1.

---

36. In the first case, suppose that each cell contained 3 nuns, then there would be 9 in each row, and 24 in all; and the arrangement would be as is represented opposite.

3	3	3
3		3
3	3	3

In the second case, when the 4 men came in, suppose that 1 of them went into each corner cell, and that 2 nuns removed from thence to each of the middle cells; if so, each corner cell would contain 1 person fewer than before, and each middle cell 2 more, and the arrangement would be as is represented opposite.

2	5	2
5		5
2	5	2

In the last case, when 4 men were gone, and 4 nuns with them, each corner cell would contain 1 nun more than in the first case, and each middle cell 2 fewer, and the arrangement would be as is represented opposite.

4	1	4
1		1
4	1	4

---

37. Put down the series of odd numbers from 1 to 100, thus: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 85, 87, 89, 91, 93, 95, 97, 99.

1. Count all the terms of the series from the number 3 by threes, rejecting* every third number following, and thus all the multiples† of 3 will be exterminated.

*The numbers rejected, may, by way of distinction, have a dot placed over them.

†A multiple of any number is the product of that number by some other whole number.

2. The next unrejected number after 5 being 7, reject it, and its square 49, and every 7th figure following (if not rejected before,) and then all the multiples of 7 will be exterminated.

In the present case, it will be found, that all the multiples are rejected, and that those numbers which have not dots over them, are all the prime numbers, except the number 2, from 1 to 100, viz: 1, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

From the above, it appears, that if we continue the expunctions, till the square of the first uncancelled number, next to that whose multiples have last been rejected, is greater than the last and greatest number to which the series is extended; the numbers which then remain unrejected, are all the primes, except the number 2, which occur in the natural progression of numbers from 1 to the limit of the series.

This operation, Dr. Horsley imagines to be the one performed by "*The Sieve of* ERASTOSTHENES."

38.

$$\begin{array}{r} 123456789 \\ 8 \\ \hline 987654312 \end{array}$$

39. The arrangement is so simple, that it seems unnecessary to give the solution; the

question would be more difficult if one-fifth or one-half of the men were to be selected.

---

40. $\frac{3}{3 \times 3}$, 3^{3-3}, $\frac{3 \times 3}{3}$ denoting $\frac{1}{3}$, 1 and 3 respectively.

---

41. If the nine digits be placed in their natural order, viz: 123456789; and then be multiplied separately by 8, 7, 5, 4, and 2, the respective products will still consist of the nine digits, but not in the natural order.

---

42. $\frac{3}{3^{3+3}}$, $\frac{3}{3^3 \times 3}$, $\frac{3 \times 3}{3^3}$, $\frac{3^3}{3 \times 3}$, $\frac{3^3 \times 3}{3}$, $3^3 \times 3 \times 3$ denoting $\frac{1}{243}$, $\frac{1}{27}$, $\frac{1}{3}$, 3, 27, and 243 respectively, the common ratio being 9.

---

43. $\frac{3}{3^3 \times 3^3}$, $\frac{3}{3 \times 3 \times 3 \times 3}$, $\frac{3 \times 3}{3 \times 3 \times 3}$, $\frac{3 \times 3 \times 3}{3 \times 3}$, $\frac{3 \times 3 \times 3 \times 3}{3}$, $\frac{3^3 \times 3^3}{3}$, denoting $\frac{1}{243}$, $\frac{1}{27}$, $\frac{1}{3}$, 3, 27, and 243 respectively, the common ratio being 9.

44.

$$77\,\frac{77}{77}.$$

---

45. The number is 987654321,
which multiplied by 8, produces 7901234568

8	7901234568
7	6913580247
5	4938271605
4	3950617284
2	1975308642

46.

Multiply two of these.	Given two of these to be multiplied.	Add two of these.
1	9	40
2	8	30
3	7	20
4	6	10
5	5	

If it were required to find the product of 9 times 8:

	In the middle.	To the right.	To the left.	Total.
Against	9 & 8 are	40 & 30 = 70;	and 1 & 2 = 2;	72.
Of 8 times 6:	8 & 6 are	30 & 10 = 40;	and 2 & 4 = 8;	48,

and so on.

To find the square of any number in the middle row, as of 9.

Against 9	in the middle stands	40 on the right and	1 on the left	
9		40	1	
		80	1	Total 81.

To find the square of 7.

Against 7	stand	20 and 3		
7		20	3	
		40	9;	Total 49.

and so on.

---

47. The whole artifice consists in making choice of the numbers 1, 12, 23, 34, &c.; a series in Arithmetical Progression, the first term of which is 1, the common difference 11, and the last term 100.

Suppose the first person makes choice of 1; then his adversary, as he must count less than 11, can at most but reach 11, by adding 10 to it. The first will then take 1, which makes 12; and whatever number the second may add, the *first* will always win, provided he continually add the number which forms the complement of that of his adversary to 11; i. e. if the latter say, and 8 make so much, he must add 3.

By this method he will infallibly attain to 89; and it will be impossible for the second to prevent him from getting first to 100. For, whatever number the second takes, he can attain at most to only 99; after which, the first may say, and 1 makes 100. If the second took 1 after 89, it would make 90; and his adversary would finish by saying, and 10 make 100.

---

48. From a number consisting of as many nines as there are digits in the smallest number, sub-

tract the smallest. Let another person add that difference to the largest number; and then taking away the first figure of the amount, let him add it to the last figure, and that sum will be the difference of the two numbers.

For example: Matthew, who is 22, tells Henry, who is older, that he can discover the difference of their ages; he therefore privately deducts 22 from 99, and the difference which is 77, he tells Henry to add to his age, and to take away the first figure from the amount, and add it to the last figure, and that last sum will be the difference of their ages; as thus:

The difference between Matthew's age and 99 is	77
To which Henry adding his age .	35
The sum is	112
Take away the first figure, and this becomes	12
Add the first figure . .	1
The sum is	13
The difference of their ages; to which add Matthew's age . . .	22
The sum is Henry's age . . .	35

---

49. The number is 123456789, which

divided by	2 gives	61728394·5
	5	24691357·8
The sum of which is		86419752·3

All which numbers contain just the nine digits.

50. The numbers offered must be divisible by 9 without a remainder; and when any two of them are added together, there must be no cipher in the amount: and the figures of that amount must make either 9 or 18. Such are the numbers following; 36, 63, 81, 117, 126, 162, 207, 226, 252, 261, 306, 315, 360, and 432.

These numbers may be written on pasteboard, &c.; and when any two of them are added together, if a figure be struck out of the sum, it will be that which would make the other figures either 9 or 18. For example:

If a person choose 126 and 252, their sum will be 378, from which, suppose he strikes out the 7, then the remaining figures 3 and 8 make 11, which to become 18 must have 7 added to it.

---

51. As there are in all 12 casks of wine, and 24 casks, it is evident that each person must have 4 casks of wine, (i. e. 8 half-casks) and 8 casks.

	casks full.	half full.	empty.
Therefore, the 1st must have	3	2	3
2nd	3	2	3
3rd	2	4	2
Or thus: the 1st	2	4	2
2nd	2	4	2
3rd	4	0	4
Or thus: the 1st	1	6	1
2nd	3	2	3
3rd	4	0	4

52. Out of the 12 pint bottle fill the 5 pint bottle, and 7 pints will remain in the former; then empty the 5 into the 7. Again, out of the 12 (which had 7 in it) fill the 5, and 2 will remain in the 12; then out of the 5 fill the 7 (which had 5 in it) and 3 will be left in the 5. Empty the 7 into the 12 (which had 2 in it) and there will be 9 in it; then empty the 5 (which had 3 in it) into the 7. Out of the 12 (which had 9 in it) fill the 5, then 4 will remain in it; and from the 5 fill the 7, (which had 3 in it) and 1 will be left in the 5. Empty the 7 into the 12, (which had 4 in it) and there will be 11 in it; also empty the 5 (which had 1 in it) into the 7. Lastly, out of the 12 (which had 11 in it) fill the 5, and there will 6 remain in it; and out of the 5 fill the 7 (which had 1 in it) and it will also contain 6, as was required.

A KEY TO THE

MAGIC SQUARES.

I. Three Rows.

1.

2	9	4
7	5	3
6	1	8

Or thus: placing the vertical rows laterally.

2	7	6
9	5	1
4	3	8

2.

9	14	13
16	12	8
11	10	15

This and the following ones may be varied as above.

3.

6	14	16
22	12	2
8	10	18

4.

17	22	21
24	20	16
19	18	23

5.

128	1	32
4	16	64
8	256	2

II. Four Rows.

6.

1	10	7	16
15	8	9	2
14	5	12	3
4	11	6	13

Or thus:

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

7.

1	18	17	4
14	7	8	11
9	12	13	6
16	3	2	19

8.

6	23	22	9
19	12	13	16
14	17	18	11
21	8	7	24

9.

8	22	21	11
19	13	14	16
15	17	18	12
20	10	9	23

10.

3	31	29	9
25	13	15	19
17	21	23	11
27	7	5	33

Which make 72 every way.

A KEY

TO THE CURIOUS

ARITHMETICAL QUESTIONS

Requiring regular Calculation.

1. $\frac{1}{3}$ of 2*d.* = $\frac{2}{3}$ of 1*d.* = $\frac{2}{9}$ of 3*d.*

---

2. As 7 : $\frac{5}{2}$ :: 11 : $\frac{55}{14}$ of 1; which = $\frac{55}{126}$ of 9.

---

3. Suppose the expense and the profit to be the same, and each = 2; then double the expense = 4, and half the profit = 1, and therefore the difference is 3, and the ratio of the former to the latter is as 4 to 1.

