Richard

RECREATIONS
MATHEMATICAL
AND
PHYSICAL;

Laying down, and Solving

Many *Profitable* and *Delightful Problems*

OF

Arithmetick, Geometry, Opticks, Gnomonicks, Cosmography, Mechanicks, Physicks, and **Pyrotechny.**

By Monsieur *OZANAM*,
Professor of the MATHEMATICKS at *Paris*.

Done into *English*, and illustrated with very Many CUTS.

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To the
READER.

THE Author of the following Treatise, Monsieur OZANAM, is a Person so well known, and deservedly esteem'd, amongst the Learned who understand him in his Native Language, that, if all others were alike acquainted with his Worth, his Name would be a sufficient Recommendation: However, having so well acquitted himself in the Preface, in giving a true Representation of his Design, with the Uses and Advantages thereof; nothing remains to be added, but a general Idea of the Subject, and Method; with a Word or two concerning the Translation.

As to the first; This Book is such a Collection of the most curious, most surprizing, most useful, and most agreeable Performances of the Arts and Sciences under which they are severally rang'd, as may prove a Spring of Invention to the Ingenious, furnishing 'em with Hints of innumerable other

Discoveries and Contrivances serviceable to the Necessity, or the Conveniency, or the Pleasure of human Life. It is parted into Eight Divisions or Sections, according to the Number of general Heads under which the Problems are reduc'd. *Problems of Arithmetick* make the first Class, being the most useful, most pleasant, and least embarrassing of those that belong to that Art; with certain and never-failing Rules of Solution: The Demonstrations, which would have interrupted the designed Pleasure, are here, and every where else, omitted. Under this first Head the Reader will find the Substance of what is contain'd in Dr. *Arburthnet*'s *Laws of Chance*; with Variation of Examples. The *Second* sort are *Problems of Geometry*, which are very numerous; but here only the most uncommon, most curious, and, withal, most entertaining, are to be found. To *Problems of the Opticks*, being a *Third* Head, pertain those of *Perspective*, of *Dioptricks*, and *Catoptricks*, all extreamly diverting. *Gnomonicks*, or *Dialling*, is a most pleasant part of Mathematicks, depending on a very profound Theory, handled at large by the Author in his *Mathematical Course*; but under *Problems of Dialling*, in the *Fourth* Rank, are placed only such as may be perform'd with Ease and Delight. *Problems of Cosmography* are the *Fifth* in order, and include those of *Astronomy*, *Geography*, *Navigation*, and *Chronology*. The

Problems

Problems of Mechanicks follow in the *Sixth* place, being generally more uſeful than curious, becauſe converſant about Things neceſſary to Life; and to theſe are referred thoſe of *Staticks* and *Hydroſtaticks*. *Problems of Phyſicks*, which are a *Seventh* Kind, comprehend not only thoſe of *Natural Philoſophy*, which is nearly ally'd to the Mathematicks, but alſo thoſe of *Chymiſtry*, *Surgery*, and *Medicine*, which admit of Experience only for their Demonſtration. The *Problems of Pyrotechny* come laſt of all, where is to be ſeen what is moſt uſeful and diverting in *Artificial Fire-works*, whether for Service or Recreation.

But to come to the preſent Tranſlation; the Reader is to know, That thoſe concern'd in the Publication, conſidering the great Uſe and Excellency of Mathematical Sciences, upon which, whatever is of Certainty in others, purely Human, generally depends, thought they could do nothing of more univerſal Advantage, than to promote the Acquiſition of a Knowledge ſo vaſtly beneficial, by all Methods within the Sphere of their Buſineſs. To this Purpoſe nothing appear'd more proper, than ſome entire Syſtem of Mathematicks, that might lead the Studious of ſuch Knowledge, from the very firſt Principles, to the higheſt Pinnacle of Perfection, without being oblig'd to interrupt their Progreſs, by turning aſide after other Books and Authors. Many Treatiſes on ſome particular Parts of Mathematicks occur'd, ſome in *Engliſh*, ſome in *Latin*, and other Languages, accurately compos'd, and excellent in their Kind; but none ſeeming ſo peculiarly adapted to the Deſign, as the *Mathematical Courſe* of Monſieur *Ozanam*, it was reſolv'd to publiſh it in *Engliſh*. However, it was thought fit firſt to make Tryal, in a ſmaller Undertaking, what Entertainment this Author might here receive, and to that End his *Mathematical Recreations* were pitch'd on; the Care of Tranſlating being committed to a Gentleman of great Ingenuity, and well-vers'd in theſe Sciences; who had not yet compleated the Copy, and had ſeen but a few Sheets from the Preſs, when he was ſnatch'd from hence by untimely Death. This melancholy Event put a tedious Pauſe to the Work, and is the Cauſe it appears ſo late in publick, tho' Notice of it was given ſome conſiderable Time ago.

In this one *Engliſh* Volume, the Reader has all that's contain'd in the two *French* ones of the Original, that is Monſieur *Ozanam*'s: Where he will find whatever is in *Van Eton*, *Oughtred*, and others that have writ on this Subject: All that belongs thereto being herein comprehended, and much better explain'd than any where elſe.

Theſe *Mathematical and Phyſical Recreations* were deſign'd by the Author, to ſerve, in ſome ſort, as a Supplement to his *Mathematical*

tical Course, where many Problems, which are here to be found, were left out, that it might not make above Five Volumes in *Octavo*; of which we will here give the General Contents.

The First Volume contains an Introduction to the Mathematicks, with the Elements of *Euclid*. The Introduction begins with the Definitions of Mathematicks, and their most general Terms; which are followed by a little Treatise of Algebra, for understanding what ensues in the *Course*; and ends with many Geometrical Operations, perform'd both upon Paper with Ruler and Compasses, and upon the Ground with a Line and Pins. The Elements of *Euclid* comprehend the first Six Books, the Eleventh, and Twelfth, with their Uses.

In the Second Volume we have Arithmetick and Trigonometry, both Rectilineal and Spherical, with the Tables of Sines and Logarithms. Arithmetick is divided into Three Parts; the *First* handles whole Numbers, the *Second* Fractions, and the *Third* Rules of Proportion. Trigonometry has also Three Divisions or Books; the First treats of the *Construction of Tables*, the Second of *Rectilineal*, and the Third of *Spherical Trigonometry*.

The Third Volume comprehends Geometry and Fortification. Geometry is distributed into Four Parts, of which, the First teaches Surveying or Measuring of

Land; the Second Longimetry, or Measuring of Lengths; the Third Planimetry, or Measuring of Surfaces; and the Fourth Stereometry, or Measuring of Solids. Fortification consists of Six Parts: in the First is handled Regular Fortification; in the Second, the Construction of Out Works; in the Third, the different Methods of Fortifying; in the Fourth, Fortification Irregular; in the Fifth, Fortification Offensive; and in the Sixth, Defensive Fortification.

The Fourth Volume includes the Mechanicks and Perspective. In Mechanicks are Three Books; the First, is of Machines Simple and Compounded; the Second, of Staticks; and the Third, of Hydrostaticks. Perspective gives us first the General and Fundamental Principles of that Science, and then treats of Perspective Practical, of Scenography, and of Shading.

The Fifth Volume consists of Geography, and Dialling. Of Geography there are Two Parts; the First, concerning the Celestial Sphere; and the Second, of the Terrestrial. Gnomonicks or Dialling hath Five Chapters; the First, contains many Lemma's necessary for understanding the Practice and Theory of Dials; the Second, treats of Horizontal Dials; the Third, of Vertical Dials; the Fourth, of Inclined Dials; and the Fifth, of Arches, of Signs, and of other Circles of the Sphere.

If

To the READER.

If the preſent Undertaking meet with a ſuitable Encouragement, thoſe concern'd deſign, with all poſſible Expedition, to publiſh, in *Engliſh*, this *Mathematical Courſe*, in Five Volumes, in 8vo, as it is in the Original; each containing more Sheets, and Cuts than are in this Treatiſe. It is propos'd by *Subſcription*, at 1 *l.* 2 *s.* 6 *d.* in Quires: Any Perſon that enters his Name with any of thoſe concern'd in this Book, laying 5 *s.* down, ſhall receive, on paying 17 *s.* 6 *d.* more, a compleat Set of the Volumes, which, conſidering the vaſt Charge of the Cuts, and what it contains, is cheaper than any thing ever yet offered: And thoſe that ſubſcribe, ſhall have their Names printed before the ſame, as Encouragers of ſo uſeful a Work.

THE

THE
AUTHOR's
PREFACE.

IT *has been an Opinion of long ſtanding, That there was ſome ſecret Art amongſt the moſt learned of the* Jews, *of the* Arabians, *and of the Diſciples of that antient Academy, which was in* Egypt *when* Moſes *was there educated, and ſtill flouriſh'd in the Time of* Solomon; *inſomuch, that it hath excited the Curioſity of the fineſt Wits to endeavour the Diſcovery of it: But is it poſſible to learn an Art without a Maſter, and without Books? The Learned of that Time committed nothing to Writing; or if they did, it was enigmatical, and ſo remote from what a Reader did expect, that of them it may be ſaid, Their Silence was more inſtructive than their Diſcourſes.*

Father

Father Schott *ſaith there are Three Sorts of* Cabala, (*ſo is that ſecret Art of the Orientals call'd;*) *that of the Rabbies, that of* Raimond Lully, *and that of the Algebriſts. The firſt he knows not what it is; the two laſt are Recreations in Numbers and Figures: and no doubt is to be made but the firſt is of the ſame Sort.* Joſephus, *who was a* Levite, *writes with Confidence, That by Right of his Birth he had been inſtructed in all the Myſteries of the* Jews, *and had been taught all the Secrets of their Art. He boaſted, from a Courtly Principle which ſway'd him more than his Conſcience, That, by his Art he had fore-told the Elevation of* Titus *to the Imperial Dignity. He conceal'd his Game, as Men of Cunning ſhould, and as our Maſters teach us. He gives out himſelf for a Miraculous Perſon; and when he relates the Adventure where he ſhould have loſt his Life by the Deſpair of the Soldiers, reſolv'd to cut one another's Throat rather than ſurrender to the* Romans, *he attributes his Deliverance to Chance and a Miracle. Notwithſtanding* Hegeſippus, *who wrote the ſame Hiſtory, ſays, That* Joſephus *did that Miracle by the Knowledge of Numbers and Figures: For he made theſe Deſperado's to be rang'd in ſuch an Order, that the Lot fell upon thoſe, whom the Commander deſir'd to have deſtroy'd: He ſav'd his own Life, not by reaſon of being a Levite, but becauſe he was a Mathematician. Monſieur* Bachet, *in his* 23. Probl. *deſcribes* *this Secret; who, had he then liv'd, would have been accounted as great a Magician as* Josephus. *Hence it appears, that the most abstracted Knowledge may be reduc'd to Practice, and what seems most remote may become of Use.*

'Tis most astonishing to find, that in the Time of the Emperours Dioclesian *and* Constantin, *the Mathematicks were prohibited by the Laws, as a Dangerous Science, under the same Penalties as Sorcery or Magick; being reputed equally criminal and pernicious to civil Society; as appears from the 17th Title of the 9th Book of* Justinian's Code. *No doubt this was an Effect of the Ignorance which at that Time reign'd; and because of the great Number of Impostors, who us'd the Mathematicks to cheat, and deceive the Credulity of the Illiterate. Nevertheless, the Stupidity of those is to be blam'd, who suffer'd themselves to be gull'd; and their Negligence is not to be allow'd, who will not sufficiently improve their Understanding, so as to be in a Condition not to be abus'd. There have been States wherein Tricks and little Thefts, cleverly perform'd, were permitted, that all might be on their Guard, and accustom'd to a requisite Precaution.*

Ignorance keeps the World in perpetual Admiration, and in a Diffidence, which ever produces an invincible Inclination to blame and persecute those that know any Thing above the Vulgar; who, being unaccustom'd to raise their Thoughts beyond Things sensible, and unable to imagin that Nature imployeth Agents that are invisible

invisible and impalpable, ascribe most an end to Sorceries and Demons, all Effects whereof they know not the Cause. To remedy these Inconveniencies is the Design of these Mathematical Recreations, and to teach all to perform these Sorceries which were dreaded by the Council of Justinian: *And hereby will be vindicated the Fame of* Thomas Aquinas, Albertus Magnus, Solomon, *and many other great Men, who had never been accus'd for Magicians, but because they knew something more than others; more effectually than has been done by the Learned, who have been satisfi'd, by Dint of Argument only, to plead their Cause.*

It will, perhaps, be here objected, That by the Pastimes of Mind, presented to the World in the ensuing Book, the Reader is diverted from that Study and Application, to which he might have been engag'd by Treatises of a serious Nature, which fix the Thoughts, rendring 'em penetrating and inquisitive. To this it might suffice to alledge the Example of Men famous for Learning, whose like Practice in this Matter, may seem a Justification beyond any other could be brought. The learned Bachet, *Sieur de* Meziriac, *famous for his excellent Works, began to make himself known to the learned World, by a Collection which he intitled,* Pleasant Problems perform'd by Numbers; *by which he design'd to make Trial of his own Ability, and the Opinion of the World, before he publish'd his Commentaries on the Arithmetick of* Diophantus *of* Alexandria, *and his other Works by which he*

hath

hath purchased to himself immortal Glory: Many other Authors of this Age, as the famous Father Kircher, *the Fathers* Schott *and* Bettin, *have gain'd no less Renown by the diverting Problems in their Works, than by their Reasonings, and more serious Observations.*

But lest these illustrious Men, adduc'd as Precedents, should themselves be expos'd to the Censure of those who would accuse them of Novelty; Instances much more ancient, grounded on solid Reason, shall be here produc'd, whereby it will appear, that in all Times this has been done by the greatest Men; being persuaded, that the same Source of Reason that makes Men take Pleasure in Admiration, causes 'em, in like manner, to find Delight in things which are the Object of that Passion.

The Enigmatical Sentences and Propositions, so much admir'd and promoted by the Kings of Syria, *which occasion'd the Continuance of the Parabolical Stile so long after, were nothing else but Pastimes of Mind, and Entertainments equally fitted to excite Pleasure, and to give Enlargement of Understanding. Persons of higher Birth and Rank were of the same Make at that Time, as those of our own are now: What was painful and laborious did discourage 'em: To engage them to Studiousness and Reflexion, by Pleasure and Curiosity, was a Piece of extraordinary Skill and Dexterity. Doubtless, the Education* Nathan, *by this means, gave to* Solomon, *did mightily conduce to that Grandure of Soul, and to that admirable Wisdom which con-*

stitutes the Character, and is the Glory of that Prince.

It was also by way of Diversion the Chaldeans *and* Egyptians, *the Inventers of Astronomy, did fore-tell to their Friends the Time, and other Circumstances, of Eclipses, and erected Systems which shewed the Length of the Days, demonstrated the Course of the Stars, and represented all the Varieties of the Celestial Motions; being persuaded, no less than the* Grecians, *that the first intellectual Pleasures are those which proceed from Mathematical Sciences, in which they educated their Children. They were convinc'd, that Childrens Reason, tho' not yet in Action, was not without its Strength, and wanted only to be put in Motion, in order to its Progress towards Perfection; which might be effected by exciting in 'em a Curiosity, that would do the same with them, which a long Train of Necessities does in those of more advanced Years. Herein lay the Secret of* Socrates, *who taught Children to resolve the greatest Difficulties of* Geometry *and* Arithmetick: *This was the Key with which he laid open their Understanding, knew its Strength, and predicted their Destiny: This was instead of that* Demon *or* Genius *he is said to have consulted, and which is reported ever to have accompanied him.*

Tho' these Plays of the Intellect, here spoken of, seem only Amusements to pass away the Time; yet are they possibly of no less Advantage than those Exercises in which the Youths of Quality are bred up at Academies, which fashion as well

as

as invigorate their Bodies, and give them a graceful Air in their Deportment: For to be accustom'd to discern the Proportions, and the Force of Mixtures; to find out an unknown Point requir'd, amongst a confus'd Infinity of others; to take a right Method in resolving the most intricate and perplexing Propositions; is to have the Mind fitted for Business, to be arm'd against Surprizes, and prepared to overcome unexpected Difficulties; Things of no less Consequence, one would think, than Adjusting the Motions of the Body by the Instructions of a Dancing Master, or the Tone of the Voice by that of a Musician.

Besides, are not Diversions sometimes necessary? And can any one be diverted by what he despises, or is asham'd of? Would a Statesman choose to be performing at Dancing Matches, in the Intervals of Councils, and of important Business? Or were it becoming for him to be found in those Exercises wherein he spent the time of his Youth? Decency, Business, and Health, would in no wise allow it. But Pastimes of Mind are for all Seasons and all Ages: They instruct the Young, and divert the Old; They are not beneath the Rich, nor above the Ability of the Poor: They may be used by either Sex without transgressing the Bounds of Modesty. Those Diversions have this further and peculiar Advantage, that there can be no Excess in them: For seeing there is a regular Conduct of Reason therein, through all the Steps it should take, it can't be conceiv'd how it should touch upon any Extreme, its Exercise being within the due Medium, where the Solution

lution of the proposed Problem is to be found.

Those who have had the Curiosity to observe the Conduct of great Men in their private Actions, have found that they are distinguisht as well in their Recreations as in their Business. Augustus *us'd to exercise himself in the Evenings with his Family at these Diversions, not judging it beneath him; and recorded with no less Exactness the Particulars of his Recreations, than those of his important Affairs. That learned Lawyer* Mutius Scevola, *after his Consultations were over, diverted himself by Playing at* Chess, *and became one of the best Players of his Time. Pope* Leo X. *one of the greatest Men of his Age, play'd sometimes at* Chess, *if we may believe* Paulus Jovius, *to recreate himself after the Fatigues of Business.*

'Tis certain the Game of Chess *was invented for Instruction as well as Diversion. The Attacks and Defences, the diverse Steps and Advantages of the different Pieces, may furnish the Considerate with Political and Moral Reflexions. By the Disaster of the King, we may learn, that a Prince must infallibly fall under his Enemies Power, when depriv'd of his Soldiers; and that he cannot neglect the Preservation of 'em, without exposing himself and his Dominions.*

All Games that are, or may be invented, may be reduc'd to three Ranks. The First is of those that depend altogether on Numbers and Figures; as the Chess, *the* Draughts, *and some others: The Second of those that are govern'd by Chance; as the* Dice, *and such like: The Third Sort is*

of

of those that are subjected to the Laws of Motion, and require an Exactness and Regularity thereof; such as Shooting with Guns, and with Bows, the Tennis, *and* Billiards. *There are some Plays of a mixed Nature, depending partly on Skill, partly on Chance; as the* Tables, *the* Cards, *and most others. But 'tis certain, there is none of 'em which might not be so far subjected to the Rules of the Mathematicks, that one might be assured of the Victory, had he but all the Understanding requisite. Games of Dexterity depend so much upon Principles of Staticks and Mechanicks, that 'tis only the Want of a due Knowledge of their Rules, or of the Way of reducing 'em to Practice, that makes a Man fall short of Conquest.*

In all Plays of Chance whatever, the Victory depends upon the coming up of a certain Number, upon Weight, or upon the Dimensions of a Figure. The Gamester that gives the Motion, might at pleasure determine the End of it, were his Skill and Dexterity perfect; and tho' this does not seem to be possible, there being none to be found Master of so much Cunning; yet 'tis true that this might be done, and that an infallible Method of Winning, at Chess *for instance, is not absolutely impossible: But no Body has hitherto found it out; nor perhaps ever will, seeing it depends on too great a Number of Combinations. 'Tis enough that the Point of Perfection is possible, to encourage the Labour of the Curious.* " A perfect Orator, *said* Tully, never was, " and yet is possible. *His Picture drawn by*

 " *that*

that famous Master, may be a Pattern for the Imitation of those who study to excel in Eloquence. The like may be said of a Poet, a Painter, an Architect, a Physician, and all others. In like manner, tho' 'tis true that no one has attained an infallible Method in all Plays, nor perhaps in any one; this ought to hinder none from endeavouring to become as skilful as he can, and to come up as near as may be to the Idea of that Method, which, because founded upon Principles of Mathematicks, must participate of a Mathematical Certainty.

It may possibly be thought an Extraordinary Attempt to endeavour to proselyte Gamesters to this Opinion, and to engage Statesmen and great Commanders in the Study of Mathematical Recreations: Notwithstanding *there can be no Harm in Carrying the Light, let who will follow after it: Yea, is it possible to hinder Mankind from learning what is built on the most natural Principles, and on Truths flowing from the Essence of Things? Should they be deprived of Pleasures so inviting by their Utility; and which are so familiar, so easie, and so suited to all endowed with Reason, that to bereave Men of them, were to rob 'em of what is most agreeable in Life.*

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PROBLEMS

Mathematical and *Physical* RECREATIONS.

Arithmetical PROBLEMS.

PROBLEME I.

A blind Abbess, visiting her Nuns, who were equally distributed in eight Cells built at the four Corners of a Square, and in the Middle of each Side; finds an equal Number of Persons in each Row or Side containing three Cells: At a second Visit, she finds the same Number of Persons in each Row, tho' their Number was enlarg'd by the Accession of four Men: And coming a third time, she still finds the same Number of Persons in each Row, tho' the four Men were then gone, and had carry'd each of 'em a Nun with 'em.

TO resolve the first Case, when the four Men were got into the Cells, we must conceive it so, that there was a Man in each Corner-Cell, and that two Nuns remov'd from thence to each of the Middle-Cells: At this rate, each Corner-Cell contain'd one Person less than before; and each Middle-Cell two more than before. Suppose then, that at the first Visitation, each Cell contain'd 3 Nuns; and so, that there were nine in each Row, and twenty-four in all; at the second Visit, which is the first Case in question, there must have been five Nuns in each

3	3	3
3		3
3	3	3

2	5	2
5		5
2	5	2

Middle-Cell, and two Perſons, *viz.* a Man and a Nun in each Corner-Cell; which ſtill makes nine Perſons in each Row.

To account for the ſecond Caſe, when the four Men were gone, and four Nuns with them; each Corner-Cell muſt have contain'd one Nun more than at the firſt Viſit, and each Middle-Cell two fewer: And thus, according to the Suppoſition laid down, each Corner-Cell contain'd four Nuns, and there was only one in each Middle-Cell; which ſtill make nine in a Row, tho' the whole Number was but twenty.

4	1	4
1		1
4	1	4

PROBLEME II.

To ſubſtract, with one ſingle Operation, ſeveral Sums, from ſeveral other Sums given.

Operation of *Subſtraction* ſhortned.

TO ſubſtract all the Sums which are under the Line at B, from all the Sums above the Line at A; begin by adding the Numbers or Figures of the Right-hand Column under the Line, ſaying, 8 and 4 is 12, and 2 makes 14; which taken from the neareſt Tens, *viz.* 20, there remains 6; which we add to the correſponding Column above, ſaying, 6 and 8 make 14 and 2 is 16, and 4 make 20, and 3 make 23: here we write 3 underneath; and, in regard there are juſt two Tens, as before, we retain or carry nothing. This done, we add after the ſame manner, the Numbers of the next lower Column, ſaying, 0 and 5 is 5, and 4 make 9; which taken from the neareſt Ten, leaves 1; which we add, as above, to the ſuperior correſponding Column, ſaying, 1 and 4 make 5, and 5 makes 10, and 6 makes 16, and 4 makes 20: here we ſet 0 underneath; and there being here two Tens, whereas in the inferior correſponding Column there was but one, we keep or carry the Difference 1 to be taken from

56243	
84564	A
3252	
26848	
———	
2942	
3654	B
2308	
———	
162003	

from the next inferior Column, because we found more Tens in A than in B: For had we found fewer in A than in B, we must have added the Difference; and if it should so fall out, that this Difference can not be taken from the inferior Column, for want of significant Figures, as it happens here in the fifth Column; we must add it to the superior Column, and write the whole Sum under the Line. Thus in the Example propos'd, we have 162003, for the Remainder of the Substraction.

PROBLEME III.

Compendious Ways of Multiplication.

Compendious ways of Multiplication.

TO multiply any Number, 128 for instance, by a Number that 's the Product of the Multiplication of two other Numbers; 24 for instance, the Product of the Multiplication of 4 and 6, or of 3 and 8: we multiply the Number propos'd 128 by 4, and the Product 512 by 6, (or else 128 by 3, and the Product by 8) and have 3072 for the requir'd Multiplication.

Hence it follows, that to multiply a Number propos'd by a square Number, we must multiply the Number propos'd by the Side or Root of the Square, and then the Product by the same Side again. Thus to multiply 128 by 25, we multiply it by 5, and the Product by 5 again.

To multiply any Number, 128 for instance, by a Number that 's the Product of the Multiplication of *three* other Numbers, as 108 the Product of 2, 6, and 9, or of 3, 6, and 6: we multiply 128 by 2, the Product by 6, and the second Product by 9; or else 128 by 3, the Product by 6, and the second Product by 6.

The Consequence of this is, that to multiply any Number propos'd, by a Cube-Number, we must multiply it first by the Side or Root of the Cube; then the Product of that Multiplication by the same Root, and the second Product by the Side again. As, to multiply 128 by 125, the Cube-Root of which is 5, we multiply 128 by 5, and the Product 640 by 5 again, and the second Product 3200 by 5 again. Thus to find how many Cubical Feet are in 32 Cubical Toises, we multiply 32 by 6, the Product of that by 6, and the second Product by 6.

 To

To multiply any Number by what Power you will of 5, add to the Number propos'd, on the Right-hand, as many Cyphers as the Exponent of the Power contains Unites, as, one Cypher for 5, two for its Square 25, three for its Cube 125, and so on; and divide the Number thus augmented by the like Power from 2; that is, 2 for 5, 4 for its Square 25, 8 for its Cube 125, and so on.

Thus to multiply 128 by 5, we divide 1280 by 2, and the Quotient 640 is the Product of the Multiplication: But to multiply 128 by 25 the Square of 5, we divide 12800 by 4 the Square of 2, and the Quotient is the Product demanded; and to multiply the same Number 128 by 125 the Cube of 5, we divide 128000 by 8 the Cube of 2. And so on.

To know how many Inches are in 53 Foot, we multiply 53 by 12; or it might be done by multiplying 53 by 2, and the Product by 6; or 53 by 3, and the Product by 4. But there's a way of doing it without any Multiplication; *viz.* by setting down 53 under 53, and then 53 again under both, advancing it a Column to the Left, so as to make 3 stand under 5; for the Sum of these three is 636, the Number of Inches contain'd in 53 Foot, or of Pence in 53 Shillings.

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 53
 53
53
———
636
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To multiply together two Numbers compos'd of several Figures, 12, for Instance, and 18; we reduce the first Number, 12, into these three parts, each of which consists only of one Figure, 2, 4, and 6; and in like manner, the second Number, 18, into 4, 6, 8; each of which last must be multiply'd by 2, the first part of the first Number; and then by 4, the 2d Figure of the same first Number; and at last by 6, the third part: and the Sum of all these Products answers the Demand.

PRO-

PROBLEME IV.

Division shorten'd.

TO divide a large Number by a smaller, by only *Addition* and *Substraction*, as 1492992 by 432; we commonly put the Divisor to the Left, under 1492, to know how many times 'tis contain'd in that Number. But yet we may save our selves that Labour, by making a Tariff of the Divisor; for which end we place it on the Right over-against 1; then add it to itself, or double it, and place that over-against 2: Then we add it to the Double, and place the Sum opposite to 3; adding it to the Triple, we have its Quadruple opposite to 4; as the Additional of itself to the Quadruple, gives the Quintuple opposite to 5; and so of the other Multiples opposite to 6, 7, 8, 9, 10: The last of which, *viz.* the Multiple corresponding to 10, ought, if the Table is right done, to be the single Divisor with a Cypher on the Right-hand. *Division shorten'd.*

1	432
2	864
3	1296
4	1728
5	2160
6	2592
7	3024
8	3456
9	3888
10	4320

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1492992 ( 3456
1296...
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 1969
 1728
 ----
 2419
 2160
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 2592
 2592
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  000
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Having thus prepar'd your Table, proceed in the common way of *Division*; and every time you have occasion to know how often your Divisor is contain'd in the corresponding Number, look in your Table for the nearest Number that does not exceed; and the Number to which that is opposite gives you at one view the Figure you're to put in your Quotient. As, in the beginning of the Division here exemplify'd, you want to know how often 432 is to be found in 1492;

in your Table, you find 1296 (the nearest Number to 1492 and not exceeding it) opposite to 3, and accordingly 3 is the first Figure of your Quotient; and so of all the rest.

This Way is very convenient, when we have occasion to divide large Numbers by a smaller Number; for the Tariff of our Divisor keeps us from being at a stand, by resolving us readily upon all our Divisions. This is frequently the Case of Surveyors of Land, who have occasion to divide large Numbers by 144, when they want to reduce square Inches into square Feet; or by 1728. when they want to reduce cubical Inches into cubical Feet.

To divide any Number by what Power you will of 5, multiply it by the like Power of 2, and cut off from the right hand of the Product as many Figures as there are Unites in the Degree of the Power; the remaining Figures on the left, will represent the Quotient of the Division, and those struck off, will be the Numerator of a Fraction, the Denominator of which will be the like Power of 10.

To divide any Number by a smaller, that is the Product of the Multiplication of two yet smaller Numbers, divide the Number propos'd, by one of the two smaller, and the Quotient by the other; and the second Quotient arising from the last Division, is what you want.

Thus to divide 20736 by 24, the Product of 3 and 8, or of 4 and 6, we take the 8th part of it's 3d, or the 6th part of it's 4th, or, (which is the same thing) we take the 3d of it's 8th part, or the 4th of it's 6th, and our Quotient proves 1728.

Hence to reduce square Feet to square Toises, (a Toise is 6 Foot) we must take the 6th part of the 6th part of the Number propos'd of square Feet, because a square Toise is 36 square Foot, and 6 times 6 is 36. Thus to reduce 542 square Feet to square Toises, we must take the 6th part of $90\frac{2}{6}$ (the 6th part of 542) and so have 15 square Toises and 2 square Feet, as the Value of 542 square Feet.

PRO-

PROBLEME V.

Of some Properties of Numbers.

I. NUmber 9 has this Property; that when it multiplies any number of Integers whatsoever, the Sum of the Figures in the Product is divisible by 9: Thus 53, multiplied by 9, makes the Product 477; the Figures of which, added together, *viz.* 7 and 7 and 4 make 18, which is exactly divisible by 9. Propertie of Nmmbers.

II. Take any two Numbers whatsoever, either one of the two, or their Sum, or their Difference is divisible by 3: Thus, of the two Numbers 6 and 5, 6 is divisible by 3; of 11 and 5 the Difference 6 is divisible by 3; of 7 and 5 the Sum 12 is divisible by 3.

III. The Product arising from the Multiplication of two Numbers, the Squares of which make a joint square Number, is divisible by 6: Thus 12 the Product of 3 and 4 the Squares of which, *viz.* 9 and 16, make together the square Number 25; this 12, I say, is divisible by 6.

To find two Numbers, the Squares of which make together a square Number, multiply any two Numbers, the one by the other, and the Double of the Product will be one of the two Numbers demanded, and the Difference of their Squares will be the other. Thus in 2 and 3, the Double of their Product 12, and 5 the Difference of their Squares (4 and 9) are two Numbers of that Quality, that their Squares 144 and 25 make together the square Number 169, the Root of which is 13. *See Prob. 6 and 7.* To find two Numbers, the Squares of which make together a square Number.

IV. The Sum and the Difference of any two Numbers, the Squares of which differ by a square Number, are, each of 'em, either a square number or the half of one: Thus, take the Numbers 6 and 10, their Squares 36 and 100 differ by the square Number 64; their Sum is 16, and their Difference 4, each of which is a square Number: Then take 8 and 10 for the two Numbers, their Squares 64 and 100, differ by the square Number 36; and the Sum 18, and the Difference 2, are the Halfs of the two square Numbers 36 and 4.

To find two Numbers, the Sum and Difference of which, are both square Numbers.

To find two Numbers, the Sum and Difference of which, are, each of 'em, a square Number, In which Case, the Squares of these two Numbers will likewise differ by a square Number: pitch upon any two Numbers, as 2 and 3, the Product of their Multiplication is 6, their Squares are 4 and 9; 13 the Sum of the two Squares, and 12 the Double of the Product of their Multiplications, are the Numbers we look for; for their Sum 25, and their Difference 1 are both square Numbers; and further, their Squares 169 and 144 differ by the Square Number 25.