---

4. First, find what A and B together, A and C together, and B and C together can severally earn in 1 day.
Thus: A and B = $\frac{40}{6}$ = 6*s.* 8*d.*; A and C = $\frac{54}{9}$ = 6*s.*; and B and C = $\frac{80}{15}$ = 5*s.* 4*d.*; then the sum of the two first

		s.	*d.*	
gives 2 A + B + C	=	12	8	
subtract B + C	=	5	4	
there remains 2 A	=	7	4	
then A	=	3	8	
Hence A can earn		3	8	per day.
B - - - -		3	0	
C - - - -		2	4	

5. Since he gained 4 feet in height every day and night, he will have advanced 12 feet in 3 days; and therefore in 4 days he will reach the top.

---

6. First, A is to have 129 } more than B.
C - - - 178 }
Sum, 307

Then $\frac{1000-307}{3} = 231 =$ B's } share.
$231+129 = 360 =$ A's
and $231+178 = 409 =$ C's

---

7. First, B is to have 72 } more than A.
C $72+112 = 184$ }
Sum, 256

Then $\frac{1500-256}{3} = 414\frac{2}{3} =$ A's } share.
$414\frac{2}{3}+72 = 486\frac{2}{3} =$ B's
and $486\frac{2}{3}+112 = 598\frac{2}{3} =$ C's

---

8. The true weight is a mean proportional between the two false ones, and is found by extracting the square root of their product.

Thus $16\times9=144$; and sq.r. $144=12$ *lbs.* the weight required.

---

9. First 6*s.* $\frac{1}{4}$*d.* = 289 farthings; and this must be equal to the number of persons multiplied into the sum spent by each. In the present case, the

multiplicand and the multiplier are equal, and therefore we have only to find what number multiplied into itself, will produce the given sum, 289; or, in other words, to find the square root of 289; this = 17, the number of persons. Whence 17 farthings or $4\frac{1}{4}d.$ is the money spent by each.

---

10. It is evident that at the last gate he had
2 oranges;
and therefore at the sixth - 4
at the fifth - 8
at the fourth - 16
at the third - 32
at the second - 64
at the first - 128
which is the number required.

---

11. In 1 hour A can count $\frac{1}{5}$ } of the sum
Ditto B $\frac{1}{11}$ }

Ditto A and B together $\frac{1}{5}+\frac{1}{11}=\frac{16}{55}$, therefore in 7 days they will count $\frac{16\times 7}{55}=\frac{112}{55}=2\frac{2}{55}$ sums; and $2\frac{2}{55}-1=1\frac{2}{55}$, the answer.

---

12. Widow's part 1
4 Daughters' 8
5 Sons' - 30

Sum, 39 : £8000 : : 1 : £205 : 2s. : $6\frac{10}{13}d.$

the widow's share; this doubled, gives £410:5:1$\frac{7}{13}$ for each daughter's share; and this trebled, gives £1230:15:4$\frac{8}{13}$ for each son's share.

---

13. It is evident that before he gave to the last person, he had 8*s*.; and this 8*s*. was 4*s*. less than what he gave the second person; he therefore gave him 12*s*.; and consequently before this he had 20*s*., and this was 2*s*. less than what he gave to the first person; he therefore gave the first 22*s*.; consequently $20+22=42$, the number of shillings he had at first.

---

14. They each sold their eggs as long as they could in pennyworths, at 7 for a penny, and the remaining ones at 3*d*. per egg; thus:

	d.	*d.*
the eldest sold 49 at 7 per penny, and had	7	10
1 at 3*d*. - - - - - -	3	
the second sold 28 at 7 per penny, and had	4	10
2 at 3*d*. - - - - - -	6	
the youngest sold 7 at 7 per penny, and had	1	10
3 at 3*d*. - - - - - -	9	

---

15. The least number that can be divided by 2, 3, and 4 respectively, without a remainder, is 12; and, that there may be 1 remaining, the number must be 13; but this is not divisible by 5 without a remainder. The next greater number is 24, to which add 1, and it becomes 25; this is divisible by 5 without a remainder, and is therefore the number required.

16. Since C's share falls in, we have only to divide the original number of acres between A and B, in the proportion of $\frac{1}{3}$ to $\frac{1}{4}$ or of 4 to 3;

Thus, $4+3=7 : \} 1000 \left\{ \begin{array}{l} 4 : 571\frac{3}{7} \text{ A's share.} \\ 3 : 428\frac{4}{7} \text{ B's share.} \end{array} \right.$

---

17. First, 2*s*. $2\frac{1}{4}d$ = 105 farthings, which will divide by 1, 3, 5, 7, 15, 21 and 35, without any remainder. So that

lb.		farthings.			
105	at	1			per lb. all answer the conditions of the question.
35	at	3			
21	at	5	or	$1\frac{1}{4}d.$	
15	at	7	or	$1\frac{3}{4}d.$	
7	at	15	or	$3\frac{3}{4}d.$	
5	at	21	or	$5\frac{1}{4}d.$	
3	at	35	or	$8\frac{3}{4}d.$	
1	at	105	or	2*s*. $2\frac{1}{4}d.$	

---

18. It is evident that he went into the 4th shop with 1 guinea; whence, with $1\frac{1}{2} \times 2 = 3$ guineas at the third shop; and with $3\frac{1}{2} \times 2 = 7$ guineas at the 2nd shop; therefore $7\frac{1}{2} \times 2 = 15$, the guineas he had at first.

---

19. From the 12 remaining, deduct 1, and 11 is the number she sold the last boy, which was half she had; her number at that time therefore was 22. From 22 deduct 2, and the remaining 20 was $\frac{2}{3}$ of her prior stock, which was therefore 30.

From 30 deduct 10, and the remainder 20 is half her original stock; consequently she had at first 40 apples.

---

20. A multiple of 9 is a multiple of 3
8 - - 4 and 2
8 and 9, or 72 ... 6.

Therefore $5 \times 7 \times 8 \times 9 = 2520$, is the number required.

---

21. On the first view of the question, there does not appear to be any loss; for, if it be supposed that, in selling 5 apples for 2*d.* she gave 3 of the latter sort (viz. those at 3 a penny) and 2 of the former (viz. those at 2 a penny) she would receive just the same money as she bought them for; but this will not hold throughout the whole, for (admitting that she sells them as above) it must be evident that the latter stock would be exhausted *first*, and consequently, she must sell as many of the former as remained overplus, at 5 for 2*d.* which she bought at the rate of 2 a penny, or 4 for 2*d.* and would therefore lose.

It will be readily found, that, when she had sold all the latter sort (in the above manner) she would have sold only 80 of the former, for there are as many 3's in 120, as 2's in 80; then the remaining 40 must be sold at 5 for 2*d.* which were bought at the rate of 4 for 2*d.*

A. d. A. d.
i. e. If 4 : 2 : : 40 : 20, prime cost of 40 of the first sort.
5 : 2 : : 40 : 16, selling price of ditto.

4*d.* loss.

22. The least number that will answer this question is 12; for if we suppose that each Grace gave one to each Muse, the latter would each have 3, and there would remain 3 for each Grace.

Note. Any multiple of 12 will answer the conditions of the question.

23. It would appear on the first view, that this problem is impossible; as it cannot be supposed that half an egg can be sold without breaking any. But it must be considered, that by taking the greater half of an odd number, we take the exact half + ½. It will be found, therefore, that the woman, before she passed the last guard, had 73 eggs remaining; for, by selling 37 of them at that guard, which is the half + ½, she would have 36 remaining. In like manner, before she came to the second guard she had 147; and before she came to the first 295.

24. In order to solve this problem, we have only to find a series of numbers beginning with 1; which added to, or subtracted from, each other, in every way possible, shall form all the numbers, from unity to the number proposed.

It may be solved two ways:

1. By addition alone. Here the weights may be expressed by a series of numbers in *double* progression.

For example, with weights of 1, 2, 4, 8, and 16*lbs.* any number of lbs. from 1 to 31 may be weighed.

For with 2 and 1, we may weigh		3lbs.
4 and 1	- - -	5
4 and 2	- - -	6
4, 2, and 1	- -	7
&c.		&c.

2. By addition combined with subtraction. Here the weights may be expressed by a series of numbers in *triple* progression.

For example: with weights of 1, 3, 9, 27, and 81*lbs.* all weights from 1 to 121 may be weighed: For with the two first, by putting 3*lbs.* in one scale and 1*lb.* in the other, 2*lbs.* are weighed; by putting both in the same scale, 4*lbs.* are weighed; by putting 9*lbs.* in on side, and three in the other, 6*lbs.* are weighed, and so on.

From this it appears, that it is not necessary to employ so many weights as in the first case; and it will be found that in no other progression are so few weights required; and on the other hand, it may be inferred, that, in order to weigh the *greatest* possible number of lbs. from one to any given number, the weights 1*lb.* 3*lbs.* 9*lbs.* &c. in triple progression must be employed.

It must be acknowledged that the former method is not so simple as the latter, but it is given in order to form the comparison, and thence to show the superiority of the latter.

---

25. Since the labour is supposed to increase in A. P. as the depth, the price must increase in A. P. also. Hence, we have to divide £20 or 400 shillings into 20 terms in A. P. and then to find the sum of the first 8 of them, which will be what is due to the bricklayer. But this will depend upon the first term, or price received for the first yard. Therefore, in order to give a definite solution, suppose some fixed price for the first yard, for example 5*s.*: Now, we have the first term, the number of terms, and the sum of all the series in an A. P. to find the common difference: this, by the rule given in *Vyse's Arithmetic (Arithmetical Progression, prop. 8) is $1\frac{11}{19}$; and by prop. 7 of the same, the sum of the first 8 terms of this progression is $84\frac{2}{9}s$, or £4 4*s.* $2\frac{1}{2}\frac{2}{19}d.$ which is the sum required.

---

26. If 60*q.* : £2 8*s.* : : 1*q.* : $9\frac{3}{5}d.$ the mean price per quarter.

$$9\tfrac{3}{5}\begin{cases}12 & 3\tfrac{3}{5}\\ 6 & 2\tfrac{2}{5}\end{cases}\qquad \begin{array}{c}\\ \overline{6}\end{array}$$

Then 6 : 60 : : $3\frac{3}{5}$: 36 *q.* of wheat, and 60 — 36 = 24*q.* of barley.