To find two Numbers, the Sum and Difference of which, are each the Half or Double of a Square.

To find two Numbers, the Sum and Difference of which, are each of em the Half or the Double of a square Number, In which Case, their Squares will likewise differ by a square Number; Take any two Numbers, as 2 and 3, the Squares of which are 4 and 9; 13 the Sum of these two Squares, and 5 the Difference, are the two Numbers demanded; for their Sum 18 and their Difference 8, are the Halfs of the two square Numbers 36 and 16, and the Doubles of the two square Numbers 9 and 4; and farther, their Squares 169 and 25, differ by the square Number 144, the Root of which is 12.

How to know that a Number is not square.

V. Every square Number ends either with two Cyphers, or with one of the five Figures 1, 4, 5, 6, 9, which serves for a Rule *To distinguish when a Number propos'd is not square, viz.* when it does not end as above; nay, if it does end with two Noughts, and these are not preceded by any of the foregoing 5 Figures, we may rest assured 'tis not square.

How to know that a Fraction is not square.

VI. Every square Fraction, that is, every Fraction that has its square Root, is such, that the Product of the Multiplication of the Numerator by the Denominator is square. Thus *we know a Fraction is not square,* when that does not happen. Take the Fraction $\frac{28}{63}$, we know it to be square, because 1764, the Product of 28, multiplied by 63, is a square Number having 42 for it's Root; and so the square Root of the propos'd Fraction is $\frac{42}{63}$, retaining the same Denominator; or $\frac{28}{42}$, retaining the same Numerator, for either of these is equivalent to $\frac{2}{3}$, for the square Root of the propos'd Fraction $\frac{28}{63}$ or $\frac{4}{9}$.

When a Fraction is cubical.

VII. Any cubical Fraction, *i. e.* any that has its Cube-Root, is such, that if you multiply the Numerator by the Square of the Denominator, or the Denominator by the Square of the Numerator, the Product has its Cube-Root; and 'tis by this Rule that *we know when a Fraction*

is a

is a Cube Fraction, such is $\frac{24}{375}$ for 3375000, and 216000, the two Products of the two ways of Multiplication just mention'd, have 150 and 60 for their Cube Roots, and so the Cube Root of the Fraction $\frac{24}{375}$ is $\frac{150}{375}$ retaining the same Denominator, or $\frac{24}{60}$ retaining the same Numerator, for each of these Fractions is equal to $\frac{2}{5}$ as the Cube Root of the propos'd Fraction $\frac{24}{375}$.

VIII. Tho' 'tis not possible to find two Homogeneous Powers, the Sum and difference of which, are each of 'em a power of the same degree, that is, square Numbers if the two first are Squares, and Cube-Numbers if they are Cubical, &c. yet 'tis possible and very easy *to find two Triangular Numbers, the Sum and difference of which, are each of 'em a Triangular Number.*

To find two Triangular Numbers, the Sum and difference of which are Triangular Numbers.

Thus 15 and 21 are two Triangular Numbers, the sides of which are 5 and 6; and their Sum 36, and the difference 6, are likewise Triangular Numbers, having 8 and 3 for their sides. Again, 780 and 990 are Triangular Numbers, the sides of which are 39 and 44; and their Sum 1770 and the difference 210 are likewise Triangular Numbers, having 59 and 20 for their sides. Once more, 1747515 and 2185095 are Triangular Numbers, having 1869 and 2090 for their sides; and their Sum 3932610 and the difference 437580 are likewise Triangular Numbers, the sides of which are 2804 and 935.

What we call a Triangular Number.

By a *Triangular Number* we understand the Sum of the natural Numbers, 1, 2, 3, 4, 5, 6, beginning with Unit, and rising to what Number you will, the last and the greatest of which is call'd the side. Thus we know that 10 is a Triangular Number, the side of which is four, by reason that 'tis equal to the Sum of the first four natural Numbers, 1, 2, 3, 4, the last and greatest of which is 4. 'Twas call'd *Triangular*, because you may dispose 10 points in the form of an Equilateral Triangle, each side of which contains 4, and hence 'twas that 4 got the Name of the side of the Triangular Number 10.

To know if a Number propos'd is Triangular.

To know if a Number propos'd is Triangular, you must multiply it by 8, and add 1 to the Product, for if the Sum be Square, the propos'd Number is Triangular. Thus we know that 10 is Triangular, because 81 (the Sum of its Multiplication by 8, with the addition of 1) is a Square Number, having 9 for its Root.

IX. The difference of two Homogeneous Powers, as of two Square-numbers, of two Cube-numbers, &c. is divisible

divisible by the difference of their sides. Accordingly we find that 21 the difference of the two Square-numbers 25 and 4, the sides of which are 5 and 2, is divisible by 3 the difference of the Sides or Roots, the Quotient 7 being always equal to the Sum of the same Sides or Roots; and that 117, the difference of the Cubes 125 and 8, the Roots of which are 5 and 2, is divisible by 3 the difference of the Roots, the Quotient 39 being equal to the Product of the said Roots multiplied one into another, *viz.* 10, added to 29 the Sum of their Squares 25 and 4.

X. The difference of two Homogeneal Powers, the common Exponent of which is an even number, is divisible by the Sum of their Roots. Thus, 21 the difference of the two Square-numbers, 25 and 4, the Roots of which are 5 and 2, is divisible by 7, the Sum of the said Roots, the Quotient 3 being equal to the difference of the Roots; and 609 the difference of the Bi-quadrats 625 and 16, the Roots of which are 5 and 2, is divisible by 7, the Sum of the Roots, the Quotient 87 being equal to the Product arising from 3 the difference of the Roots, multiplied with 29 the Sum of their Squares 25 and 4.

XI. The Sum of two Homogeneal Powers, the common Exponent of which is an odd number, is divisible by the Sum of their Roots. Thus we know that 133 the Sum of the two Cubes 125 and 8, the Roots of which are 5 and 2, is divisible by 7 the Sum of these Roots, the Quotient 19 being equal to the Excess of the Sum of the Squares of the Roots (29) above the Product of the Roots (10) And that 3157 the Sum of the two Sursolids 3125 and 32, the Roots of which are 5 and 2, is divisible by 7 the Sum of the Roots; the Quotient 451 being equal to the Excess of 741 the Sum of the Bi quadrat Powers of the Roots 5 and 2 (625, 16) and of the Square of the Product of the same Roots (100,) its Excess I say above 290 the Product of the Sum of the Squares of the same Roots (29) multiplied by 10 the Product of the Roots themselves.

XII. All the powers of the natural Numbers 1, 2, 3, 4, 5, 6, *&c.* have as many Differences as their Exponents contain Units, the last Differences being always equal among themselves in each Power, that is, the second Differences, or the Differences of the Differences, in the Squares 1, 4, 9, 16, 25, 36, *&c.* for these second Differences make 2, the first being the uneven Numbers 3, 5 7, 9, 11, *&c.* The

&c. The third Differences, or the Differences of the Differences of the first Differences in the Cubes 1, 8, 27, 64, 125, 216, *&c.* for these third Differences make 6, the first being 7, 19, 37, 61, 91, *&c.* and the second Differences, i. e. the Differences of these Differences being 12, 18, 24, 30. *&c.* which rise by 6 for the third Difference, and so of the rest.

Gnomon and Pyramidial Numbers.

The same thing happens to *Polygon* Numbers form'd by the continual Addition of Numbers in continual Arithmetical Progression, which are call'd *Gnomons*, and of which the first is always an Unit, which is virtually any *Polygon* Number. The same is the case with Pyramidal Numbers, which are form'd by the continual Addition of Polygon Numbers consider'd as *Gnomons*, the first of which is always Unit: And in like manner with the *Pyramido-Pyramidal* Numbers, which are produced by the continual Addition of Pyramidal Numbers, consider'd as Gnomons, the first of which is always Unity.

When the *Gnomons* rise, or exceed one another by One, as 1, 2, 3, 4, 5, 6, *&c.* the Polygon Numbers 1, 3, 6, 10, 15, 21, *&c.* which are form'd from them are call'd *Triangular*, the Property of which is such that each of 'em being multiplied by 8, and the Product inlarged by Unity, the Sum is a Square-number, as we intimated above. And farther, 9 the Sum of the second and the third, omitting the first, is a Square-number, and 36 the Sum of the fifth and the sixth, omitting the fourth, is likewise Square, and so on.

When the *Gnomons* rise, or exceed one another by two Units, as the odd Numbers 1, 3, 5, 7, 9, 11, *&c.* the Polygon Numbers form'd from 'em 1, 4, 9, 16, 25, 36, *&c.* are Square-numbers; and when the *Gnomons* increase by three Units, as 1, 4, 7, 10, 13, 16, *&c.* the Numbers form'd from 'em, 1, 5, 12, 22, 35, 51, *&c.* are call'd Pentagons, and have this peculiar Quality that each of 'em being multiplied by 24, and 1 added to the Product, the Sum is a Square-number, by which Rule *we know when a propos'd Number is Pentagon*, and so of the others.

To find the Sum of as many Triangular Numbers as you will, commencing from Unit, of these eight for Instance, 1, 3, 6, 10, 15, 21, 28, 36, multiply the given Number 8 by the next follower 9, and the Product 72 by the next after that 10, and divide the second Product 720 by 6, the Quotient gives you 120 the Sum demanded.

The

The Sum of all these infinite Fractions $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{21}$, *&c.* the common Numerator of which is 1, and the Denominators of which are Triangular Numbers, their Sum, I say, is just 1.

To find the Sum of as many Square-numbers from an Unit as you will, of these eight, for Example, 1, 4, 9, 16, 25, 36, 49, 64, take 36 the last of as many Triangular Numbers, *viz.* 1, 3, 6, 10, 15, 21, 28, 36, from 240 the double of this Sum 120, and the remainder 204 is the Sum you want.

XIII. The Cubes, 1, 8, 27, 64, 125, 216, *&c.* of the natural Numbers, 1, 2, 3, 4, 5, 6, *&c* are such, that the first 1 is a Square-number, the Root of which 1 is the first Triangular Number; the Sum of the two first, 1 and 8, *viz.* 9, is a Square-number, the Root of which 3 is the second Triangular Number; 36 the Sum of the three first, 1, 8, 27, is a Square-number, the Root of which 6 is the third Triangular Number, and so on. And therefore if you want *to find the Sum of any Number of Cubick Numbers from an Unit,* of these six for Example, 1, 8, 27, 64, 125, 216, the Square of the sixth Triangular Number 21 (441) is the Sum you desire.

XIV. Among whole Numbers, there's only 2 that being added to its self, makes as much as when multiplied by its self, *viz.* 4, for all other Numbers make more by Multiplication than by Addition.

Tho' we can't find two whole Numbers, the Sum of which is equal to the Product of their Multiplication, yet we can easily *find two fractional Numbers, and even in a given* Ratio, *the Sum of which is equal to their Product, viz.* by dividing the Sum of the two Terms of the given *Ratio* by each of the two Terms; thus, if you give 'em the *Ratio* of the two Numbers, 2, 3, divide their Sum 5 separately by 2 and by 3, and you'll have the two Numbers $2\frac{1}{2}$, $1\frac{2}{3}$, which make as much when added together, as when multiplied together, *viz.* $4\frac{1}{6}$.

XV. Any Number is the half of the Sum of two others equally remote, the one in the way of defect, and the other in Excess. For Example, 6 is the half of 12, the Sum of the two Numbers equally remote, 5 and 7, or 4 and 8.

XVI. The Number 37 has this Property, that being multiplied by any of these Numbers, 3, 6, 9, 12, 15, 18, 21, 24, 27, which are in continual Arithmetical

Pro-

Progression, all the Products are compos'd of one Figure thrice repeated.

37	37	37	37	37	37	37	37	37
3	6	9	12	15	18	21	24	27
111	222	333	444	555	666	777	888	999

XVII. The two Numbers 5 and 6 are call'd *Spherical*, because their Powers terminate in these very Numbers. The Powers of 5, *viz.* 25, 125, 625, &c. terminate in 5, and in like manner the Powers of 6, *viz.* 36, 216, 1296, &c. end with 6.

Spherical Numbers.

5 has that peculiar Quality, that when multiplied by an odd Number (as 7) its Product terminates in 5 (as 35,) and when multiplied by an even Number (as 8) its Product ends in a Cypher, (as 40.)

The other Number, 6, has likewise this singular Quality, that 'tis the first of the Numbers which we call *perfect*, as being equal to the Sum of their Aliquot parts, for 6 is equal to the Sum of its Aliquot parts 1, 2, 3; 28 is likewise a *perfect* Number, in regard 'tis equal to the Sum of its Aliquot parts 1, 2, 4, 7, 14: And one may find an infinity of other perfect Numbers, as 496, which is equal to the Sum of its Aliquot parts 1, 2, 4, 8, 16, 31, 62, 124, 248.

What we call a perfect Number.

To find all the perfect Numbers in order, make use of the Powers of 2, *viz.* 2, 4, 8, 16, 32, &c. and see which of these Powers, when an Unit is taken from them, makes a prime Number, and you'll find in 4, 8, 32, &c. that if you subtract 1 from each of 'em, the Remainders 3, 7, 31, &c. are prime Numbers, each of which ought to be multiplied by the half of the corresponding Power, that is, 3 by 2, 7 by 4, 31 by 16, &c. in order to obtain the perfect Numbers 6, 28, 496, &c.

2.	4.	8.	16.	32.
	1	1		1
	3	7		31
	2	4		16
	6	28		496

To find all the Aliquot Parts, or all the Divisors of a propos'd Number, of which an Unit is always one. If the Number be 8128 (for Example) which is likewise a perfect Number, divide it by the least Number that offers, *viz.* 2, which is easily done, because 8128 is an even Number, so the Quotient will be 4064, which set down over against 2 for

To find all the Aliquot Parts of a Number.

for your second Divisor, which may still be divided by th first Divisor 2, and so its Square 4 may likewise be a Di visor, which set down under 2, over against the secon Quotient 2032 for another Divisor, which may still be di vided by the first Divisor 2, and therefore its Cube 8 wi likewise be a Divisor, which you are to write under the Square 4, and opposite to the third Quotient 1016 for another Divisor: Thus you go on, till you come to the last Divisor that can't be divided by 2, *viz*, the sixth Quotient 127, which being a prime Number, that is, a Number that can be divided by nothing but an Unit, gives us to know that we have traced all the Divisors of the Number propos'd 8128, and here you see the Sum of the Divisors is equal to the Number propos'd, and by consequence 'tis a perfect Number.

1	
2	4064
4	2032
8	1016
16	508
32	254
64	127
127	8001
	127
	8128

By the same Method did we find out all the Divisors of the other Number 2096128, which is likewise perfect, for as you see 'tis equal to the Sum of its Aliquot parts. You see likewise that the last Quotient 2047 which answers to 1024 the tenth Power of the first Divisor 2 is also a prime Number, for if it could have been divided by any other Number beyond 2, as by 3, it behoved us to have multiplied all the Powers of the first Divisor 2 by this new Divisor 3, and to have divided the Number propos'd and all the Quotient by this new Divisor 3, in order to have ther Divisors, as you'll see in the following Example.

1	
2	1048064
4	524032
8	262016
16	131008
32	65504
64	32752
128	16376
256	8188
512	4094
1024	2047
2047	2094081

XVIII. The Number 120 is equal to the half of 240, the Sum of its Aliquot parts 1, 2, 3 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60. The Number 672 is likewise equal to the half of 1344 the Sum of its Aliquot parts, as will appear by observing the Method above prescrib'd, which we shall not now repeat. We may find a great many other Numbers that have the same Quality; nay some may be found to be the third, or any other part of the Sum

Sum of their Aliquot parts, which we ſhall not now inſiſt upon.

XIX. The two Numbers 220 and 284 are call'd *Amiable*, becauſe the firſt 220 is equal to the Sum of the Aliquot-parts of the latter, 1, 2, 4, 71, 142; and reciprocally the latter 284 is equal to the Sum of the Aliquot-parts of the former, 1, 2, 4, 5, 10, 11, 22, 44, 55, 110. Theſe Aliquot-parts are eaſily found by what we have ſaid before, eſpecially if we conſider that all Numbers that end in 5 or in 0, are diviſible by 5. Amiable Numbers.

To find all the Amiable Numbers in order, make uſe of the Number 2, which is of ſuch a Quality, that if you take 1 from its Triple 6, from its Sexturiple 12, from the Octodecuple of its Square, 72, the remainders are the three prime Numbers 5, 11, and 71, of which 5 and 11 being multiplied together, and the Product 55 being multiplied by 4 the double of the Number 2, this ſecond Product 220 will be the firſt of the two Numbers we look for; and to find the other 284, we need only to multiply the third prime Number 71, by 4, the ſame double of 2, that we uſed before.

To find two other *Amiable* Numbers, inſtead of 2 we make uſe of one of its powers that poſſeſſes the ſame Quality, ſuch as its Cube 8; for you ſubſtract an Unit from its Triple 24, from its Sextuple 48, and from 1152 the Octodecuple of its Square 64, the Remainders are the three prime Numbers *viz.* 23, 47, 1151, of which the two firſt 23, 47 ought to be multiplied together, and their Product 1081 ought to be multiplied by 16 the double of the Cube 8, in order to have 17296 for the firſt of the two Numbers demanded. And for the other *Amiable* Number, which is 18416 we muſt multiply the third prime Number 1151 by 16 the ſame double of the Cube 8.

If you ſtill want other amiable Numbers, inſtead of 2, or its Cube 8, make uſe of its Square Cube 64, for it has the ſame Quality, and will anſwer as above.

In regard, 'tis difficult to know whether a Number is prime if it be a large Number, we ſhall at the end of this Problem Subjoyn a Table of all the prime Numbers between 1 and 10000.

XX. The Squares of the two Numbers 31, 34, *viz.* 961, 1156, are ſuch, that the firſt 961, with its Aliquot parts, 1, 31, makes a Sum (993) equal to 1, 2, 4, 17, 34, 68, 289, 578 the Aliquot parts of the ſecond 1156.

XXI. The

XXI. The two Numbers 26, 20, make, each of 'em with their Aliquot parts the same Sum; the first 26 with its Aliquot-parts 1, 2, 13, makes 42, and the second (20) with its Aliquot-parts 1, 2, 4, 5, 10, makes likewise 42.

The same is the case of 48 and 464, each of 'em with their Aliquot-parts making 930: of 11 and 6, each of 'em with their Aliquot-parts making 12; and in fine of 17 and 10, which with their Aliquot-parts make 18 a piece.

Nay, we may find three Numbers, each of which with its Aliquot-parts makes the same Sum, as 20, 26 and 41, as also 23, 14, 15, and 46, 51, 71.

We may find two Square-numbers of the same Quality, particularly 16 and 25 the Squares of 4 and 5; which are the lowest that can be, and by virtue of which we come at as many more as we will of the same Quality, *viz.* by multiplying them by some odd Square-number, that is not divisible by 5. For Example, if we multiply each of 'em by the Square-number 9, we obtain two other Square-numbers 144 and 225, each of which with its Aliquot-parts makes just 403.

XXII. 81 the Square of 9, with its Aliquot-parts 1, 3, 9, 27, makes a Square-number (121) the Root of which is 11. 400 the Square of 20, with its Aliquot-parts makes the Square of 31 (961.)

XXIII. 666 the Sum of these three Triangular Numbers 15, 21, 630, the sides of which are 5, 6, 35, is likewise a Triangular Number, the side of which is 36. The same is the case of these three Triangular Numbers 210, 780, 1711, and likewise of these 666, 2628, 5886.

XXIV. 49 the Square of 7 has this Quality, that 8 the Sum of its Aliquot parts, 1, 7, is the Cube of 2, and 343 the Cube of the same Number 7, does with its Aliquot parts, 1, 7, 49, make the Square-number 400, the Root of which is 20. I do not here pretend to direct you how to find out others of the same Quality, for unless you light on them by chance, 'tis very difficult to trace 'em without Algebra, which I propose not to mention in this Performance.

XXV. 9 the Square of 3 has this Quality, that 4 the Sum of its Aliquot-parts 1, 3, is the Square of 2. 2401 the Square of 49 has the same Quality, for 400 the Sum of its Aliquot-parts 1, 7, 49, 343 is the Square of 20.

XXVI. The

XXVI. The two Numbers 99, 63, have this Quality, that (57) the Sum of the Aliquot-parts of the first, 1, 3, 9, 11, 33, surpasses (41) the Sum of the Aliquot-parts of the second, 1, 3, 7, 9, 21, by the Square-number 16, the Root of which is Four. The same is the condition of 325 and 175; for the Sum of the Aliquot-parts of the first exceeds that of the Aliquot-parts of the other, by the Square-number 36.

XXVII. The Sum of Two-numbers that differ by Unity, is equal to the Difference of their Squares; and the Sum of the Squares of their Triangular-numbers is likewise a Triangular-number. Thus 5 and 6 make the Sum 11 equal to the difference of their Squares 25, 36, and their Triangular-numbers 15, 21, are such, that 666 the Sum of their Squares, 225, 441, is likewise a Triangular-Number, the side of which is 36.

XXVIII. The two Triangular-numbers, 6, 10, of the Two-numbers, 3, 4, the Difference of which is likewise an Unity, have this Quality, that their Sum 16, and their Difference 4, are Square-numbers, having 4 and 2 for Roots; and 136 the Sum of heir Squares (36, 100) is a Triangular-number, the side of which 16 is likewise a Square-number, the Root of which is at the same time a Square-number, having 2 for its side or Root.

The same is the Quality of the two other Triangular Numbers, 36, 45, the sides of which, 8, 9, differ only by Unity, for their Sum 81, and their Difference 9, are Square-numbers, the Roots of which are 9 and 3, and 3321 the Sum of their Squares (1296, 2025) is a Triangular-number, the side of which is 81, and that has its Square Root 9, which again is the Square of 3.

There are many other Triangular-numbers of this Quality, that may be found out by substracting and adding any Square-number to its Square, the halves of the Remainder and of the Sum being the two Triangular-Numbers demanded. For Example, if you subtract 8 the Square-number 16 from and add it to, its Square 256, half the Remainder 240, and half the Sum 272, present us with 120, and 136, for the two Triangular-numbers thought for, the sides of which are, 15, 16, the difference consisting still in Unity.

These two Triangular-numbers thus found, have this farther Quality, that the greatest of their Sides is always a Square-number, and the Difference of their Squares is likewise a Square-number; and withal their Sum is a Biquadrate, equal to the Square of their Difference, and at the same time to the side of the Triangular-number that composes the Sum of their Squares.

XXIX. The Difference of the Squares of two Numbers in a duplicate *Ratio*, is equal to the Sum of their Cubes divided by the Sum of their Two-numbers, and that very Sum of their Cubes is the third of a Cube.

Accordingly, 4 and 8 being in a duplicate *Ratio*, the difference 48 of their Squares, 16, 64, is equal to the Quotient resulting from the Division of 576 (the Sum of their Cubes, 64, 512) by 12 the Sum of the Two-numbers, and the very Sum of their Cubes 576 is the third part of the Cube 1728, the Root of which 12 is always equal to the Sum of the Two-numbers.

I should never have done, if I pretended here to fetch in all the Properties of Numbers, which indeed are infinite, and upon that consideration I shall now conclude this Problem with the Table of the Prime-numbers that I promis'd above.

Table

Table of the Prime Numbers between 1 and 10000.

2	193	433	691	991	1283	1579	1889	2213	2539	2837
3	197	439	—	997	1289	1583	—	2221	2543	2843
5	199	443	701	—	1291	1597	1901	2237	2549	2851
7	—	449	709	1009	1297	—	1907	2239	2551	2857
11	211	457	719	1013	—	1601	1913	2243	2557	2861
13	223	461	727	1019	1301	1670	1931	2251	2579	2879
17	227	463	733	1021	1303	1609	1933	2267	2591	2887
19	229	467	739	1031	1307	1613	949	2269	2503	2897
23	233	479	743	1033	1319	1619	1951	2374	—	—
29	239	487	751	1039	1321	1621	1973	2281	2609	2903
31	241	491	757	1049	1327	1627	1979	2287	2617	2909
37	251	499	761	1051	1361	1637	1987	2293	2621	2917
41	257	—	769	1061	1367	1657	1993	2297	2633	2927
43	263	503	773	1063	1373	1663	1997	—	2647	2939
47	269	509	787	1069	1381	1667	1999	2309	2657	2953
53	271	521	797	1087	1399	1669	—	2311	2659	2957
59	277	523	—	1091	—	1693	2003	2333	2663	2963
61	281	541	811	1093	1409	1697	2011	2339	2671	2969
67	283	547	821	1097	1423	1699	2017	2341	2677	2971
71	293	557	823	—	1427	—	2027	2347	2683	2999
73	—	563	827	1103	1429	1709	2029	2351	2687	—
79	307	569	829	1109	1433	1721	2039	2357	2689	3001
83	311	571	839	1117	1439	1723	2053	2371	2693	3011
89	313	577	853	1123	1447	1733	2063	2377	2699	3019
97	317	587	857	1129	1451	1741	2069	2381	—	3023
—	331	593	859	1151	1453	1747	2081	2383	2707	3037
101	337	599	863	1153	1459	1753	2083	2389	2711	3041
103	347	—	877	1163	1471	1759	2087	2393	2713	3049
107	349	601	881	1171	1481	1777	2089	2399	2719	3061
109	353	607	883	1181	1483	1783	2099	—	2729	3067
113	359	613	887	1187	1487	1787	—	2411	2731	3079
127	367	617	—	1193	1489	1789	2111	2417	2741	3083
131	373	619	907	—	1493	—	2113	2423	2749	3089
137	379	631	911	1201	1499	1801	2129	2437	2753	—
139	383	641	919	1213	—	1811	2131	2441	2767	3109
149	389	643	929	1217	1511	1823	2137	2447	2777	3119
151	397	647	937	1223	1523	1831	2141	2459	2789	3121
157	—	653	941	1229	1531	1847	2143	2467	2791	3137
163	401	659	947	1231	1543	1861	2153	2473	2797	3163
167	409	661	953	1237	1549	1867	2161	2477	—	3167
173	419	673	967	1249	1553	1871	2179	—	2801	3169
179	421	677	971	1259	1559	1873	—	2503	2803	3181
181	431	683	977	1277	1567	1877	2203	2521	2819	3187
191			983	1279	1571	1879	2207	2531	2833	

Table of the Prime Numbers between 1 and 10000.

3191	3533	3877	4229	4597	4967	5323	5683	6053	6379
—	3539	3881	4231	—	4969	5333	5689	6067	6389
3203	3541	3889	4241	4603	4973	5347	5693	6073	6397
3209	3547	—	4243	4621	4987	5351	—	6079	—
3217	3557	3907	4253	4637	4993	5381	5701	6089	6421
3221	3559	911	4259	4639	4999	5385	5711	6091	6427
3229	3571	3917	4261	4643	—	5393	5717	—	6449
3251	3581	3919	4271	4649	5003	5399	5737	6101	6451
3253	3583	3923	4273	4651	5009	—	5741	6113	6469
3257	3593	3929	4283	4657	5011	5407	5743	6121	6473
3259	—	3931	4289	4663	5021	5413	5749	6131	6481
3271	3607	3943	4297	4673	5023	5417	5779	6133	6491
3299	3613	3947	—	4679	5039	5419	5783	6143	—
—	3617	3967	4327	4691	5051	5431	5791	6151	6521
3301	3623	3989	4337	—	5059	5437	—	6163	6529
3307	3631	—	4339	4703	5077	5441	5801	6173	6547
3313	3637	4001	4349	4721	5081	5443	5807	6197	6551
3319	3643	4003	4357	4723	5087	5449	5813	6199	6553
3323	3659	4007	4363	4729	5099	5471	5821	—	6563
3329	3671	4013	4373	4733	—	5477	5827	6203	6569
3331	3673	4019	4391	4751	5101	5479	5839	6211	6571
3343	3677	4021	4397	4759	5107	5483	5843	6217	6577
3347	3391	4027	—	4783	5113	—	5849	6221	6581
3359	3697	4049	4409	4787	5119	5501	5851	6229	6599
3361	—	4051	4421	4789	5147	5503	5857	6247	—
3371	3701	4057	4423	4793	5153	5507	5861	6257	6607
3373	3709	4073	4441	4799	5167	5519	5867	6261	6619
2389	3719	4079	4447	—	5171	5521	5869	6269	6637
3391	3727	4091	4451	4801	5179	5527	5879	6271	6653
—	3733	4093	4457	4813	5189	5531	5881	6277	6659
3407	3739	4099	4463	4817	5197	5557	5897	6287	6661
3413	3761	—	4481	4831	—	5563	—	6299	6673
3443	3767	4111	4483	4861	5209	5569	5903	—	6679
3449	3769	4127	4493	4871	5227	5573	5923	6301	6689
3457	3779	4129	—	4877	5231	5581	5927	6317	6691
3461	3793	4133	4507	4889	5233	5591	5939	6313	—
3463	3797	4139	4513	—	5237	—	5953	6 23	6701
3467	—	4153	4517	4903	5261	5623	5981	6329	6703
3469	3803	4157	4519	4909	5273	5639	5987	6337	6709
3491	3821	4159	4523	4919	5279	5641	—	343	6719
3499	3823	4177	4547	4931	5281	5647	6007	635	6733
—	3833	—	4549	4933	5297	5651	6011	6353	6737
3511	3847	4201	4561	4937	—	5653	6029	6359	6761
3517	3851	4211	4567	4943	5303	5657	6037	6361	6763
3527	853	4217	4583	4951	5309	5659	6043	6 67	6779
3529	3863	4219	4591	4957		5669	6047	6373	6781

Table of the Prime Numbers between 1 and 10000.

6791	7103	7459	7723	8089	8419	8737	9049	9397	9719
6793	7109	7477	7727	8093	8423	8741	9059	—	9721
—	7121	7481	7741	—	8429	8747	9067	9403	9733
6803	7127	7487	7753	8101	8431	8753	9091	9413	9739
6823	7129	7489	7757	8111	8443	8761	—	9419	9743
6827	7151	7499	7759	8117	8447	8779	9103	9421	9749
6829	7159	—	7789	8123	8461	8783	9109	9431	9767
6833	7177	7507	7793	8147	8467	—	9127	9433	9769
6841	7187	7517	—	8161	—	8803	9133	9437	9781
6857	7193	7523	7817	8167	8501	8807	9137	9439	9787
6863	—	7529	7823	8171	8513	8819	9151	9461	9791
6869	7207	7537	7829	8179	8521	8821	9157	9463	9791
6871	7211	7541	7841	8191	8527	8831	9161	9467	—
6883	7213	7547	7853	—	8537	8837	9173	9473	9803
6899	7219	7549	7867	8209	8539	8839	9181	9479	9811
—	7229	7559	7873	8219	8543	8849	9187	9491	9817
6907	7237	7561	7877	8221	8563	8861	9199	9497	9829
6911	7243	7573	7879	8231	8573	8863	—	—	9833
6917	7247	7577	7883	8233	8581	8867	9203	9511	9839
6947	7253	7583	—	8237	8597	8887	9209	9521	9851
6949	7283	7589	7901	8243	8599	8893	9221	9533	9857
6959	7297	7591	7907	8263	—	—	9227	9539	9859
6961	—	—	7917	8269	8609	8923	9239	9547	9871
6967	7307	7603	7927	8273	8623	8929	9241	9551	9883
6971	7309	7607	7933	8287	8627	8933	9257	9587	—
6977	7321	7621	7937	8291	8629	8941	9277	—	9887
6983	7331	7639	7949	8293	8641	8951	9281	9601	9901
6991	7333	7643	7951	8297	8647	8963	9283	9613	9907
6997	7349	7649	7963	—	8663	8969	9293	9619	9923
—	7351	7669	7993	8311	8669	8971	—	9623	9929
7001	7369	7673	—	8317	8677	8999	9311	9629	9931
7013	7393	7681	8009	8329	8681	—	9319	9631	9941
7019	—	7687	8011	8353	8689	9001	9323	9643	9949
7027	7411	7691	8017	8363	8693	9007	9337	9649	9967
7039	7417	7699	8039	8369	8699	9011	9341	9661	9973
7043	7433	—	8053	8377	—	9013	9343	9677	—
7057	7451	7703	8059	8387	8707	9029	9349	9679	
7069	7457	7717	8069	8389	8713	9041	9371	9689	
7079			8081	—	8719	9043	9377	9697	
—			8087		8731		9193	—	

PRO-

PROBLEM VI.

Of Right Angled Triangles in Numbers.