* That author is particularly referred to, as he has

27. Their gains being evidently as 2, 3, and 4, we have

$$\text{As } 2+3+4=9 : 420 :: \begin{cases} 2 & \frac{280}{3} = 93:6:8, \text{ X's gain in } 4 \\ 3 & 140:0:0, \text{ Y's } \ldots 6 \\ 4 & \frac{560}{3} = 186:13:4 \text{ Z's } \ldots 9 \end{cases}$$

(£ s. d. — Months)

Then find each of their gains for the same time,

Thus: $\frac{280}{3} \div 4 = \frac{70}{3} = \frac{630}{27}$, X's gain in 1 month

$140 \div 6 = \frac{70}{3} = \frac{630}{27}$, Y's

$\frac{560}{3} \div 9 = \frac{560}{27}$, Z's.

Which numbers are as 630, 630, and 560, or (dividing each by 70,) as 9, 9, and 8.

Whence

$$9+9+8=26 : 4262 :: \begin{cases} 9 : 1475\frac{4}{13}, \text{ X's Stock.} \\ 9 : 1475\frac{4}{13}, \text{ Y's} \\ 8 : 1311\frac{5}{13}, \text{ Z's.} \end{cases}$$

---

28. Add the two sums together, and we shall have 10 crowns and 10 dollars = 103*s*. 4*d*.; hence 1 crown and 1 dollar = 10*s*. 4*d*. Multiply this by 6, and 6 crowns and 6 dollars = 62*s*.; from which take 6 crowns and 4 dollars, and there remain 2 dollars = 8*s*. 6*d*.; hence 1 dollar = 4*s*. 3*d*. and consequently 1 crown = 10*s*. 4*d*. — 4*s*. 3*d*. = 6*s*. 1*d*.

---

29. First, if in ½ an hour, the lion could eat

given the former proposition in question, whereas some writers on Arithmetic have omitted it.

the sheep; in $\frac{1}{8}$ of an hour, he would eat $\frac{1}{4}$, and consequently $\frac{3}{4}$ would be left. Next, find what part each of them could eat in 1 hour: thus

$$\text{If } \overset{h.}{\tfrac{1}{2}} : \overset{s.}{1} :: \overset{h.}{1} : \overset{s.}{2} \text{ the lion could eat in hour}$$
$$\tfrac{3}{4} : 1 :: 1 : 1\tfrac{1}{3} \text{ the wolf}$$
$$1 : 1 :: 1 : 1 \text{ the dog.}$$

The sum is $4\frac{1}{3}$ or $\frac{13}{3}$; then if all together could eat $\frac{13}{3}$ in one hour, find in what time they could eat the remaining $\frac{3}{4}$: thus, $\overset{s.}{\frac{13}{3}} : \overset{h.}{1} :: \overset{s.}{\frac{3}{4}} : \overset{h.}{\frac{9}{52}}$; to which add $\frac{1}{8}$ h (the time the lion ate before they all began) and the sum is $\frac{31}{104} = \overset{h}{}\ \overset{m}{17}\ \overset{sec.}{53\frac{1}{13}}$ the time required.

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30. By the question 5 cobblers, 4 taylors, 3 weavers, and 4 combers all paid alike, therefore

1 Cobbler paid	$\frac{1}{5}$	part, and consequently	25	paid	5
1 Tailor	- $\frac{1}{4}$	- - -	20	-	5
1 Weaver	- $\frac{1}{3}$	- - -	18	-	6
1 Comber	- $\frac{1}{4}$	- - -	12	-	3
				Total	19

(Parts of the Reckoning.)

$$\text{Then } \overset{Parts.}{19} : \overset{s.}{133} : \overset{Parts.}{5} : \overset{s.}{35}, \text{ the cobblers paid}$$
$$5 : 35, \text{ the tailors}$$
$$6 : 42, \text{ the weavers}$$
$$3 : 21, \text{ the combers.}$$

---

31. As the word *mala* is still to retain the same place, and there are 12 words in the whole, we have to change the situations of 11 of them in every way possible; which, by the rules of *permutation*, may be done

$1\times2\times3\times4\times5\times6\times7\times8\times9\times10\times11=39916800$ different ways.

---

32. From the nature of the question, it appears that the increase in the first year will be 0, in the second 1, in the third 1, in the fourth 2, in the fifth 3, in the sixth 5, and so on to 20 years or terms (each being = the sum of the two next preceding ones;) whence the two last terms are 2584 and 4181, and the sum of them all (or the whole series) $=2\times4181+2584-1=10945$, the increase required.

A KEY TO THE

GEOMETRICAL PUZZLES.

1. TOBACCO.

2. Let *a b c d* (*plate* I. *fig.* I.) represent the well; then the figures marked 1, 1, 1, 1, and 2, 2, 2, 2, of the form of a frame, are the two first square stones; and the square marked 3, in the middle, is the third square stone.

3. See *plate* I. *fig.* 2.

4. Two: for the ground being square, it will consist of 5 rows (each 5 acres) as in *plate* I. *fig.* 3.

5. The men must be placed in the periphery of a regular dodecagon, 9 along each side, and 1 at each corner; and the general must be in the centre.

6. A shoe.

7. See *plate* I. *fig.* 4, *No.* 1. Divide the length into four equal parts, and the breadth into 3, and draw the lines, joining the points of divi-

sion, as in the figure; then the dark line *a b c d e f* is the direction in which the board must be cut; and by transferring *a* to the place of *c*, as in *fig.* 4, *No.* 2, the required square will immediately be formed.

---

8. Let *plate* I. *fig.* 5, *No.* 1, represent the plank. Cut it into four equal triangular pieces marked *a*, *b*, *c*, *d*, the bases of each measuring 4 feet, and the height 2 feet; leaving a piece marked *e* in the middle, 2 feet square. Then by applying these pieces, as is represented by *fig.* 5, *No.* 2, the table is formed as was required.

---

9. Let *a b c d*, (*plate* I. *fig.* 6, *No.* 1.) represent the board, *e* being the centre. Draw any two diameters *a c* and *b d* perpendicular to each other. With the same centre *e*, and radius half that of the board, describe the circle marked 5, 6, 7, 8, (the outer circle being marked 1, 2, 3, 4). Then cut the board in the direction of the lines described upon it, and apply the respective pieces, as exhibited in *fig.* 6, *No.* 2 and 3, and then two ovals will be formed, with holes in the middle, as was required.

---

10. Let A B C D (*plate* 1, *fig.* 7, *No.* 1) be the square.

Draw lines from	A	to the middle of	BD
	B	- -	CD
	C	- -	AB
	D	- -	AC

then the square will be divided into 4 equal triangles, marked *a, a, a, a*, and 4 equal trapeziums, marked *b, b, b, b*, and the square *c* in the middle. Apply *a* to *b*, as is represented in *fig*. 7, *No*. 2, and 4 squares will be formed, which, with the middle square *c*, will present 5 equal squares as was required.

---

11. Draw A B a diameter of the given circle, (*plate* I, *fig*. 8) and divide it into 4 equal parts, at the points *a, b, c*. On Aa, Ab, and Ac, as diameters, describe semicircles on one side of the diameter AB; and on Bc, Bb, and Ba, describe semicircles on the other side of the diameter.

Then the corresponding semicircles joining, as is represented in the figure, form the lines A a B, A b B, and A c B, which will divide the diameter and area of the circle as was required.

If the diameter and area were to be divided into any other number of equal parts; first divide the diameter into the required number, and then proceed as above.

---

12. See *plate* I, *fig*. 9. Divide the length into 3 equal parts, and the breadth into 2, and draw the lines joining the points of division, as in the figure; then the dark line ***a b c d*** is the direction in which the board must be cut; and, by transferring ***a*** to the place of *c*, a square board will immediately be formed of the size required.

---

13. Cut each of the squares through the two opposite corners, and then their diagonals will form the sides of the required square; but the

application of the pieces is too simple to require elucidation.

---

14. Let *plate* I. *fig*. 10, *No*. 1, represent the carpet; divide the length into 4 equal parts, and the breadth into 3, and draw the lines joining the points of division as in the figure; then the dark line *a b c d e f* is the direction in which the carpet must be cut; and, by transferring *a* to the place of *c*, as in *fig*. 10, *No*. 2, a carpet will be formed of the dimensions required.

---

15. Cut each of the given squares (*plate* II, *fig*. 11, *Nos*. 1 and 2) diagonally into the four triangles marked *a*, *a*; *b*, *b*. Then, by applying them, as is represented in *fig*. 11, *No*. 3, we shall have three-fourths of a square of the form required.

---

16. The piece of wood, &c. must be in the form of a wedge, except that its base must be a circle, instead of a parallelogram.

---

17. As each side of the table is to be increased 12 inches, it is evident that if rectangular pieces of wood, 6 inches broad, be made to surround it, as is represented in *plate* II. *fig*. 12, the table will be of the size required.

In order to prove the same:

$30 \times 30 = 900$ Sq. In.		the content of the original table
$6 \times 6 \times 4 = 144$	- -	4 corners added
$30 \times 6 \times 4 = 720$	- -	4 sides ditto
1764	- -	required table

$= 42 \times 42$, as it ought to be.

It will likewise be found that the total of the contents of the four sides and corners is 864 sq. inches, or 6 sq. feet, as per question.

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18. Cut each of the six squares diagonally, and twelve triangles will be formed; then, by applying these as is represented in *plate* II. *fig.* 13, we shall have *three-fourths* of a square of the form required.

---

19. Let AB (*plate* II. *fig.* 14) be the given line: divide it into any number of equal parts; for example 5, viz; A1, 12, 23, &c. Through A, draw a line aP (of any length) perpendicular to AB. From P, through the points of division 1, 2, 3, &c. draw Pb, Pc, Pd, &c.; and make 1b, 2c, 3d, &c. each equal to Aa. Then the line **af*, drawn with a steady hand, from *a*, through *b*, *c*, &c. is the line required.

From mere inspection, the *least* distances of the lines AB and *af* from each other, represented by the several perpendiculars *bg*, *ch*, *di*, *ek*, and *fe* (meeting AB produced,) *appear* to continually decrease, especially near the commencement; but it must be proved, first, that they *really* decrease, and then, that (although they continually approach) they will *not meet*.

By the construction, the angle at A is a right angle, and therefore (*Euclid* I. 32.) the sum of the angles

* The line *af* is a *curve* called the "Conchoid of Nicomedes," and AB is its *asymptote*.

AP1 and A1P AP2 and A2P &c. &c.	of the triangles	PA1 PA2 &c.	must be a right angle,

and therefore a *constant* quantity; hence the *greater*, one of those triangles is, the *less* must its complement be. Now, the *greater* side of every triangle is opposite the *greater* angle *(Euclid* I. 18.*)* therefore the angle at P must increase, as the opposite side A1, A2, and A3, &c. increases; whence, on the other hand, A1P, A2P, A3P, &c. must continually *decrease.* Again, if two straight lines cut one another, the opposite angles shall be equal *(Euc.* I. 15*;)* therefore the angle A 1 P is equal to the angle b 1 B, and A 2 P to c 2 B, &c.; whence the angles A 1 P, A 2 P, &c. (being equal to the former) must also continually *decrease;* and, therefore, the sides opposite, viz.: b1, c 3, &c. must likewise continually *decrease.* Q. E. D.

From the above, it must be evident, that, how far soever AB is produced (so that the two lines continually approach) and the division continued, and lines drawn as before; the perpendiculars of the triangle formed as above would still continually decrease; but as the hypothenuse, which is a constituent part of every triangle, must, by the construction, still exist, and remain of the same length as Aa, the triangle itself must exist, and consequently its *perpendicular*; and therefore the two lines *cannot meet.*

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20. See *plate II. fig.* 15.

21. See *plate II. fig.* 16.

22. See *plate II. fig.* 17.

23. See *plate II. fig.* 18.

24. See *plate II. fig.* 19.

25. See *plate III. fig.* 20. *Nos.* 1, 2 *and* 3.

26. See *plate III. fig.* 21, *Nos.* 1, 2 *and* 3.

27. See *plate III. fig.* 22, *Nos.* 1 *and* 2.

28. On a *plane* surface, the solution of the problem would be impracticable, but on a *curved* surface it may be easily done. Fold a sheet of paper round a cylinder, and with a pair of compasses describe a circle upon it, assuming any point as a centre; then, when the paper is unfolded, and extended on a plane surface, an *oval will be presented, the shortest diameter of which will be in the direction corresponding with the axis of the cylinder.

29. Take a piece of wood &c., in the form of a right† cylinder, having the diameter of its

* The oval must not be confounded with the *ellipse*: the latter has no part of the curve of a circle in its composition; but, being described on two points, called its foci, it is continually varying.

† By a *right* cylinder, is meant one that is upright

base, and the altitude, equal; then, if it be cut through its axis, the section will be a square; if cut through a plane perpendicular to the axis (or parallel to the base,) the section will be a circle; and, if cut obliquely to the axis, the section will be an oval. Then, making three holes in a piece of wood, pasteboard, &c., the first equal in size to the above square; the second to the circle, and the third to the oval; it is evident that the cylinder may be made just to pass through the first of these holes, in a direction perpendicular to its axis; through the second in the direction of its axis; and, through the third, when situated in the proper degree of obliquity.

---

30. In *Simpson's Geometry*‡ it is demonstrated that, of all right-lined figures contained under the same perimeter, and number of sides, the greatest is when the sides are equal. From this it may be inferred, that the content of B's field is less than that of A's, and therefore the latter ought not to accept the offer.

And, farther, the greater the inequality that exists between the sides of the oblong, the less will be its content, and the greater would be the loss sustained by A were he to exchange.

---

31. A circle that has its circumference double

when standing on its base; if not upright, it is called an *oblique* cylinder.

‡ Maxima and Minima of Geometrical quantities, Theorem 10th.

that of another, has its diameter double also; and, as circles are to one another as the squares of their diameters (*Euclid* xii. 2,) it follows, that a circle having its diameter double that of another, must have its area four times that of another. Therefore, the man ought to have offered four times the former price; consequently his offer was not reasonable.

---

32. Let A, C, B be three points in the required circle or arc (*plate* III. *fig*. 23.) Draw the lines AC, CB, and make an angle equal to ACB of any solid substance, and fix two pegs in A and B; then, by making the sides of the determinate angle slide between these pegs, the vertex (being furnished with a spike or pencil,) as it revolves between A and B, will trace out the required arc.

If another angle of the like kind were constructed, forming the supplement of ACB to two right angles, and if it were made to revolve with its sides always touching the points A and B, but with its vertex in a direction *opposite* to C, it would describe the *other* segment of the circle, which, with the arc ACB, would make up the *whole* circle.

The above construction evidently depends upon Euclid, III. 21; viz, "The angles in the same segment of a circle are equal to one another."

---

33. See *plate* III. *fig*. 24. Suppose A, B, and C, the three given points. With the centre A, and radius AB, describe an arc; and with centre C, and radius BC, describe another, intersecting

the former in D. With the centre B, and radii BA, BD, describe arcs AGH and EF; on the latter of which make the chord or distance EF = BC. From E and F, with any radius, greater than the difference of AB and EB, and less than their sum (about the midway between these limits is best in practice,) describe arcs cutting AGH in the points G, H. Lastly, from A and C, as centres, with radius = chord GH, describe arcs intersecting in I, and I will be the centre sought.

For the demonstration, see the *Gentleman's Diary* for 1806, from which work the problem is taken.

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34. See *plate* III. *fig*. 25. Draw the diameter AB, which divide into as many parts at the points, D, C &c. and in the same ratio as those proposed; then, on the several distances of these points from the two ends A and B, as diameters, describe the alternate semicircles on the different sides of the whole diameter AB, and they will divide the whole circle in the manner proposed; i. e. the spaces TV, RS, PQ, will be as the lines AD, DC, CB.

---

35. A right line perpendicular to the plane of the two given lines, at the point of their concourse, will be perpendicular to them both.

---

36. That this cannot be a *plane* triangle must be evident from *Euclid* I. 32; it must therefore be *spherical*, and may be easily described thus: From any point A, as a centre (*plate* III. *fig* 26,)

with any distance, describe an arc BC; and from B, with the same distance describe an arc AC; Lastly, from C, with still the same distance, describe the arc AB; then an equilateral spherical triangle will be formed, having each of the three angles 90°, as was required.

---

37. See *plate III. fig.* 27. With any distance AB, as a radius, and centre B, describe a circle, in which inflect AB four times from A, to C, D, E and F. From the points A and E, with the distance AD describe arcs intersecting each other in G and H. And from the points D and F, with the same distance AD, describe arcs intersecting in I. From the points A and E, with the distance BG, describe arcs intersecting the circumference of the circle at K. Lastly, from the points A and K, with the distance AB, describe arcs intersecting in L. Then, supposing AB to be 1; the distance

AK = sq. r. 2
AD = - 3
AE = - 4 = 2
IK = - 5
IG = - 6
IC = - 7
GH = - 8
IA = - 9 = 3
IL = - 10.

The Demonstration is too long for a work of this nature.

---

38. As the radius of a circle is equal to the

side of the inscribed hexagon; from any point A in the circumference of the given circle, *(plate III. fig. 28)* the centre of which is O, apply the radius three times round, to the point B, the *first* point of division being at C. With the centres A and B, and the distance BC, describe arcs intersecting each other in D; the distance DO is the side of the square sought, which apply 4 times round the circle from A to E, B, F, and A, and it is done.

Demonstration. Conceive lines to be drawn from A to C, D, E and B respectively, also from D to O through E; then, since twice the square of the side of a square is equal to that of its diagonal: we have to prove that $2OD^2 = AB^2$.

First: $OD^2 = AD^2 - AO^2 = BC^2 - AC^2$, (since $AD = BC$, and $AC = AO = \frac{1}{2} AB$ by the construction;) and, as ACB is manifestly a semicircle, and the angle ACB in it a right angle *(Euclid,* III, 31,*)* $BC^2 = AB^2 - AC^2 = 3 AC^2$; $\therefore OD^2 = (BC^2 - AC^2) = 2 AC^2$, and $2OD^2 = 4AC^2 = 2AB.^2$ Q. E. D.

In order to render this problem, and the next still more curious; suppose the centre of the circle *not to be known*. Then, by assuming any three points in the circumference, the centre of the circle (passing through them) may be found by *problem* 33.

---

39. Using the same construction as in the last problem, the distance CE *(plate III. fig. 28)* will be a side of the dodecagon sought.

Demonstration. By the solution to the last problem, the arc AE divides the whole circle into 4 equal parts, and the arc AC divides it into 6; therefore the difference of these arcs, viz. CE, must be $\frac{1}{4}$ circumference — $\frac{1}{6}$ circumference = $\frac{1}{12}$ circumference. Q. E. D.

---

40. In the circle inscribe a square; then thrice the diameter of the circle, added to $\frac{1}{5}$ of the side of the square will give the circumference very nearly.

Demonstration. Since twice the square of the side of a square is equal to that of the diagonal; we have (supposing the diameter of the circle, or diagonal of the square = 1) $\frac{1}{2}$ sq. r. 2 for the side of the square, $\frac{1}{5}$ of which is $\frac{1}{10}$ sq. r. 2 or 1·4142, to which add thrice the diameter, or 3; the sum is 3.14142, which is, within about the 18,000th part, equal to the circumference of a circle, the diameter of which is 1.