BY a Rectangular Triangle in Numbers, we mean three unequal Numbers, the greatest of which is such that its Square is equal to the Square of the other two. Such are 3, 4, 5, for 25 the Square of 5 the greatest, which we call the *Hypothenuse*, is equal to the Sum of 9 and 16, the Squares of the other Two-numbers, 3, 4, which we call the Sides, taking one for the *Base* of the Right-Angled-Triangle, and the other for the Altitude, or Height. Half the Product of the *Base* and the *Altitude*, is call'd the *Area*, and is always divisible by 3. The Reader will observe all along that by the Product of Two-numbers, we understand the Number arising from their mutual Multiplication.

There's an infinite number of Right-Angled Triangles, of divers sorts, both in whole and in broken or Fractional-numbers, but we generally conceive them in integers, among which the first and the least of all is that now mention'd, 3, 4, 5, which has an infinity of fine Properties, but 'twould be tedious to enumerate 'em, and therefore I shall content my self with observing, that the Sum (216) of the Cubes (27, 64, 125) of the two sides, 3, 5, and of the *Hypothenuse* (5) is a Cube, the Root or Side of which (6) is equal to its Area.

To find in Numbers as many Right-Angled Triangles as you will: Take any Two-numbers, for Example 2 and 3, which we call *Generating-Numbers*, multiply 'em the one by the other, and (12) the double of their Product (6) is the side of a Right-lined-Triangle, the other side being equal to (5) the difference of the Squares (4, 9) of the Generating-numbers, 2, 3, and the *Hypothenuse* being equal to (13) the Sum of the same Squares, 4, 9. And thus you have this Right-Angled-Triangle 5, 12, 13, for 169, the Square of the *Hypothenuse* 13, is equal to the Sum of 25, 144, the Squares of the two Sides 5, 12.

The first Right-Angled-Triangle, having 1, 2, for its Generating-numbers is such, that the difference of the two Sides 3, 4, is 1; and if you want to *find another of the same Quality*, take 2 the greatest of these Generating-Numbers

Numbers for the least of the Two in the Triangle demanded; and in order to find the greatest for this second Triangle, add 1 the least of the first to 4 the double of the greatest of the first, and so you have 5 for your greatest Generating-number of the second Right-angled Triangle, which consequently is 20, 21, 29, where the difference of the two Sides 20, 21, is again 1.

If you desire a third Right-angled-triangle of the same Quality, make use of the last 20, 21, 29, after the same manner as you did the first, taking its greatest Generating Number for the least of the Third, and adding its least to the double of the greatest, for the greatest of this your Third Triangle; and so observing the same Method you may find a fourth, fifth, *&c.* as appears by this Table.

Sides		Hypoth.	Generat-numb.	
3	4	5	1	2
20	21	29	2	5
119	120	169	5	12
696	697	1025	12	29
4059	4060	5741	29	70
23660	23661	33461	70	169

The first Right-angled-triangle 3, 4, 5, has likewise this Quality, that the Excess of the Hypothenuse 5 above the Side 4, is also 1, for as much as the difference of the two Generating-numbers is 1, and for this reason you may *find an infinite number of other Right-angled-triangles of this Quality*, if for their Generating-numbers you take two that differ only by Unity, as you see in this Table.

Bases	Altitude.	Hypoth.	Generat-numb.	
3	4	5	1	2
5	12	13	2	3
7	24	25	3	4
9	40	41	4	5
11	60	61	5	6
14	84	85	6	7

Here you see the first Differences of the Bases, 3, 5, 7, 9, *&c.* are equal, and the second Differences of the Altitudes, 4, 12, 24, 40, *&c.* are likewise equal; and the same is the case of the Hypothenuses, 5, 13, 25, *&c.*

Here the Bases are odd Numbers, and if you *would have 'em the Squares of these odd Numbers*, only take the Altitudes and Hypothenuses for the Generating-numbers of the Triangles you propose, which by consequence will run thus

Bases	Heights	Hypoth.	Gen. numb.	
9	40	41	4	5
25	312	313	12	13
49	1200	1201	24	25
81	3280	3281	40	41
121	7320	7321	60	61
169.	14280.	14281.	84.	85.

If instead of one side you would have the *Hypothenuse to be the Square-number*, then your Generating-numbers must be the Sides of a Right-angled-triangle, as in the following Scheme, where you see the Hypothenuse is the Square of the greatest Generating-number, with the addition of 1.

Sides		Hypoth.	Gen. numb.	
7	24	25	3	4
119	120	169	5	12
336	527	625	7	24
720	1519	1681	9	40
1320	3479	3721	11	60
2184	6887	7225	13	84

The Right-angled Triangle, 21, 28, 35, has this Quality, that the two Sides 21, 28, are Triangular-numbers, the Sides of which, 6 and 7, differ only by Unity, and the Square (1225) of the Hypothenuse (35) is likewise a Triangular-number, the Side of which is 49.

The same is the Quality of the Triangle 820, 861, 1189, as also of the Triangle 28441, 28680, 40391. and of others.

The following Right angled Triangles, which may be continued *in Infinitum*, are such that their Bases and Hypothenuses are Triangular-numbers, and their Heighths Cubick-numbers.

Bases

Bases	Heighths	Hypoth.	Gen.numb.	
6	8	10	1	3
36	27	45	3	6
120	64	136	6	10
300	125	335	10	15
630	216	666	15	21
1176	343	1225	21	28

You may *find as many such Triangles as you will*, by adding and subtracting a Square-number from its Square, for in the addition half the Sum is the Hypothenuse, and in subtracting half the Remainder is the Base, the Heighth being equal to the Cube of the Root of the first Square-number: or, which is the same thing, by taking for the Generating-numbers the Triangular-numbers in order, as you see in the Scheme before us, where the least Generating-numbers of one Right Angled Triangle is the greatest of the preceding Triangle.

PROBLEM VII.

Of Arithmetical Progression.

BY Arithmetical Progression, we mean a Series of Quantities call'd *Terms*, that rise continually by an equal Excess, as 1, 3, 5, 7, 9, 11, *&c.* where the Excess is 2, or 1, 4, 7, 10, 13, 16, *&c.* where the Excess is 3; or 2, 6, 10, 14, 18, 22, *&c.* where they rise by 4 at a time. And so of the rest.

The principal Property of Arithmetical Progression, is this. Take three continual *Terms*, as 6, 10, 14, the Sum (20) of the two Extremes (6, 14,) is equal to the double of the Middle-term (10.) Take four continual Terms, as 6, 10, 14, 18, the Sum (24) of the two Extremes (6, 18) is equal to that of the two Middle-terms, (10, 14.) In fine, in a larger Number of continual Terms, six for Instance, as 2, 6, 10, 14, 18, 22, the Sum (24) of the two Extremes (2, 22) is equal to that of any two Terms that lie at an equal distance from them, as 6, 18, and 10, 14. From whence 'tis easie to conclude, that when a multitude of Progressive Terms

is

is an odd Number, the Sum of the Extremes, or of those equally remote, is the double of the Middle-term, as in these five Terms, 2, 6, 10, 14, 18; for the Sum (20) of the Extremes 2, 10, 18, or of the two equally remote, 6, 14, is the double of the Middle-term, 10.

You may readily find such Numbers as have this Quality, that the sum of their Squares makes a Square-number, or, which is the same thing, the Sides of a Right-angled Triangle in Numbers; and that by vertue of this double Arithmetical Progression, $1\frac{1}{3}$, $2\frac{2}{5}$, $3\frac{3}{7}$, $4\frac{4}{9}$, &c. where the Excess is 2 in Fractions, and 1 in Whole-numbers, for if you reduce the Integer with its Fraction to a Fraction only, as $1\frac{1}{3}$ to $\frac{4}{3}$, the Numerator 4 and the Denominator 3 will be the Sides of the Right-angled Triangle 3, 4, 5; and in like manner if you reduce $2\frac{2}{5}$ to $\frac{12}{5}$ (which is done by multiplying the Whole-number 2 by the Denominator 5, and adding to the Product 10 the Numerator 2) the Denominator 5 and the Numerator 12, will be the Sides of the Right-angled Triangle 5, 12, 13. And so of the rest. Here you may see any odd Number may be one of the Sides of a Right angled Triangle in Whole-numbers.

Instead of the double Arithmetical Progression, you may make use of this, $1\frac{7}{8}$, $2\frac{11}{12}$, $3\frac{15}{16}$, $4\frac{19}{20}$, $5\frac{23}{24}$, &c. where the Excess is 4 in Fractions, and 1 in Whole-numbers, for if you reduce $1\frac{7}{8}$ to $\frac{15}{8}$, the Denominator 8, and the Numerator 15, will be two Sides of the Right-angled Triangle 8, 15, 17; and in like manner if you reduce $2\frac{11}{12}$ to $\frac{35}{12}$, the Denominator 12, and the Numerator 35, will be two Sides of another Triangle 12, 35, 37. And so on. Here you see any odd Number may be one of the Sides of a Right-Angled Triangle in Whole-numbers.

In an Arithmetical Progression, the Sum of the Terms is equal to the Sum of the two Extremes, multiplied by half the number of all the Terms. And for this Reason, in order *to find the Sum of any number of Terms in Arithmetical Progression*, for Example, the Sum of these eight, 3, 5, 7, 9, 11, 13, 15, 17, you must multiply the Sum (20) of the two Extremes (3, 17) by the number of the multitude of the Terms (8) for then half the Product (80 the half of 160) is the Sum you inquire for.

If on the other hand you know the Sum of the Terms, the first Term it self, and the number or multitude of the Terms, you may find out what the Terms are, by tracing the Excess in this manner. Suppose the given Sum of the Terms

Terms to be 80, the Number of 'em 8, and the first Term given 3, divide (160) the double of the Sum given (80) by the Number given (8) then subtract from 20 the Quotient, 6 the double of the first Term given 3, and at last divide the remainder 14 by the given Number wanting 1, that is 7, and the Quotient 2 is the Excess you look for, which added to the first Term gives you 5 for the second, and added to the second 7 for the third, and so on.

If the Sum of the Terms, their Number, and the Excess be given, we find out the first Term, and by consequence all the rest after the manner of the third Question ensuing.

Question I. *A Gentleman bargains with a Bricklayer to have a Well sunk upon these Terms; he's to allow him three Livres for the first Toise (a Toise is 6 Foot) of depth, 5 for the second, seven for the third, and so on, rising two Livres every Toise till the Well is twenty Toises deep: Query, how much will be due to the Bricklayer, when he has dig'd twenty Toises deep?*

To resolve this Question, multiply the 2 Livres Augmentation-Mony at every Toise, by the number of the Toises, bating 1, that is by 19, to the Product 38 add 6 the double of 3 the number of Livres promis'd for the first Toise, then multiply the Sum 44 by half the number of all the Toises, *viz.* 10, and the Product shews you 444 Livres due to the Bricklayer for sinking the Well 20 Toises deep.

Quest. II. *A Gentleman travell'd* 100 *Leagues in eight Days, and every Day travell'd equally farther than the preceding Day. Now it being discover'd that the first Day he travell'd two Leagues, the Question is how many Leagues he travell'd on each of the other Days.*

To resolve this Question, divide 200 the double of the Leagues given 100, by 8 the number of Days given, and from the Quotient 25, subtract 4, the double of 2 the given number of Leagues that he travell'd the first Day. Divide the Remainder 21 by 7, the given number of Days wanting one; and the Quotient 3 shews that he travell'd every Day three Leagues more than the Day before, from whence 'tis easy to conclude, that since he travell'd 2 Leagues the first Day, he travell'd 5 the second, 8 the third, and so on.

Quest. III. *A Traveller went* 100 *Leagues in* 8 *Days, and every Day three Leagues more than the preceding Day. 'Tis ask'd how many Leagues he travell'd a Day?*

Divide

Divide 200 the double of the Leagues given 100, by 8 the number of Days given, and from the Quotient 25 subtract 21, the Product of 3 the number of the daily increase multiplied by 7 the given number of Days bating one. The Remainder being 4 half it, and that shews you he travel'd 2 Leagues the first Day; from whence 'tis easy to gather that he travell'd 5 the second, 8 the third, and so on.

Quest. IV. *A Robber being pursued travell'd 8 Leagues a Day; an Archer, who was the pursuer, made but 3 Leagues the first Day, 5 the Second, 7 the third, and so on increasing 2 Leagues every Day. The Question is in how many Days the Archer will come up with the Robber, and how many Leagues they will have travel'd?*

To resolve this and such like Questions, add 2 the number of the daily increase of Leagues, by the Archer, to 16 the double of 8 the number of Leagues made every Day by the Robber: From the Sum 18 subtract 6 the duplicate of 3 the number of Leagues that the Archer travel'd the first Day. The Remainder 12, divide by 2 the number of the Archer's daily increase; and the Quotient 6 will shew you, that the Archer will come up with the Robber at the end of six Days, and consequently both of 'em must by that time have travel'd 48 Leagues, for six times 8 is 48, and the same is the Sum of these six Terms of Arithmetical Progression, 3, 5, 7, 9, 11, 13.

Quest. V. *We'll suppose, 'tis 100 Leagues from* Paris *to* Lions, *and that two Couriers set out at the same time, and took the same Road; one to go from* Paris *to* Lions, *making every Day 2 Leagues more than the Day before, and the other from* Lions *to* Paris *travelling every Day 3 Leagues farther than the preceding Day; And that they met exactly half way, the first at the end of 5 Days, and the other at the end of four Days. Query, how many Leagues these two Couriers travell'd each Day?*

To find how many Leagues the Courier travel'd every Day that was 5 Days upon the Road before he met the other; subtract 5 the number of Days from 25 the Square of it, and having multiplied the Remainder 20 by 2 the number of the daily increase of Leagues for this Courier; subtract the Product 40 from 100, the number of Leagues between *Paris* and *Lions*; and divide the Remainder 60 by 10 the double of 5 the number of Days; and the Quotient 6 will shew you, that the Courier travel'd vel'd 6 Leagues the first Day, and consequently 8 the second, 10 the third, 12 the fourth, and 14 the fifth.

In like manner with reference to the other Courier, that arriv'd half way in 4 Days, subtract 4 the number of Days from 16 its own Square, and having multiplied the Remainder 12 by 3 the number of his daily increase of Leagues, subtract the Product 36 from 100, the distance of Leagues from *Paris* to *Lions*; and divide the Remainder 64 by 8 the double of 4 the number of Days, and the Quotient 8 will shew you that this Courier travel'd 8 Leagues the first Day, and consequently 11 the second, 14 the third, and 17 the fourth.

Quest. VI. *There's a hundred Apples and one Basket, ranged in a strait Line at the distance of a Pace one from another; the Question is, how many Paces must he walk that pretends to gather the Apples one after another, and so put 'em into the Basket, which is not to be mov'd from its place?*

'Tis certain, that for the first Apple he must make 2 Paces, one to go and another to return; for the second 4, two to go, and two to return; for the third 6, three to go, and so on in this Arithmetical Progression, 2, 4, 6, 8, 10, &c. of which the last and greatest Term will be 200, that is, double the number of Apples. To 200 the last Term, add 2 the first Term, and multiply the Sum 202 by 50, which is half the number of Apples, or the number of the multitude of the Terms; and the Product 10100 will be the Sum of all the Terms, to the number of Paces demanded.

PROBLEM VIII.

Of Geometrical Progression.

BY *Geometrical Progression* we understand a Series of several Quantities that grow or rise continually thro' the multiplication of one and the same Number, as 3, 6, 12, 24, 48, 96, &c. where each Term is the double of the precedent Term; or, as 2, 6, 18, 54, 162, 486, &c. where each Term is the triple of its Antecedent. And so of others.

The principal Property of *Geometrical Progression*, is, that in three Terms continually proportional, as 3, 6 12, the

the Product 36 of the two Extremes, 3, 12, is equal to the Square of the middle Term 6: And that in four Terms in continual Proportion, as 3, 6, 12, 24, the Product 72 of the two Extremes 3, 24, is the same with the Product of the two means, 6, 12: And in fine, That in a greater number of Terms in continual proportion, as in these six, 3, 6, 12, 24, 48, 96, the Product 288 of the two Extremes 3, 96, is the same with that of 12, 24, two equally remote from it. From hence 'tis easy to conclude that when the number of the Terms is odd, this Product is equal to the Square of the Mean, as in these five Terms, 3, 6, 12, 24, 48; for 144 the Product of the two Extremes 3, 48, or of the two equally remote, 6, 24, is the Square of the Mean 12.