---

41. Let BADC (*plate* III. *fig.* 29) be the given circle, of which AC is the diameter, and AB the quadrant. Make the chords AE, ED, DC, each equal to the radius. Draw DB, EF intersecting the diameter in the points G and F; then BG+FG, or BF+FG = arc AB very nearly.

Demonstration. Drawing DO, EO, the angle DOE (by the construction 60°) = double the angle at B (*Euclid*, III. 20,) ∴ the angle at B =30°; bisect this by the line BO, which will also bisect the base GF, and we have the angle OBF = 15°.

As OB is the radius of the circle, OF must be the tangent to an arc of 15°, and BF, the secant; hence we have only to show that the secant of 15°+ twice tangent of 15° (to radius 1) = $\frac{1}{4}$ of circumference of the circle (to radius 1) very nearly.

By Dr. Hutton's Tables, the natural secant of 15° = 1.0352762, and the natural tangent = ·2679492; twice this, added to the former, gives 1·5711746, which is, within about the 8000th part, the same as $\frac{1}{4}$ of the circumference of a circle, the radius of which is 1.

---

42. Let ABCD (*plate III. fig.* 30.) be the given square:

With the centre $\left.\begin{matrix}A\\C\end{matrix}\right\}$ and radius $\left\{\begin{matrix}AC\\BC\end{matrix}\right\}$ describe arcs intersecting each other in the point E.

Again:

With the centre $\left.\begin{matrix}C\\A\end{matrix}\right\}$ and radius $\left\{\begin{matrix}AC\\AB\end{matrix}\right\}$ describe arcs intersecting each other in the point F.

Lastly: with the centres E and F, and radius AF or CE, describe arcs intersecting in O. O is the centre required.

Demonstration. Conceive the necessary lines to be drawn, and a semicircle to be described with the centre A, and radius AC; then (*Euclid, III,* 31) the angle at E, would be a right angle. From E, let fall the perpendicular EG; then, by similar triangles: As the diameter of the semicircle : CE : : CE : CG; i. e. (supposing the side of the square = 1,) As 2sq. r. 2 : 1 :: 1 : $\frac{1}{4}$ sq. r. 2. But, by the construction, OEC is an isosceles triangle, ∴ OG = GC, and consequently OC = 2OG or 2GC = $\frac{1}{2}$ sq. r. 2 = $\frac{1}{2}$ CA. Q. E. D.

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43. Let AB *(plate III, fig.* 31,*)* be the given line. With the centres A and B, and radius AB, describe the arcs* aBb, cAd. Find the points C, E, perpendicular to A; and D, F, perpendicular to B, by the first part of *Problem* 38. Now it must be evident, by the construction, that the points A, B, C, D, and A, B, E, F, are the extremities of two *squares*.† Find their respective centres G and H by the *last* problem. Again, the points A, G, B, H, are also the extremities of a square; find the centre O as before: this will be the middle of the line AB as was required.

As the method of finding the centre of a square is demonstrated in the *last* problem, no farther demonstration seems necessary.

---

44. Find the extremities *m*, *n*, of the diameter, *(plate III. fig.* 32,*)* and the point A perpendicular to the centre‡ *O*, by *Problem* 38.

* For want of room in the plate, the arcs *only* are described, instead of the whole circles (as should have been the case,) in order that the construction may be adapted to the figure.

† Here we have a method of describing a square upon a given line, by the compasses *only*.

‡ If the centre of the circle is not *known*; assume

Find, also, the middle point r between o and n by the *last* problem*. Apply the radius from n to c; then, with the centre r, and radius Ar describe an arc. Also, with the centre c and radius Am or An, describe another arc intersecting the former in the point s. The distance As being carried *five* times round the circle, the angular points of the required pentagon will be determined; and, by joining those, the figure is drawn as was required.

Demonstration. By employing a *ruler*, it will be found that the above method of construction agrees with the one *commonly* adopted, with this difference, that, when the diameter mn is drawn, we have not to apply the distance Am from c to s, in order to intersect the arc described with radius Ar and centre r; for the line mn intersects it in s: which must here be proved.

Imagining lines to be drawn; if mn passes through s, ros is a right line, and $rs^2 + cr^2 = cs^2$; i. e. $Ar^2 + (Ao^2 - or^2) = Am^2$, since, by construction, $rs = Ar$, and $cs = Am$. Now $Ar^2 = Ao^2 + or^2$, hence $Ao^2 + or^2 + (Ao^2 - or^2) = 2\,Ao^2 = Am^2$, as it ought to be. ∴ &c.

☞ For a demonstration of the *common* method above alluded to, see the *Ladies' Diary for* 1786; or *Creswell's Supplement to Euclid, Book IV, Prop.* 17, *Cor.* 30.

three points in the circumference, and proceed as in problem 33.

* For, by that problem, it is evident that the centre of the shortest distance between two points, may be found without drawing a line to join them.

A KEY TO

TREES PLANTED IN ROWS.

1. See *fig.* 1, *plate* IV.

2. See *fig.* 2, *ditto.*

3. See *fig.* 3, *ditto.*

4. See *fig.* 4, *ditto.*

5. See *fig.* 5, *ditto.*

6. See *fig.* 6, *ditto.*

7. See *fig.* 7, *ditto.*

8. See *fig.* 8, *ditto.*

9. The trees must stand in the periphery of a regular decagon, 8 along each side, and 1 at each corner.

10. Let ABCD (*plate* IV. *fig.* 9.) represent the quadrangle. Produce the opposite sides DA and BC, till they meet in the point E, also AB and DC, till they meet in F; join EF, and through the point D draw a line parallel to it; produce

BA and BC, till they meet this line in G and H. Divide GD and DH into the same number of equal parts; for example, four. Join the points of division in the line GD to the point F, and those in the line DH to E; then those points where the lines intersect the sides of the quadrangle, and where they intersect each other within it, are the places where the trees are to be planted.

Then, to have a quincunx; in each small quadrilateral of the plantation, draw two diagonals intersecting each other: in each of these points of intersection, the fifth tree is to be planted.

A KEY TO THE

GEOGRAPHICAL PARADOXES.

1. The two places must be directly under the two poles; for to the north pole the sun rises about the 21st of March, and does not set till the 23rd of September: and the ensuing twilight continues till the sun is 18 degrees below the horizon, i. e. till about the 13th of November; then dark night continues till about the 29th of January, at which time day-break commences, and the morning twilight continues till sun-rise, on the 21st of March.—Hence, between sun-rise and sun-set, 6 months elapse, between day-break and the end of twilight about 288 days, but total darkness continues only 77 days.

N. B. When the sun rises to the north pole, he sets to the south, et contra; and because he rises but once, and sets but once in the year, there can be but one day and one night during the whole year.

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2. If by neither day nor night is mean twilight, the places may be in any part of the Frigid Zones. But, if we are to understand that the sun neither rises nor sets for 24 hours, the places must be 90 degrees from the sun. Thus: if the sun be in the equator, then the places are directly under the poles; for, at those times, the sun circuits about their horizon for 24 hours, half above and half below it; hence, during that time, it is there neither day nor night.

3. Directly under the south pole; which has not only the least, but the greatest, and all intermediate degrees of longitude, since they all meet in the poles. * Or thus: all places that lie under the first meridian, have both the greatest and least degree of longitude; for when the utmost extent of longitude ends, its least denomination begins.

4. Suppose one place to lie directly under either of the poles, a second 10 degrees on this side, and a third 20 degrees on the other, under the same meridian circle, then they will all differ in latitude, and likewise in longitude, since the pole contains all degrees of longitude.

5. The paradox seems to refer to the difference made by Geographers in fixing their first meridian. Thus, the British have it at Greenwich; the Dutch, at the Peak of Teneriffe, one of the Canary Isles; and the French, at Ferro, another of the Canary Isles. Now take, in the same latitude, three places; suppose 10 degrees from each of those meridians. Then these will also all agree in longitude, with respect to their first meridians, though they lie under three different ones in reference to the globe.

6. Not only in Asia, but in every place where Christians and Jews dwell together, the latter

* This only refers to the *old* way of reckoning the longitude; it is *now* made to end at 180 degrees both ways.

have their sabbath one day every week earlier than the former.

Or thus: the two places may be Macao, a sea-port in China, possessed by the Portuguese; and the Philippine Isles, said to belong to the Spaniards of Castile; places near each other and under the same meridian. Now, when the Spaniards have their last Saturday in Lent, the Portuguese, in Macao, eat flesh, it being their first Sunday in Easter.

The cause of this difference is, that the Spaniards sailed thither westerly, and lost half a day, and the Portuguese easterly, and gained half a day. [*See Varenius's Geography, chap.* 29. *prop.* 12, *corol.* 3.]

To illustrate this: suppose the persons who travel westward should keep pace with the sun, it is evident that they would have continual day, or it would be the same day to them during their voyage round the earth; but the people who remained at the place those departed from, have had night in the same time, and therefore reckon a day more than the former.

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7. Directly under the south pole.

---

8. Suppose the island to be Negropont, in the Ægean sea, where both Christians and Turks dwell, the latter following the Lunar year, which is 11 days shorter than the Solar, observed by the former. Now, if the children (one of whom may be born of Christian, and the other of Turkish

parents) should live together 30 Solar years, and then die at the same moment of time, the Turk, according to the Turkish reckoning, would be 10 months older than the Christian, according to the Christian reckoning.

Or thus: if one of the children sail directly east, and the other directly west; when they have encompassed the globe, there will (according to the preceding solution) be two days difference in their ages; and, as that might be accomplished in one year, there would, after 50 years sailing, be a difference of more than 3 months.

---

9. If a burning-glass be made the nodus of the dial, and be so situated that the focus may fall on an iron or brass plate or ring, on which the figures are deeply cut; a blind man (on any part of the globe) may feel that part which is heated by the sun, also, upon what figure it is, and to which it is nearest.

---

10. Horizontal dials within the Tropics cast no shadows at noon twice every year, because the sun is vertical: and no universal ring-dial will shew the hour when the sun is in either equinox.

---

11. The first meridian (from whence longitude is reckoned east and west) passes midway between the ship and the island, and therefore regard is had to the east and west longitude, and not to the points of the compass.

12. Directly under the poles, where all the points of the compass meet.

---

13. The earth is surrounded by a body of air, called the atmosphere, through which the rays of light come to the eye from all the heavenly bodies; and since these rays are admitted through a vacuum, or at least through a very *rare* * medium, and fall obliquely upon the atmosphere, which is a *dense* medium, they will, by the laws of optics, be *refracted* in lines, approaching nearer to a perpendicular from the place of the observer (or nearer the zenith) than they would be were the medium to be removed. Hence, all the heavenly bodies appear higher than they really are, and the nearer they are to the horizon, the more obliquely the ray falls, and, consequently, the greater is the *refraction*, or difference between their *apparent* and *true* altitudes.

The above may be elucidated by the following simple experiment. Put a piece of silver at the bottom of an empty vessel, and then stand at such a distance from it as to cause the silver to be just out of sight. Then, by filling the vessel with *water*, which is a denser medium than *air*, and standing at the same distance from it as before, the silver may be plainly seen.

---

14. When the sun is in the horizon of any place, (whether Naples or elsewhere) he is the length of

* A medium is a fluid or substance, through which a ray of light can penetrate.

the earth's semidiameter more distant from that place than in his meridian at noon. Now, there being but a very small proportion between the depth of the lowest valley in the world and the earth's semidiameter, which is nearly 4000 miles, the sun must be considerably more than 3000 miles nearer at noon than at his rising, there being no valley even the hundredth part of 1000 miles deep.

---

15. It is very probable that there may be one or more villages, not only in the south of England, particularly Sussex, but in a more southern latitude, so situated on the north side of a hill, as to be prevented by it from seeing the sun, several days before and after the winter solstice.

---

16. Directly under the south pole.

Or thus: by a metonymy, taking the sun for sunshine, it may be any place beyond the Antarctic circle, and, then it is not to be understood that the sun stands still in the meridian, but that he enlightens it for so many days, as he is above the horizon; as when we say that the sun does not move from such a wall or dial, for so many hours. Thus, in latitude 68 degrees south, the sun shines upon its meridian constantly for 30 days.

---

17. Directly under the north pole.

---

18. Directly under either of the poles.

---

19. Directly under the north pole, when the

sun enters aries, and under the south pole when he enters libra; at both which times, when it is full moon, though they both rise on opposite sides of the horizon, i. e. in the equinoctial points; yet they are both upon the same points of the compass, i. e. under the north pole due south, and under the south pole due north.

---

20. There are several places in England, and in other parts, where rivers run a considerable way under ground; as the Mole, in Surry, and the Guadiana, in Spain; the former runs from Dorking to Leatherhead, under ground, which is upwards of 4 miles, or about 7000 yards; which would evidently afford sufficient room for more than 3000 men to walk abreast. Or thus: if a common bridge is meant, the men may pass over it sideways.

---

21. In all places, persons standing near any still and clear water, will, *by the refracted vision*, appear with their heads downwards to others that are looking into it.

Or thus: China being situated in a meridian, almost opposite to ours, the feet of its inhabitants must be very nearly opposite our feet; and, therefore, it may seem to us as if they walked on their heads.

---

22. Suppose the three places are in Sweden, Norway, and Russia, where their day is about two months long near the summer solstice, and

let the fourth place be *equidistant from the other three. Now, a person walking but 12 miles a day, may, in 2 months, perform upwards of 700 miles, which is evidently more than the distance required.

---

23. Admitting the earth to be without any irregularities as to its surface, it would not even then (as is generally imagined) be a sphere, but an † oblate spheroid, flattened at the poles.

In the former case, the length of a degree in every latitude is the same; but in the latter, it will in no two places be alike.

Or thus: suppose London, Paris, and Bourbon, all under the same Brazen meridian; equally different in latitude, yet the distance from London to Paris, will exceed that of Bourbon to Paris, nearly 100 miles; because London is about 2 degrees west of the latter place, which is about the breadth of the Brazen meridian: whereas Bourbon and Paris are in the same longitude, and consequently, nearer by almost 2 degrees.

---

24. If any two places be in the same parallel of latitude, respecting the rhumb, the first must bear of the second east and west; and yet the second, respecting the angle of position, (or the

* If so, it will be found that the fourth place is 571·35 ($=\frac{1000}{3}$ sq. r. 3) miles from each of the other three, which is about $\frac{4}{7}$ of the distance that they are from each other.

† An oblate spheroid is a solid formed by the revolution of a semi-ellipsis about its conjugate diameter.

bearing of one place from the zenith of the other) on the globe, may be much short of due west. Thus: Lisbon, in Portugal, and Smyrna, in Natolia, are in the same parallel of latitude, viz. 39 degrees north; and, therefore, by the rhumb line, they bear east and west. But, on the globe, Smyrna bears off the zenith of Lisbon 75 degrees N.E. and Lisbon bears off the zenith of Smyrna 80 degrees S.W.

N.B. A rhumb line makes equal angles with all meridians on the globe; and an equal part thereof alters the latitude equally: but, in the circle of position, it makes unequal angles, i.e. greater angles with all other meridians than with that from which it was drawn.

---

25. In a deep well or coal-pit, those stars which are in, or near the zenith, will, at all times, in a clear sky, become visible to an eye that is for a short time steadily directed towards them. The reason of this is, that the surrounding light which the atmosphere diffuses in the open air, is prevented from pressing on the sight.

---

26. This refers to the artificial globe and the hour index. Thus: the difference of longitude between London and Messina is 16 degrees, which, being reduced into time by the hour-circle and index, makes no more than 1 h. 4 m. As to the time of the year, north wind, &c. &c. they appear to be inserted only to render the paradox more intricate and obscure.

27. The two places must each be 50 miles distant from one of the poles, and both in one great circle or meridian; but on different sides of the globe. Then, supposing two such places, one on each side of the north pole, the first 50 miles of the course will be due north, directly towards the north pole, and the remaining 50 miles will be due south, or directly from the north pole.

---

28. Supposing the two places to be denoted by A and B, the former situated 60 miles or 1 degree east of the latter, and consequently, differing in time scarcely 4 minutes; yet, if a ship, instead of sailing from B to A eastward, should sail directly westward, the distance run would be 359 degrees, amounting to more than 23 hours difference in time, although the shortest distance between the two places would be only 1 degree.

---

29. Not only three, but all places under the equator: for, during the first part of the night, one hemisphere is visible, which, in twelve hours, entirely sets against morning, when the other half appears, that was entirely hid the evening before.

---

30. This may arise from the diurnal advance of the sun, moon, and planets, in their particular orbits, which exceed the 15 degrees hourly, or 360° of the equinox, by the said diurnal motion of the planets. Or, it may be a floating island.

---

31. Directly, or nearly under the north pole,

because the sun, in consequence of the earth's unequal motion round him, continues about seven* days longer, in the artificial day, under the north pole, between Aries and Libra, than in that, under the south pole between Libra and Aries.

---

32. This appears to have reference to the variety of risings of the planets and stars, as to one, the Cosmical, the other, the Achronical, the third, the Heliacal, the fourth, the true or apparent diurnal rising of the moon. Each astronomer, though exactly true with regard to that particular rising, will differ from the rest, not only in minutes and seconds, but in hours and days.

---

33. The former place must be under the tropics when the sun is vertical. The latter place under the poles or polar circles.

---

34. Directly or nearly under the poles.

---

35. From the sun which is considered the centre of the world's system.

---

36. The planets are Venus and Mercury, which have a two-fold conjunction with the sun, both in the superior, and inferior, or opposite points of their orbits; in the first, they are between the sun and us, and then, sometimes, viz. at their transits,

*See the note to the 6th Geographical Theorem in Keith's Use of the Globes.

appear like spots in his disk; and in the last, the sun is between us and them.

---

37. The first place is on Venus or Mercury, and the last on Saturn or Jupiter.

---

38. The north side of the wall is perpendicular to the horizon, and the sun, having north declination all the summer, will shine on the north side before six in the evening. The south side has a reclining plane, the acclivity of which is greater than the latitude of the place, and, therefore, the sun will shine on that place before six in the morning all the summer.

---

39. This may be understood two ways.

1st. With regard to the poles: all places are south of the north pole, and north of the south pole.

2nd. With regard to the spiral or rhumb line, made by a ship or person travelling on any one point of the compass, between the cardinals, which suppose S. E. from Madrid 2000 miles, the second place will be S. E. of the first, and differ both in longitude and latitude from Madrid; and, if he travel 2000 miles more on the same rhumb, the third place will be S. E. of the second, yet differ in latitude and longitude from them both.

---

40. The planets are Venus and Mercury; the orbits of which being within that of the earth, never form either a trine, square, or sextile aspect with the sun.

41. The place may be the wharf of Greenwich, the Isle (so called) of Dogs over against it, and the appearance caused by refraction, when the water is high. See solution of paradox 13th.

---

42. Each of those places must be under the equator, and each considered as the first meridian.

---

43. Guernsey lies between England and France, and is about 26 miles from France; and England is but 21 miles from France, between Dover and Calais.

---

44. Any two places not next to one another, must be more distant than those that are; and the first and last the most distant of all.

---

45. Under any part of the equator, when the sun is in the equinoctial, he may be said to be due east, at every hour before noon, and due west every hour after noon.

Or thus: at any place where the declination is greater than the latitude; for then the sun will be upon the same vertical circle, twice in the morning, and twice in the afternoon.

---

46. They must first form themselves into a circle, at the south pole, the centre of which will be behind every man; and when they have travelled 5400 miles, they will come to the equator, where they will stand sideways to each other, 300 miles distant, in a circle, the centre

of which will be directly under them, in the centre of the earth; and when they have travelled 5400 miles more, they will all meet again with their faces inward, directly under the north pole; and, consequently, the centre of that circle must be before them.