Thus you see that what Arithmetical Progression has by Addition, Geometrical Progression has it by Multiplication: But there's another considerable difference between these two Progressions, consisting in this; that in Arithmetical Progression the Differences of the Terms are equal, and in Geometrical Progression they are always unequal, and keep up among themselves the same Geometrical Progression, by continuing *in infinitum*, the Differences of Differences, without ever coming to equal Differences. Accordingly we see in this Geometrical Progressions 2, 6, 18, 54, 162, 486, the Differences of the Terms make just such another Geometrical Progression, 4, 12, 36, 108, 324; and in this last Progression the Differences of the Terms make again the like Geometrical Progression, 8, 24, 72, 216, and so on.

In three Proportional Terms, such as 2, 6, 18, the Cube 216 of the Mean 6, is equal to the solid Product of the three Terms multiplied together: And in four Numbers in continued proportion, such as 2, 6, 18, 54, the Cube 216 of the second 6, is equal to the solid Product arising from the Multiplication of 54 the fourth Term, by the Square of the first 2; and in like manner 5832 the Cube of the third Term 18, is equal to the solid Product of the first Term 2 multiplied by 2916 the Square of the fourth 54.

From what has been said 'tis easy *to find a Geometrical Mean proportional between two Numbers given*, by multiplying the one Number by the other, and extracting the square Root for the Mean proportional: And 'tis equally easy *to find two Means in continued Geometrical Proportion to two Numbers given*, as 2 and 54; by multiplying the last

laſt 54 by the Square of the firſt, and extracting the Cube Root (6) of the Product (216) for the firſt Mean proportional, which multiplied by the ſecond Number 54, makes 324, and 18 the Square Root of that Product is the ſecond Mean proportional.

But *to find an Arithmetical Mean Proportional to two Numbers given*, take half the Sum of the two Numbers for the Mean required; as in 2, 8 given, 5 the half of 10 is the Mean: And *to find two Arithmetical Means in continued Proportion* as between 2 and 11, we ſubtract the leaſt Number 2 from the greateſt 11, and add 3 a third part of the Remainder 9, to the leaſt Number 2, which gives us 5 for the firſt Mean; as the addition of 6, the double of that third part, to the ſame leaſt Number 2, does 8 for the ſecond. Or, if you will, you may add 4, the double of the leaſt 2, to 11 the greateſt, and reciprocally 22, the double of 11 the greateſt, to 2 the leaſt, and the thirds of the two Sums make 5 and 8 for the two Means demanded.

'Tis evident that all the Powers of the ſame Number, as 2, riſing in order, make a Geometrical Progreſſion, ſuch as this, where you ſee the Exponents of the Powers

(1)	(2)		(4)				(8)
2,	4,	8,	16,	32,	64,	128,	256, &c.
1	1		1				1
3	5		17				257

(1) (2) (4) (8) are the Terms of a Geometrical Proportion, *viz.* 2, 4, 16, 256, &c. and all the Powers are ſuch that if you add an Unit to each of 'em, the Sums 3, 5, 17, 257, &c. are prime Numbers: And ſo 'tis eaſy to find a prime Number greater than any Nmuber given.

If you continue a Geometrical Progreſſion upon the decreaſe *in infinitum*, as 6, 2, $\frac{2}{3}$, $\frac{2}{9}$, $\frac{2}{27}$, &c. the Difference 4 of the two firſt Terms 6 and 2 is to the firſt 6, as the ſame Number 6 is to the Sum of all the infinite Terms. And therefore, *to find the Sum of all the infinite Terms of a decreaſing Geometrical Proportion*, as that above, you muſt divide 36 the Square of the firſt Term 6, by 4 the Difference of the two firſt Terms, and the Quotient 9 is the Sum you want. If you take from this Quotient, 8 the Sum of the two firſt Terms 6 and 2, the remainder 1 is the Sum of the infinite Fractions continually proportional, $\frac{2}{3}$, $\frac{2}{9}$, $\frac{2}{27}$, &c. And by the ſame means we are taught that the Sum of other infinite Fractions in continued

tinued Proportion, amounts likewise to 1. This Rule gives the Solution of the following Question: But before I propose it, I must acquaint you, that,

When we speak of Quantities in Proportion, without specifying, we always mean Geometrical Proportion. Here I must observe by the by, that taking an Unit for Numerator, and the natural Numbers, 1, 2, 3, 4, 5, *&c.* for Denominators, if you make the following Series of Fractions, $\frac{1}{1}\ \frac{1}{2}\ \frac{1}{3}\ \frac{1}{4}\ \frac{1}{5}$, *&c.* which still decrease, these three taken consecutively from three to three at pleasure, will be in Harmonick Proportion; that is, the first of the three will be to the third, as the difference of the two first is to the difference of the two last; as will better appear by reducing these Fractions to the same Denomination, or to Integers, by multiplying them by the Number 60, which is divisible by all the Denominators 2, 3, 4, 5, for instead of the five Fractions you have the five Whole-numbers, 60, 30, 20, 15, 12; of which the three first 60, 30, 20, are fairly in Harmonick Proportion, for the first 60 is to the third 20 which is its third part, as 30 the difference of the two first is to 10 the difference of the two last, which is likewise the third part of 30. By the same consideration you will perceive that these three, 30, 20, 15 are in Harmonick Proportion as well as the other three 20, 15, 12.

Question, *A great Ship pursues a little one, steering the same way, at the distance of four Leagues from it, and sails twice as fast as the small Ship. 'Tis ask'd how far the great Ship must sail before it overtakes the lesser.*

The distance of the two Ships being 4, and their Celerities being in a double *Ratio*, continue *in infinitum*, the double Geometrical Progression, 4, 2, 1, $\frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}$, *&c.* the first and the greatest Term of which is 4; and find the Sum of all the infinite Terms, by dividing 16 the Square of the first 4, by 2 the difference of the two first, and the Quotient 8 directs that the great Ship must make 8 Leagues before she can come up with the other.

PRO-

PROBLEM IX.

Of Magical Squares.

BY a *Magical Square* we underſtand a Square divided into ſeveral other ſmall equal Squares, fill'd with Terms of an Arithmetical Progreſſion, ſo tranſpos'd, that all of the ſame Line or Rank, whether longitudinal, tranſverſe, or diagonal, make the ſame Sum.

This is the Square here annext, divided into 25 little Boxes or Squares, in which the firſt 25 natural Numbers are ſo tranſpos'd, that the Sum of each Rank from above downward, or from the right to the left, or along the Diagonals or Diameters of the Squares, is every way 65; which Sum 65 is in all an odd ſquare Number, that is, it contains an odd ſquare Number of Places, *viz.* 25. and is equal to the Product ariſing from 5, the Root of the ſquare Number 25; multiply'd by 13 the middle Term of the Arithmetical Progreſſion, 1, 2, 3, 4, &c.

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

This Sum is likewiſe found, by diſpoſing the given Terms of the Arithmetical Progreſſion, according to their natural Series 1, 2, 3, 4 &c. in the ſquare places, as you ſee here; for then the Sum of each diagonal Rank, that is, the Rank extending from one corner of the Square to the other, is the Sum demanded. This will likewiſe hold

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

in even Squares, or those which contain an even square Number of Boxes.

In order to dispose magically in the Boxes of an odd Square: For Instance, that of 25 Boxes, having 5 for its Side; to dispose, I say, as many given Numbers in Arithmetical Progression, as, 1, 2, 3, 4, 5, and so on till you come to the last, and greatest 25: Write the first and the least immediately under the middle Box, or that which possesses the Center of the Square; and moving Diagonal-wise to the Right, write the second Term 2 in the adjacent Box, the lower-most of the next Right-Hand Rank. Here proceeding in the course of the Diagonal from Left to Right you find no place for Number 3, and so are to place it in the opposite or uppermost Box of the Rank into which it should have fallen. In like manner, finding no place for 4, you are to place it in the opposite Box of the Rank that it falls to on the outside.

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

Thus you continue proceeding still diagonal-wise to the right; but in regard 6 falls to a place that's already fill'd with 1, you must there take a retrograde diagonal-Course from the right to the left, and write 6 in the lowermost station of the Rank in which the foregoing Term 5 was plac'd, and so there will remain an empty place between 5 and 6. This retrograde Course must always be observ'd when you fall in with a Station already possess'd. Continue to place the rest in order, according to these Rules till you come to the Angle of the Square, where in this Example stands 15: Then forasmuch as you can no longer move diagonalwise to the right, you must place the Term

Term 16 in the ſecond place (from the top) of the ſame Rank; this done, the reſt may be placed as the former, without any Difficulty.

There are ſeveral Magical Diſpoſitions both for odd and even Squares; but theſe being difficult to underſtand, we reckon them improper for *Mathematical Recreations.*

This Square was call'd *Magical*, from its being in great Veneration among the *Egyptians*, and the *Pythagoreans* their Diſciples, who, to add more Efficacy and Virtue to this Square, dedicated it to the Seven Planets divers ways, and engrav'd it upon a Plate of the Metal that ſympathiz'd with the Planet. The Square thus dedicated, was incloſ'd with a regular Polygon, inſcrib'd in a Circle divided into as many equal Parts as there were Units in the ſide of the Square; with the Names of the Angels of the Planet, and the Signs of the Zodiack written upon the void Spaces between the Polygon and the Circumference of the Circle circumſcrib'd. Through vain Superſtition they believed that ſuch a Medal or Taliſman would befriend the Perſon that carried it about him upon occaſion.

They attributed to *Saturn* the Square of 9 Places or Boxes, 3 being the ſide, and 15 the Sum of Numbers in each Row or Column; to *Jupiter* the Square of 16 places, 4 being the ſide, and 34 the Sum of the Numbers in each Row; to *Mars* the Square with 25, 5 being the ſide, and 65 the Sum of Numbers in each Rank; to the *Sun* the Square with 36, 6 being the Side, and 111 the Sum of each Row; to *Venus* that of 49, 7 being the Side, and 175 the Sum of Numbers in each Rank or Column; to *Mercury* that of 64, 8 being the Side, and 260 the Sum of each Column; to the *Moon* the Square with 81 lodges, having 9 for its Side, and 369 for the Sum of each Column.

In fine, they attributed to imperfect Matter, the Square with 4 Diviſions, having 2 for the Side; and to *God* the Square of only 1 Lodge, the Side of which is an Unit, which multiplied by it ſelf, undergoes no Change. By virtue of this Problem, we are taught to reſolve the following Queſtion.

Queſtion, *To draw up in three Ranks the Nine firſt Cards, from an Ace to a Nine, in ſuch a manner that all the Points of each Rank, taken either length-wiſe or breadth-wiſe, or diagonal-wiſe, may make the ſame Sum.*

4	9	2
3	5	7
8	1	6

Dispose the Nine first natural Numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, Magically, according to the Directions laid down above, and as you see it done here, and place the Cards according to their Number, answerable to these Figures.

8	256	2
4	16	64
128	1	32

Instead of an Arithmetical Progression, you may take a Geometrical; for instance, this double Progression, 1, 2, 4, 8, 16, 32, 64, 128, 256, &c. and placing them Magically, as above, you'll find the Product of each Rank will be equal, *viz.* 4096. which is just the Cube of the Middle Term 16.

1260	840	630
504	420	360
315	280	252

Here we shall add by the by, one Square more of 9 Stations, in which the Numbers of each Rank taken any way, as above, are in harmonical Proportion; and you may find as many other Numbers of the same quality, as you will, if instead of the foregoing Numbers you put Letters, as you see it done underneath, where the literal Magnitudes of each Rank are Harmonically proportional; and so by giving different Value to the three undetermin'd Letters, a, b, c, you'll have, instead of literal Quantities, Numbers that will always preserve an Harmonick Proportion in each Rank.

a	$\frac{2ac}{a+c}$	c
$\frac{2ab}{a+b}$	$\frac{2bc}{b+c}$	$\frac{2abc}{2ab+ab-bc}$
b	$\frac{2abc}{2ac+ab-bc}$	$\frac{abc}{ab+ac-bc}$

PROBL.

PROBLEM X.

Of an Arithmetical Triangle.

BY an *Arithmetical Triangle* we mean the half of a Square, like a Magical Square, divided into several small and equal Stations or Points, which contain the Natural Numbers 1, 2, 3, 4, *&c.* the Triangular Numbers 1, 3, 6, 10, *&c.* which are form'd by the continual addition of the foregoing Numbers; the Pyramidal Numbers 1, 4, 10, 20, *&c.* form'd by the continual addition of the Triangular; the Pyramido-Pyramidals 1, 5, 15, 35, *&c.* form'd by the continual addition of the Pyramidal; and so on, as you see in the following Cut.

1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	
1	3	6	10	15	21	28		
1	4	10	20	35	56			
1	5	15	35	70				
1	6	21	56					
1	7	28						
1	8							
1								

F

Among the different Uses of the Arithmetical Triangle, I shall only single out those relating to *Combinations*, *Permutations*, and the *Rules of Game*; the rest being too speculative for Mathematical Recreations.

By *Combinations* we understand all the different Choices that can be made of several things, the Multitude of which is known, by taking them divers ways, one by one, two and two, three and three, *&c.* without ever taking the same twice.

Of *Combinations.* For *Example*, If you have four things express'd by these four Letters, *a*, *b*, *c*, *d*; all the different ways of joyning two of them, as *ab*, *ac*, *ad*, *bc*, *bd*, *cd*; or three of them, as *abc*, *abd*, *acd*, *bcd*; these, I say, are call'd *Combinations*. And from hence 'tis easie to apprehend, that when Four things are propos'd, you may take 'em one by one four ways; two and two six ways; three and three four ways; and by fours only one way; so that 1 in 4 combines four times; 2 six times; 3 four times; and 4 only once.

To find in a greater number of different things, such as Seven; the divers Combinations that may be made by taking them divers ways, whether by *Addition* or *Multiplication*; as, if you would know all the possible Conjunctions of the Seven Planets, taking them two by two; that is to say, if you would know how often 2 combines in 7; add an Unit to each of the two Numbers given, 2, 7, and so you have 3, 8, which gives us to know, that in the third Station (reck'ning from below upwards, or from above downwards) of the eighth Diagonal of the Arithmetical Triangle, you'll have the Number of Combinations demanded, *viz.* 21.

Or else, the two Numbers given being 2 and 7, add together all the Numbers of the second Rank, till you come at the seventh Diagonal, *viz* 1, 2, 3, 4, 5, 6, and the Sum 21 is what you want.

When the Number of things proposed goes beyond 9, the Triangle here delineated can't serve you; and therefore we shall give this General Rule for any Number whatsoever.

The two Numbers given being 2 and 7, to know how often 2 the least will combine in 7 the greatest; make of them these two Arithmetical Progressions 2, 1, and 7, 6, which decrease by an Unit, and ought to have but two Terms, that is, as many as the least Number 2 has Units.

Then

Then multiply together all the Terms of each Progression, that is, 7 by 6, and 2 by 1; and divide the first Product 42 by the second 2, and the Quotient 21 satisfies the Demand.

By this, or the foregoing Method, you'll discover, that 3 combines in 7, 35 times; 4 likewise 35 times; 5, 22 times; and 6 only 7 times. Whence it follows, that the Number of all the Combinations possible of seven different things, taken one by one, by two's, by threes, by fours, by fives, by sixes, and sevens, amounts to 127, as appears by the addition of all the particular Combinations, 7, 21, 35, 35, 21, 7, 1, which answer the Numbers 1, 2, 3, 4, 5, 6, 7. But you may find this Total yet easier, by forming this double Geometrical Progression, 1, 2, 4, 8, 16, 32, 64, consisting of seven Terms, answerable to the number of things combined, *viz.* 7; for the Sum of these Terms, 127, is the Number you look for; which may still be found yet an easier way, *viz.* Subtract 1 from the propos'd number of things 7, and the Remainder, 6, directs you to take the sixth Power (64) of the Number 2; and the double of that Power, bating an Unit, 127, is the Number desired.

Before I dismiss this Subject, I shall here set down two Methods peculiar to 2 and 3, for finding out how often these two Numbers may be combin'd in any number of things. Suppose the number of things given is 7, you'll find how often 2 will combine in it, by subtracting the given Number 7 from its Square 49; and taking (21) the half of the remainder, 42, for the Number desired. You'll find how often 3 may combine in 7, by adding 14, the double of 7 to 343, the Cube of the same given Number 7, and subtracting from the Sum (357) the triple (147) of the Square (49) of the same Number (7) for then the sixth part (35) of the remainder (210) shews you, that 3 will combine in 7 35 times.