---

47. *From the Jews' abode let the other two set out, the Christian due east and the Turk due west, on the globe; then their reckoning will agree with the Jews' when they meet, and on the Saturday they will all keep their proper sabbaths.

---

48. If a person be born in Bissextile, on the 29th of February, and travel westward round the globe, he may see twelve Bissextile years before he is completely 44 years old.

---

49. The man and his wife disagree on board a ship: he goes to one end, and she to the other; whence the Paradox is evident.

---

50. In some high latitude, about the longest day: when the time from sun-rise to sun-set amounts to several months.

---

51. The person might first see the sun rise, in a part of the horizon that was level; and then, by changing his situation, perhaps a mile or less, he might again see the sun rise in a part that was

* See Emerson's Geography, Prop. 19. and its Corol.

hilly; as, during the intermediate time, the sun had been hid by the hills. With regard to his setting, the effect would be exactly the reverse of the former.

---

52. According to Euclid, a plane can touch a circle only in one point; and that person only, who stands on that point, with respect to the centre of the sphere or earth, can stand upright.
See Whiston's Euclid, Book 1, Prop. 37, Schol. (1.)

---

53. All walls are endeavoured to be built perpendicularly to the tangent, and point to the earth's centre, to which, if continued, they would meet in a point, and consequently are not parallel; and on one point only can a perpendicular be raised, as appears from the preceding solution.

Or thus: By the European city may be meant Edinburgh, which is noted for stone buildings, that are very high, and strong, and that stand on an inclined plane.

---

54. According to Whiston's Euclid, Book 3, Prop. 37, Schol. (3.). "If any one should travel over the whole circumference of the earth, the way gone over by his head would exceed that gone over by his feet by the difference of circumferences; or by the circumference of a circle, the radius of which is equal to the man's height."

Now, supposing that each of the circumnavigators, Anson, Drake, and Cook was 6 feet high; then the diameter of a circle, of that radius, will be 4 yards, and the circumference a little more than 12 yards.

---

55. The man must travel in the circumference of a circle, (which must not measure less than 100 miles,) in the centre of which the church must be supposed to stand.

---

56. The horse must be put in a mill, where he will travel in a circle, without any impediment; and, as circles increase the farther they are from their centres, so the near legs of the horse, making a larger circle than the other, must pass over more ground.

---

57. The places must lie in the circumference of a circle.

---

58. The surface of water is always spherical; and the *greater* any sphere is, the *less* is its convexity. Hence, the top diameter of any vessel at the *summit* of a mountain, will form the base of the segment of a greater sphere than it would at the *bottom*. This sphere, being *greater*, must (from what has been already said) be *less* convex; or, in other words, the spherical surface of the water must be *less* above the brim of the vessel; and, consequently, it will hold less in the *former* case than in the *latter*.

59. As grain, trees, &c. are always considered to grow perpendicularly to the horizon: the surface of a *hemispherical* hill, measuring *twenty* acres, being just *twice* that of the horizontal and circular base on which it stands, will bear no more grain than a *level* field measuring *ten* acres.

---

60. The fire must be the sun, which imparts more heat to the earth in summer than in winter, although it is nearer* the sun in the latter season than in the former.

Here are two positions to be demonstrated:

1st. That the earth is nearer the sun in winter than in summer.

2nd. That the sun imparts more heat in summer than in winter.

With regard to the former: The apparent diameter of the sun is longer in winter than in summer, and therefore the earth must then be nearer him, since all objects appear larger as we approach them.

With regard to the latter: As the earth has just been shown to be nearer the sun in winter than in summer, it might be supposed that it must receive the greater heat. But as their *farthest* distance exceeds their *nearest* only about a *thirtieth* part of their *mean* distance from each

* The orbit of the earth (as well as that of every other planet) is *Elliptical;* yet the sun is not supposed to be fixed in the *centre* of the Ellipsis, but in one of the *foci*. Hence it is evident that the earth is not always at the *same* distance from the sun.

other, we may conclude that a difference so small, can make little or no sensible alteration of heat.

Hence the phenomenon expressed in the paradox must be produced by more powerful causes.

The more *directly* the rays of the sun fall upon any object, the *greater* is the heat imparted, and the more *obliquely*, the *less* is the heat; because a greater number of rays falls upon the same space in the *former* case than in the *latter*.

Hence, in summer, whilst the sun is at, or near his greatest altitude, his rays fall the more directly upon the earth; and, consequently, it receives a continual increase of heat; whereas, in winter, the contrary effect must take place. Again, in summer, when the sun is longest above the horizon, the earth is heated the greatest space of time; and the nights, being the shortest, it has the least time to cool; and, consequently, there must be the greatest increase of heat; whereas, in winter, the effect must be the reverse.

From what has been said, it may be concluded: That the small difference of the distances of the earth from the sun, in summer and in winter, occasions little or no sensible difference of heat; but that the excess of heat in summer above that in winter, arises from the combined effects of these two causes: —— The more *direct* emission of the sun's rays, and the *longer* duration of the sun above the horizon in summer than in winter.

SYMBOLS.

As a few symbols are employed in the key, it may not be improper to explain, in this place, such as are not the most common.

1. ∴ *therefore.*
2. Q. E. D. *quod erat demonstrandum,* which was to be demonstrated.
3. AB^2 , AB *squared.*
4. sq. r. *the square root of.*

CONTENTS.

Rose, Printer, Bristol.

Fig 1

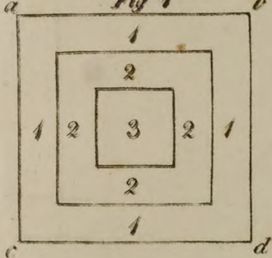


Fig 2

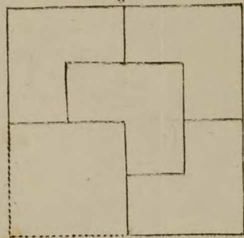
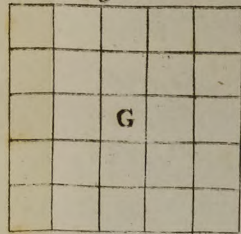


Fig 3



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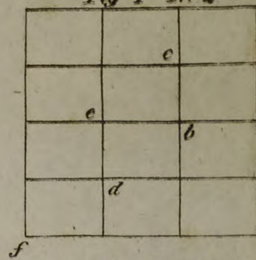

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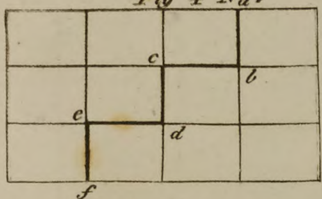

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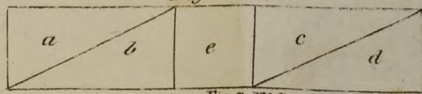
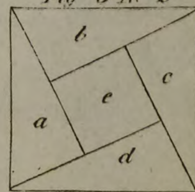


Fig 5 N.° 2



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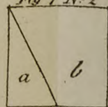

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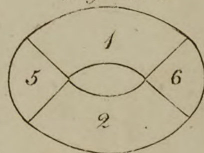


Fig 6 b N.° 1

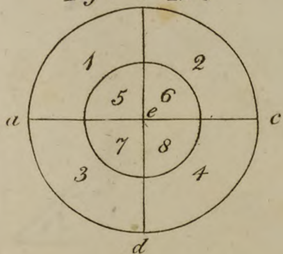
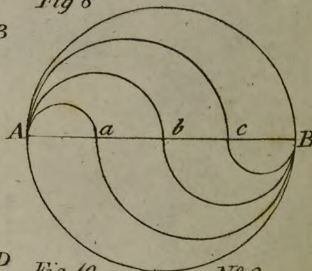


Fig 7 N.° 1

Fig 8



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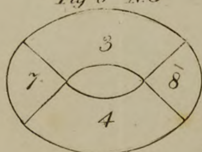

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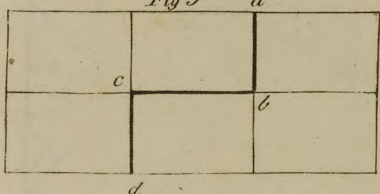

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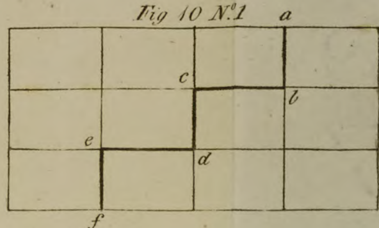

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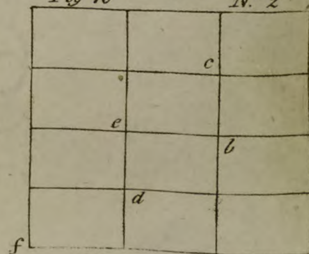

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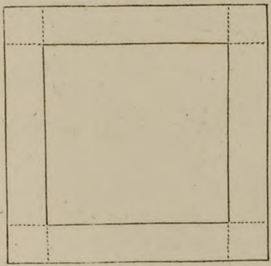

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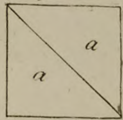

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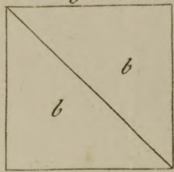

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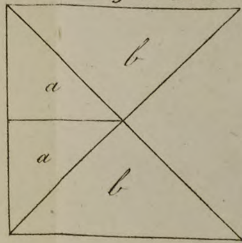

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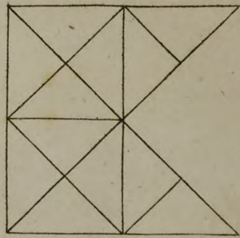

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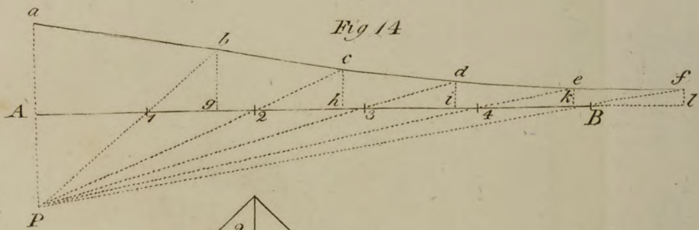

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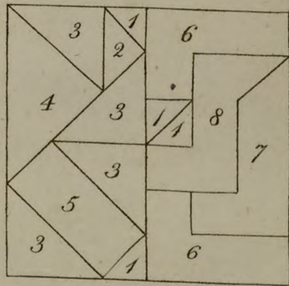

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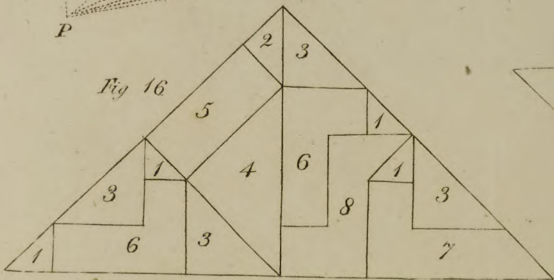


Fig 19

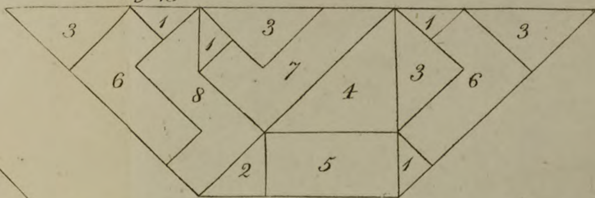
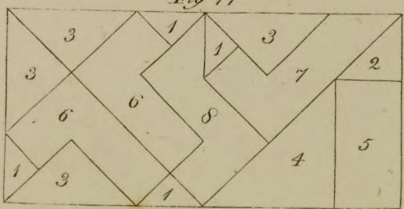


Fig 17



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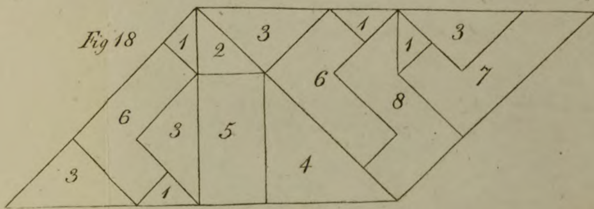


PLATE III

Fig^s 20 N.^o 1 N.^o 2 N.^o 3

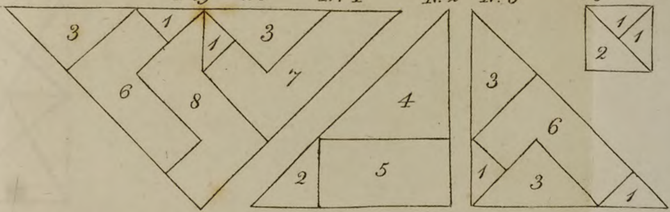
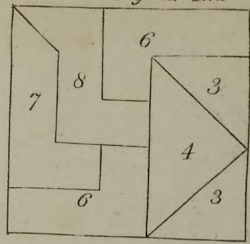


Fig 21 N.^o 1

Fig 21 N.^o 2



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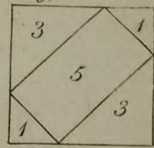

Fig. 22 N.^o 1

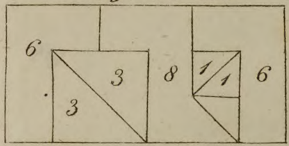

Fig 22 N.^o 2

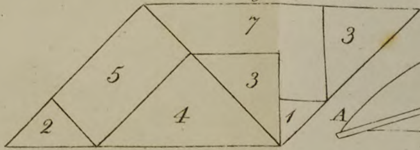

Fig 23

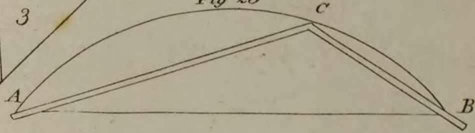

Fig 24

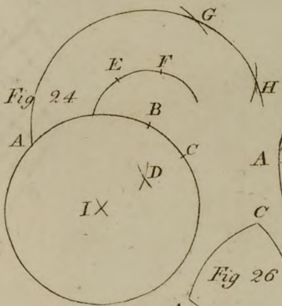

Fig 25

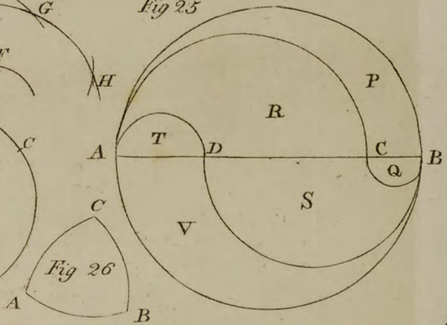

Fig 26

Fig 27

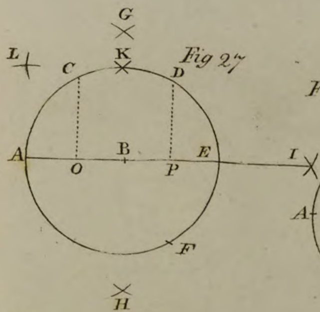

Fig 28

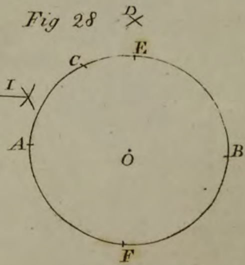

Fig 29

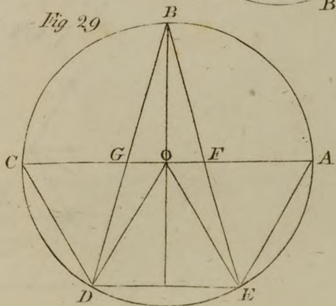

Fig 30

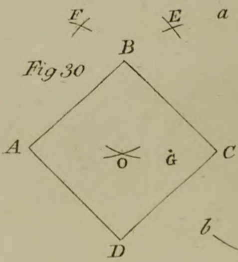

Fig 31

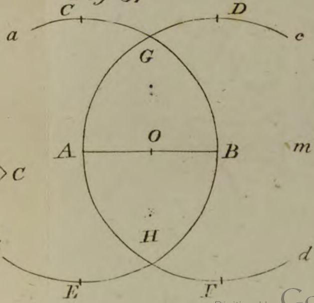

Fig 32

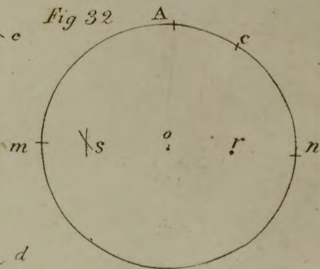

PLATE IV

Fig 1

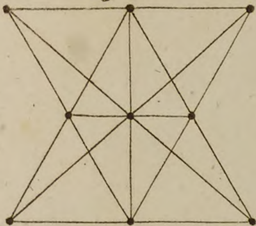

Fig 2

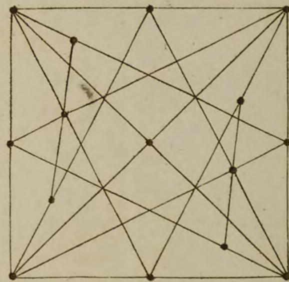

Fig 3

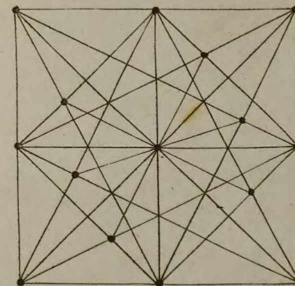

Fig 9

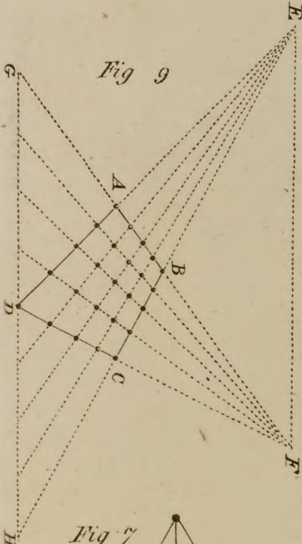

Fig 4

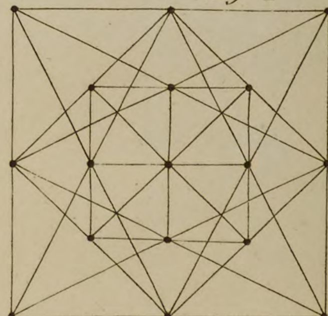

Fig 5

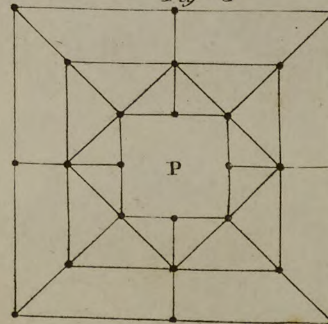

Fig 6.

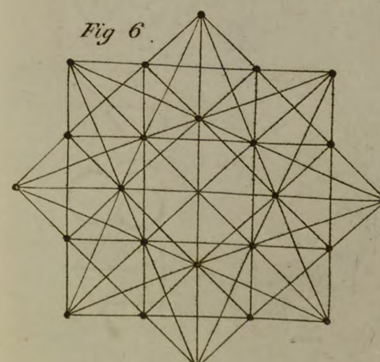

Fig 8

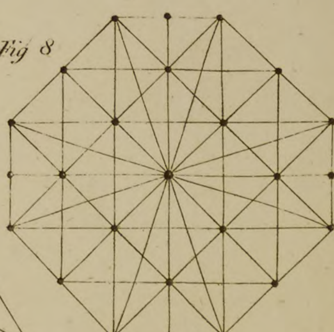

Fig 7