There's another sort of *Combinations*, that may be call'd Of *Permutation*, in which we take the same thing twice; as, if you would combine these three Numbers by two's, 2, 5, 6, in order to know what different Quantities they can produce, if you consider the two first thus, 25, you'll call 'em twenty five; if thus, 52, you'll call 'em fifty two; in like manner, the first and third taken thus, 26, is a quite different Quantity from the same two taken thus, 62; and so of all others. From whence it ap-

Of Permutations.

C 4 pears

pears, that the Multitude or Number of Permutations is the Double of that of Combinations.

Permutations are of very good use in making Anagrams, and sometimes give very lucky Hits; as in the Word *ROMA*, the Letters of which being transposed make this other Word *AMOR*; but 'tis a much luckier Hit that we meet with in these two Latin Verses;

Signa te, signa, temere me tangis & angis,
Roma tibi subito motibus ibit amor.

the Letters of which being read backwards, form the same Verses.

We likewise make use of Permutations in playing at Dice, to know the Number of Chances that attend the engaging to throw with two Dice, 9 for Instance; it being certain, that the Person who engages has four Chances for it; for 9 may come up four ways, by *quatre cinque*, by *cinque quatre*, by *tres six*, and again by *six tres* (according as the first or second Dye happens to appear.)

To give the joynt Combinations of several Letters; for example, these four *AMOR*, that is, to find the Number of their simple Permutations, by transposing them all possible ways; make this Arithmetical Progression, consisting of as many Terms as there are Letters to combine together, which in this Example are Four; so that the first Term is always an Unit, and the last denotes the Number of Letters; then multiply together all the Terms, and the Product 24, is the Number of Permutations or different Changes that these four Letters *AMOR* can undergo, as you see here;

AMOR	MARO	OAMR	ROMA
AMRO	MAOR	OARM	ROAM
AOMR	MOAR	OMAR	RMAO
AORM	MORA	OMRA	RMOA
ARMO	MRAO	ORAM	RAMO
AROM	MROA	ORMA	RAOM

By

By the same way do we find the number of Permutations of any other number of Letters, *viz.* By making a Progression of as many natural Numbers as there are Letters to combine, and multiplying together all the Terms of the Progression. Thus you'll find that Five Letters may be transpos'd 120 ways; Six 720; and so on, as in the following Table, where you see the Twenty Three Letters of the Alphabet may be combined 25852016738884976640000 ways.

1	1. A.
2	2. B.
3	6. C.
4	24. D.
5	120. E.
6	720. F.
7	5040. G.
8	40320. H.
9	362880. I.
10	3628800. K.
11	39916800. L.
12	479001600. M.
13	6227020800. N.
14	87178291200. O.
15	1307674368000. P.
16	20922789888000. Q.
17	355687428096000. R.
18	6402373705728000. S.
19	121645100408832000. T.
20	2432902008176640000. V.
21	51090942171709440000. X.
22	1124000727777607680000. Y.
23	25852016738884976640000. Z.
24	620448401733239439360000.
25	15511210043330985984000000.

This Table is easily calculated; for having discover'd that Four Letters, for Example, may be combin'd or transpos'd 24 ways; if you multiply 24, the number of Combinations, by 5 the next Number, you have 120 for the Combinations of Five Letters; and that multiplied by the next Number 6, makes 720 for the Combinations of Six Letters; and so on through all the succeeding Letters.

Of the *Parti's* or Division of Game.

By *Parti*, in the way of Gaming, we understand the just Distribution or Adjustment of what Money out of the Stakes belongs to several Players, who play for it so many Games, or a certain number of Parts or Setts, in proportion to what every one has ground to hope from Fortune, upon the Setts he wants to be up.

For Example, If two Gamesters have staked down 40 Pistols, which is then no longer their Property, only by way of Retaliation, they have a right to what Chance may bring 'em, upon the Conditions stipulated at the first Agreement; suppose they were to play for these 80 Pistols three Setts, that the first had gain'd one Sett, and the second none; that is, the first wants two Setts to be out, and the second three; these Suppositions being laid down, and the Gamesters having a mind to draw their Stakes, without standing to their Chances, the just *Quota* appertaining to each, is what is call'd *Parti*, and is found out by the Arithmetical Triangle, after this manner.

Since the Supposition runs, that the first Gamester wants 2 Setts, and the other 3, and the Sum of the two Numbers 2 and 3 is 5; we must turn to the Fifth Diagonal of the Arithmetical Triangle, and there take 5 the Sum of the two first Numbers 1, 4, by reason of the two Setts that the first Gamester is short; and 11 the Sum of the other three, 6, 4, 1, by reason of the three Setts that the second Gamester is short: And these two Sums 5 and 11 give the reciprocal *Ratio* of the two *Parti*'s inquired for; so that the *Parti* or *Quota* of the one or first is to that of the second, as 11 to 5.

But to adjust these *Quota*'s, that is, to assign each Gamester his positive Share of the 80 Pistoles at stake, this Number 80 must be divided into two parts proportional to the two Terms 11, 5; and this is done by multiplying 80 by the two Sums 11, 5, separately, and dividing each of the two Products, (880, 400.) by 16, the Sum of the two Terms 11, 5; by which means you have 55 for the Number of Pistoles due to the first Gamester that gain'd a Sett; and 25 for the other that gain'd none.

In like manner, if the first wants but 1 Sett to be out, and the second 2, we add together these two Numbers, 1, 2, and their Sum being 3, turn to the Third Diagonal of the Arithmetical Triangle, and there take the first Number 1, and the Sum 3 of the two others 2, 1; from these

these two Numbers 1, 3, we learn that the first his *Quota* is to that of the second as 3 to 1; and since the Sum of these two Terms is 4, the Consequence is, that the first Gamester ought to have $\frac{3}{4}$ of the 80 Pistoles staked, and the second only $\frac{1}{4}$, that is, the first 60 Pistoles, and the other 20.

Hence it appears, that when the Game is at this pass, the first may lay upon the Square 3 to 1: And this we can likewise make out without the Arithmetical Triangle, after the following manner.

Since the first wants One Sett to be out, and the second Two, we must consider, that if they went on with the Game, and the second gain'd a Sett, then the two Gamesters would have equal Chances, and so their *Quota's* or Dividends would be equal, it being a constant and a general Rule, that the one Share of the first is to that of the second, as the Chances of the one are to those of the other. And so in this Supposition, each of 'em has a Title to an equal Half of the Money. 'Tis therefore certain, that if the first gains the Sett that's to be play'd, he sweeps all; but if he loses it, he has a Title to an equal Half; and therefore if they have a mind to draw without playing the Sett, the first ought to have half the Money at stake, and the half of the remaining Half, that is $\frac{3}{4}$ of the Whole; so that $\frac{1}{4}$ remains to the second; for 'tis evident, that if a Gamester has a Right to a certain Sum, in case he gains, and to a lesser, in case he loses, he has a Right to the Half of those two taken together, if the Game is thrown up. I. *Case.*

This first Case directs us to the Solution of the second, which supposes the first to want one Sett to be out, and the second three; for if the first gains the Sett, he sweeps all the 80 Pistoles; if he loses, it turns to the first Case, as above, that is, he has a Right only to $\frac{3}{4}$; and therefore, if the Stakes are drawn without playing that Sett, his Right is Half of these two Sums taken together, *i. e.* $\frac{7}{8}$ or 70 Pistoles, $\frac{1}{8}$ or 10 Pistoles remaining to the second. II. *Case.*

This leads us to a Resolution of a third Case. Supposing the first to be two Setts short, and the second three; for if the first gains the next Sett, he has a Right to $\frac{7}{8}$ of of the Money, by the Second Case; if he looses it, so that the second wants only two to be out, as well as he, the Money is to be equally divided between them. Upon the whole, the Game stands thus; if the first wins, he claims $\frac{7}{8}$, it III. *Case.*

if he looses, he claims $\frac{1}{2}$; and therefore, if the Game is thrown up without playing this Sett, he claims the Half of these two Sums put together, *i. e.* $\frac{11}{16}$ or 55 Pistoles, leaving $\frac{5}{16}$ or 25 to the second.

IV. *Case.* The second Case leads us likewise to the Solution of a fourth Case, in which the first is suppos'd to be one Sett short of the Whole, and the second four; for if the first gains a Sett, he carries the 80 Pistoles; if he loses it, so that the second lacks only three to be out, he claims $\frac{7}{8}$ by the second-Case. Now since, in case of winning, he takes 80 Pistoles, and in case of losing $\frac{7}{8}$ of them, his Dividend, upon throwing up, is the Half these two Sums put together, that is, $\frac{15}{16}$, or 75 Pistoles, and so he leaves $\frac{1}{16}$, or 5 Pistoles for the second.

V. *Case.* The fourth and third Cases lead us, after the same manner, to the Solution of a fifth, which supposes, that the first Gamester is two Setts short, and the second four; for if the first gains a Sett, and so lacks but one to be out, he claims $\frac{15}{16}$, by the fourth Case; and if he looses it, so that the seconds wants but three, he claims $\frac{11}{16}$, by the the third; and consequently, in case of drawing, his Due is the Half of these two Sums put together, that is, $\frac{13}{16}$, or 65 Pistoles, $\frac{3}{16}$, or 15 Pistoles being left for the second. And so of the other Cases.

Another and an easier way of solving these Cases. All these, and an infinite Number of other Cases that may happen, are solvable without the Arithmetical Triangle, after a different and an easie manner, as follows;

Case V. Take the fifth Case for Instance, which supposes the first to be two Setts short, and the second four; in this Supposition the two Gamesters want between 'em six Setts to be out: Take 1 off the *6*, and, since the Remainder is 5, suppose these five Letters of the same form *a a a a a*, to favour the first Gamester; and these five *b b b b b*, to favour the second; make Combinations of these ten Letters, as you see it here done; where, of 32 Combinations, the first *26* to the Left, having at least two *a*, are taken for the Number of Chances that can make the first to win; because he lacks two Setts; and the remaining 6 to the Right, or where there are at least four *b*, are taken for the Number of Chances upon which the second may win; because he wants four to be out.

aaaaa	*aaabb*	*aabbb*	*abbbb*
aaaab	*aabba*	*abbba*	*bbbba*
aaaba	*abbaa*	*bbbaa*	*babbb*
aabaa	*bbaaa*	*ababb*	*bbabb*
abaaa	*aabab*	*abbab*	*bbbab*
baaaa	*abaab*	*ababb*	*bbbbb*
	baaab	*baabb*	
	baaba	*babba*	
	babaa	*bbaba*	
	ababa	*babab*	

Thus it is plain, that the first his Due is to that of the second as 26 to 6, or, as 13 to 3.

In like manner to solve the third Case, which supposes the first to want two Setts to be up, and the second three, so that they want five between 'em; take 1 from the said Sum 5, and since the Remainder is 4, suppose these similar Letters *a a a a* to be favourable to the first, and these four *b b b b* to the second, and combine these eight Letters together, as you see it here done; where, of the 16 Combinations, the first 11 to the Left having at least two *a*'s, must represent the Number of Chances that the first has for Game, two Sets being what he wants; and the remaining 5 to the Right having at least three *b*'s, must be taken for the Number of Chances that can make the second up, he being three Setts short. Thus the Claim of the first is to that of the second as 11 to 5, &c. *Case* III.

aaaa	*aabb*	*abbb*
aaab	*abba*	*bbba*
aaba	*bbaa*	*bbab*
abaa	*baab*	*babb*
baaa	*baba*	*bbbb*
	abab	

The same 16 Combinations will serve for the Solution of the fourth Case, in which the first was supposed to be one Set short, and the second four; so that 5 is the Number of Setts wanted between 'em, as in the third Case. For among these 16 you will find 15 that have at least one *a*, (answerable to the one Sett that the first wants) for the Chances upon which the first will win; and only one that has four *b*'s, the second being four Setts short, which shews there is but one Chance that can save the second. Thus the first Share is to that of the second, as 15 to 1. And so of all other Cases. *Case* IV.

To

Of the Game at Dice.

To know, when two are at play, what Advantage one has, that engages to throw *6*, for Example, with one Dye, at a certain Number of Throws, and first of all, at the first Throw; we must consider, that his Case is 1 to 5; for he has but one Chance to win, and 5 to loose upon; and consequently if he lays upon one Throw, he ought to lay but 1 to 5.

To engage to throw *6* with one Dye at two Throws, is the same thing, as to throw two Dyes at a time, one of which is to be a *6*; and in that Case, he who throws has but 11 Chances to win upon, since he may throw the first *6*, and the second 1, 2, 3, 4, or 5; or the second *6*, and the first 1, 2, 3, 4, or 5; or else both Dyes sixes; whereas he has 25 to lose upon, as you see here. Where 'tis easie to conclude, that he who offers to throw with one Dye at two Throws, ought to set but 11 to 25.

1. 1	2. 1	3. 1	4. 1	5. 1
1. 2	2. 2	3. 2	4. 2	5. 2
1. 3	2. 3	3. 3	4. 3	5. 3
1. 4	2. 4	3. 4	4. 4	5. 4
1. 5	2. 5	3. 5	4. 5	5. 5

When you lay upon *6* at two Throws, take notice that 36, the Sum of all the Chances, 11, 25, is the Square of the given Number *6*; and that 25, the Number of Chance against him who throws, is the Square of the same Number, wanting 1, that is, 5. And therefore to find the Number of Chances that favour him who is to throw, you need only to take 1 from 12, the Double of the Number given, and the remainder 11 is the Number required; which being subtracted from 36, the Square of the former Number 6, leaves 25 the Remainder, which will always be a square Number, and denote the Chances against him.

To lay upon *6* at three Throws with one Dye, is the same as to lay upon *6* at one Throw with three Dice; and in that Case, he who throws has 91 favourable Chances, and 125 against him, and so ought to set but 91 to 125; thus you see he is at a loss who lays upon the Square for 6 at three Throws of one Dye.

Take notice that the Sum 216 of all the Chances 91, 125, is the Cube of the given Number *6*, when you engage to throw *6* at three Throws with one Dye; and that 125, the Number of the Chances against you, is the Cube of the same Number given, less 1, *i. e.* 5. And therefore,

to

to find the Number of Chances that favour the Person that throws, you need only to subtract 125, the Cube of the given Number 6, wanting 1 (*i. e.* 5) from 216 the Cube of the same Number given.

By the same Method we find out what Advantage he has who proffers to throw 6 with one Dye at four Throws; for if we subtract from the fourth Power or Biquadrate 1296 of the given Number 6, if we subtract, I say, from that, 625 the Biquadrate of the same Number, less one, or of 5, the Remainder shews us 671 favourable Chances for him that throws; the Biquadrate 625 being the Number of the Chances against him: So that he who lays upon 6 at four Throws has the Odds on his side.

But he has a much greater Advantage upon 6 at five Throws with one Dye, as appears by subtracting 3125, the fifth Power of 5 (the given Number, bating 1) from 7776, the fifth Power of the given Number 6; for the Remainder 4651, is the Number of favourable Chances, and 3125, the fifth Power subtracted, is the Number of those against him who throws.

If you want to know what Advantage he has, who offers, with two or several Dice, to throw at one Throw a determin'd Raffle; for Example two *Tres*; you must consider, that with two Dice he has but one Chance to save him, and 35 to loose upon, since two Dice can combine 36 different ways, that is, their 6 Faces may have 36 different Postures, as you see by this Scheme;

1	1	2	1	3	1	4	1	5	1	6	1
1	2	2	2	3	2	4	2	5	2	6	2
1	3	2	3	3	3	4	3	5	3	6	3
1	4	2	4	3	4	4	4	5	4	6	4
1	5	2	5	3	5	4	5	5	5	6	5
1	6	2	6	3	6	4	6	5	6	6	6

This Number 36, is the Square of 6, the Number of Faces, there being but two Dice; but if there were three, the Cube of 6, 216, would be the Number of Combinations; and if there were four, the Biquadrate of 6, 1296 would be the Number. And so on.

From what has been said, 'tis evident, that in engageing a determin'd Raffle at one Throw with two Dice, one ought to lay but 1 to 35; and by a Parity of Reason, that he ought to lay 3 to 213 upon a determin'd Raffle or

Pair-

Pair-Royal with three Dice; and 6 to 1290 with four; for of the 216 Chances of three Dice, there's only three that can favour him, since three things can combine by two's only 3 ways; and of 1296 Chances of four Dice, only 6 can favour the Thrower, since four things combine by two's 6 ways.

But if you want to know what Odds he lies under who proffers to throw a Raffle of one sort or t'other at the first Throw of two or more Dice; you may find, without Difficulty, that he ought to sett but 6 to 30, or 1 to 5 upon two Dice, since of the 36 Chances of two Dice, there's only 6 that can make a Raffle; and that upon three Dice, his Case is 18 to 198, or 1 to 11, since of the 216 Chances, that three Dice can fall upon, only 18 can produce a Raffle.

PROBLEM XI.

Several Dice being thrown, to find the Number of Points that arise from them, after some Operations.

SUppose three Dice thrown upon a Table, which we shall call A, B, C; bid the Person that threw 'em add together all the uppermost Points, and likewise those underneath of any two of the three: For Instance, B and C, A being set apart, without altering its Face. Then bid him throw again the same two Dice, B and C, and make him add to the foregoing Sum all the Points of the upper Faces, and withal the lowermost Points, or those underneath of one of them, C for Instance, B being set apart near A without changing its Face, for giving a second Sum. In fine, order him once more to throw the last Dye C, and bid him add to the foregoing second Sum the upper Points, for a third Sum, which is thus to be discovered. After the third Dye C is set by the other two, without changing its Posture, do you come up, and compute all the Points upon the Faces of the three Dice, and add to their Sum as many 7's as there are Dice, that is, in this Example 21, and the Sum of these is what you look for; for when a Dye is well made, 7 is the Number of the Points of the opposite Faces.

To exemplifie the matter; Suppose the first Throw of the three Dice, A, B, C, brought up 1, 4, 5; setting the

the first 1 apart, we add to these three Points 1, 4, 5, the Points 3 and 2 that are found under or opposite to the upper Points 4 and 5 of the other two Dice; and this gives me the first Sum 15. Now suppose again that the two last Dice are thrown, and shew uppermost the two Points 3 and 6, we set that with the three Points apart, near the Dye that had 1 before, and add to the foregoing Sum (15) these two Points 3 and 6, and withall 1 the Point that's found lowermost in the Dye that's still kept in service, and had 6 for its Face at this Throw; thus we have 25 for the second Sum. We suppose at last, that this third and last Dye being thrown a third time, it comes up 6, which we add to the second Sum 25, and so make the third Sum 31. And this Sum is to be found out by adding 21 to 10 the Sum of the Points 1, 3, 6, that appear upon the Faces or uppermost Sides of the three Dice then set by.

PROBLEM XII.

Two Dice being thrown, to find the upper Points of each Dye without seeing them.

MAke any one throw two Dice upon a Table, and add 5 to the Double of the upper Points of one of 'em, and add to the Sum multiplied by 5, the Number of the uppermost Points of the other or the second Dye; after that, having ask'd him the joint Sum, throw out of it 25, the Square of the Number 5 that you gave to him, and the Remainder will be a Number consisting of two Figures; the first of which to the left representing the Tens, is the Number of the upper Points of the first Dye, and the second Figure to the Right representing Units, is the Number of the upper Points of the second Dye.

We'll suppose that the Number of the Points of the first Dye that comes up is 2, and that of the second 3; we add 5 to 4, the Double of the Points of the first, and multiply the Sum 9 by the same Number 5, the Product of which Operation is 45, to which we add 3, the Number of the upper Points of the second Dye, and so make it 48; then we throw out of it 25, the Square of the same Number 5, and the Remainder is 23, the first Figure of which 2 represents the Number of Points of the first Dye, and the second 3 the Number of Points of the second Dye.

Another way of solving this Problem. Another way of answering this Problem, is this; Ask him who threw the Dice, what the Points underneath make together, and how much the under Points of one surpass those of the other; and if this Excess is, for Example, 1, and the Sum of all the lower Points is 9, add these two Numbers 1 and 9, and subtract the Sum 10 from 14; then take 2, the half of the Remainder 4, for the Number of the upper Points of one of the Dice; and as for the other Dye, instead of adding the Excess 1, to the Sum 9, subtract it out of 9, and take the Remainder 8 out of 14, 6 is the Remainder, the Half of which, 3, is the Number of the upper Points of the second Dye.

A third way. A Third Way is this; Bid the Person who threw the Dice, add together the upper Points, and tell you their Sum, which we here suppose to be 5; then give him Orders to multiply the Number of the upper Points of one Dye by the Number of upper Points of the other Dye, and to acquaint you in like manner with their Product, which we here suppose to be 6: Now having this Product 6, and the preceeding Sum 5, square 5, and from its Square 25 subtract 24, the Quadruple of the Product 6, and the Remainder is 1: Then take the square Root of the Remainder, which in this Case is 1, and by adding it to and subtracting it from the foregoing Sum 5, you have these two Numbers, 6, 4, the Halfs of which 3, 2, are the Numbers of the upper Points of each Dye.

PROBLEM XIII.

Upon the Throw of Three Dice, to find the upper Points of each Dye, without seeing them.

ORder the Person that has thrown the Dice, to place 'em near one another in a streight Line, and ask him the Sum of the lowermost Points of the first and second Dye, which we here suppose to be 9; then ask him the Sum of the Points underneath of the second and third, which we here suppose to be 5; and at last the under Points of the first and third, which we put 6. Now having these Numbers given you, 9, 5, 6, subtract the second Number 5 from 15, the Sum of the first and third 9 and 6; and the Remainder 10 from 14; so there re-

mains

mains 4, the Half of which 2 is the Number of the upper Points of the first Dye. To find the Number of the upper Points of the second, subtract the third Number 6 from 14, the Sum of the two first 9 and 5; and the Remainder 8 from 14 again; so you have a second Remainder 6, the Half of which, 3, is the Number demanded. At last for the third Dye, subtract the first Number 9 from 11, the Sum of the second and third, 5, 6, and the Remainder 2 from 14; so you have a second Remainder 12, the Half of which, 6, is the Number of the upper Points of the third Dye.

PROBLEM XIV.

To find a Number thought of by another.

ORder the Person to take 1 from the Number thought upon, and after doubling the Remainder, to take 1 from it, and to add to the last Remainder, the Number thought upon. Then ask him what that Sum is, and after adding 3 to it, take the third part of it for the Number thought of. For Example, Let 5 be the Number, take 1 from it, there remains 4; then take 1 from 8, the Double of that 4, and the Remainder is 7, which becomes 12, by the Addition of 5, the Number thought of; and that 12, by the Addition of 3, makes 15, the third part of which, 5, is the Number thought of.

Another Way is this: After taking 1 from the Number thought of, let the Remainder be tripled; then let him take 1 from that Triple, and add to the Remainder the Number thought of. At last, ask him the Number arising from that Addition, and if you add 4 to it, you'll find the fourth part of the Sum to be the Number thought of. Thus 5, bating 1, makes 4, that tripled makes 12, which loosing 1, sinks to 11, and enlarg'd by the Accession of 5, comes to 16, which, by the Addition of 4, is 20, and the fourth part of that, *viz.* 5, is the Number thought of. (Another Way of finding a Number thought of.)

Add 1 to the Number thought of, double the Sum, and add 1 more to it, and then add to the whole Sum the Number thought of. Having learn'd the Sum-Total, take 3 from it, and the third part of the Remainder is what you look for. Thus, 5 and 1 is 6, and the Double of (A *third* Way.)

that, enlarg'd by 1, is 13, which, by the Addition of 5, comes to 18; take 3 from that, the Remainder is 15, the third part of which, 5, is the Number thought of.

The Fourth Way. Or else, after adding 1 to the Number thought of, bid the Person triple the same, and add first 1 to it, and then the Number thought of. At last, ask the Sum of this last Addition, and after robbing it of 4, take the fourth part of the Remainder for the Number thought of. Thus, 5 and 1 is 6, the Triple of which and 1 is 19, which with 5 is 24, and that bating 4 is 20, the fourth part of which, 5, answers the Problem.

The Fifth Way. Take 1 from 5, the Number thought of, double the Remainder, 4, from which, 8, take 1, and likewise the Number thought of; after which, ask for the Remainder 2, and add 3 to it, so you have your Number.

The Sixth Way. Let the Person that thinks add 1 to the 5, the Number thought of, and to the Double of that, 12, 1 more; and subtract from the Sum, 13, the Number thought of; then ask for the Remainder 8, and taking 3 from it, what you leave behind, 5, is the Number thought of.

The Seventh Way. Bid the Person that thinks take 1 from 5, the Number thought of; and 1 from 12, the Triple of the Remainder; and then the Double of the Number thought of, 10, from 11, the last Remainder. This done, ask for the Remainder of the third Subtraction, *viz.* 1. and adding 4 to it, you'll find Satisfaction.

The Eighth Way. Add 1 to the Number thought of 5, adding 1 more to the Triple of that you have, 19, from which take 10, the Double of the Number thought of; then ask for the Remainder, 9, from which take 4, and so you're right.

The Ninth Way. Order the Person to triple the Number thought of (5) and out of the triple Number (15) to cast away the Half, if 'twere possible; and since in this Example 'tis not, to add 1 to it so as to make it 16; the Half of which, 8, must be tripled, and that makes 24. The Person that thinks having done this, ask him how many 9's are in the last Triple (24); he answers two; so you're to take 2 for every 9, which in this Example makes 4, and by reason of the 1 you gave to make the 15 an even Number, you're here to repay it by Addition to the 4, and so you have 5, the Number thought of. If there happen to be no 9 in the last Triple, the Number thought of is 1.

The Tenth Way. Bid him add 1 to to the Number thought of (which makes 6); then subtract it from it, and so it leaves (4) a Rea Remainder; then bid him multiply the Sum (6) into the Remainder (4) and tell you the Product. To this Product 24 add 1, and of the Sum 25 take the square Root 5.

Bid the Person that thinks add 1 to the Number thought of (which we all along suppose to be 5) and multiply the Sum (6) by the Number thought of (5); then let him subtract the Number thought of (5) from the Product (30) and tell you the Remainder (25) the square Root of which 5 is the Number thought of. *An Eleventh Way.*

After taking 1 from the Number thought of, bid him multiply the Remainder (4) by the Number thought of (5) and add to the Product (20) the same Number thought of, and tell you the Sum 25, of which you're to extract the Square Root 5. *A Twelfth Way.*

Bid him add 2 to the Number thought of, and clap a Cypher to the Right of the Sum, which makes 70; and to that add 12, to the Sum of which Addition (82) let him clap another Cypher, so as to make it 820. From this Decuple (820) let him subtract 320, and tell you the Remainder 500, from which you are to cut off the two Cyphers (each of which did still decuple the Number it was put to), and so you have the Number thought of 5. *A Thirteenth Way.*

Let him add 5 to the Double of the Number thought of; to the Sum 15 let him add a Cypher on the Right Hand to decuple it; then let him add 20 to the Sum (150) and to the last Sum (170) set another decupling Cypher; at last let him subtract 700 from the last Sum of all (1700) and discover to you the Remainder 1000, from which you are to strike off two Cyphers to the Right, and take the half of the Remainder (10) for the Number thought of. *A Fourteenth Way.*

These two last Methods are not very subtile; for the last Number being known, 'tis an easie matter, by a retrograde View, to find out the other Numbers, and by consequence the Number thought of. And upon that Consideration we shall here subjoyn two other Methods that are more mysterious.

Bid the Person that thinks add 1 to the Triple of the Number thought of, and triple the Sum (16) again; to which last Sum (48) bid him add the Number thought of (5); then ask him the Sum of all (53) and from that take off 3, and the Right Hand Cypher from the Remainder 50; which leaves you 5 to the Left for the Number thought of. *A Fifteenth and more mysterious Way.*

A Sixteenth Way. Bid him take 1 from the Triple of the Number thought of (15) and multiply the Remainder (14) by 3; and add to (42) the Product, the Number thought of (5); then ask the Sum of the Addition, 47, to which add 3, and cut off from the Sum 50 the Cypher, which must needs be on the Right-Hand, and so leaves to the Left the Number thought of.

Corollary I. From these two last Methods we may draw this Inference, that *If we add an Unit to the Triple of any Number* (as to 18 the Triple of *6*) *and the same Number* (*6*) *to the Triple of the Sum* (57) *the second Sum* (63) *will always terminate with* 3.

Coroll. II. Another Inference is, that *If we subtract an Unit from* (18) *the Triple of any Number* (*6*) *and add the same Number* (*6*) *to the Triple of the Remainder* (51 the Triple of 17) *the Sum* (57) *will always end with the Figure 7.*

Coroll. III. The last Inference is, That this double Problem is impossible, *viz. To find a Number of such a Quality, that if you add to, or subtract from its Triple, an Unit, and add the same Number to the Triple of the Sum of the Remainder, the last Sum will be a perfect square Number*; for as we shew'd at *Probl.* V. no Number ending in 3 or 7 can be a true Square. See the following *Problem.*

PROBLEM XV.

To find the Number remaining after some Operations, without asking any Questions.

LET another think of a Number at pleasure; bid him add to the Double of it an even Number, such as you have a mind to. For Example 8; then bid him subtract from half the Sum the Number thought on, and what remains is the Half of the even Number that you order'd him to add before; and so you may roundly tell him you are sure the Remainder is 4. Tho' the Demonstration of this is easie, yet those who are not apprised of the Reason will be surprised at it. However that you may light exactly on the Number thought of, conceal your Knowledge of the Remainder 4, and bid him subtract that Remainder, whatever it is, from the Number thought of, if so be it be larger; or else, if the Number be less, to subtract it from the Remainder; and then ask him for the

Remainder

Remainder of the laſt Subtraction; for, if you add this Remainder to the Half of the even Number you gave him (*i. e.* 4 the Half of 8) when the Number thought of is larger than that of the Half of the even Number; or if you ſubtract the Remainder from the ſame Half (4) when the Number thought of is leſs than it, you'll have the Number thought of. To exemplifie the matter, let 5 be the Number thought of, and 8 added to its Double 10, which makes 18; the Half of that is 9; and 5, the Number thought of, ſubtracted from 9 leaves 4, the Half of the additional Number 8; and if you take this Half 4 from the Number thought of 5, there will remain 1, which being added to the ſame Half 4 (the Number thought of being greater than that Half) gives 5, the Number thought of. In like manner, if to 10, the Double of 5, the Number thought of, you add 12, you'll have 22, the Half of which is 11; and from thence taking the Number thought of 5, there remains 6, the Half of the additional Number 12; and if from that Half 6 you take the Number thought of, 5, (which in this Example is leſs than the ſaid Half) there will remain 1, which being taken from the ſame Half, ſince the Number thought of is leſs than that Half (6) leaves 5 for the Number thought of.

But an eaſier Way to anſwer the *Problem* is this: Bid the Perſon that thinks, take from the Double of the Number thought of, any even Number you will that is leſs, for Example 4; then let him take the Half of the Remainder from the Number thought of, and what remains will be 2, the Half of the firſt Number ſubtracted 4; and therefore to find the Number thought of, bid him add the Number thought of to that Half 2, and then ask the Sum, 7, from which you're to take the ſame Half, and ſo there will remain 5 for the Number thought of.

But another, and yet eaſier, way is this: Bid him add what Number you will to the Number thought of, and multiply the Sum by the Number thought of; for if you make him ſubtract the Square of the Number thought of from the Product, and tell you the Remainder, you have nothing to do but to divide that Remainder by the Number you gave him to add before; for the Quotient is the Number thought of. Thus 4 added to 5 (the Number thought of) makes 9, which being multiplied by 5, makes

49; from which take 25, the Square of the Number thought of, and there remains 20, which being divided by 4, leaves 5 in the Quotient.

Or else, bid the Person that thinks, take a certain lesser Number from the Number thought of, and multiply the Remainder by the same Number thought of; for if you make him take the Square of the Number thought of from the Product, and tell you the Remainder; by dividing that Remainder by the Number you ordered to be taken from the Number thought of, you have the Number thought of in the Quotient.

But of all the Ways for finding out a Number thought of, the following is certainly the easiest; make him take from the Number thought of what Number you pitch upon that's less than it, and set the Remainder apart; then make him add the same Number to the Number thought upon, and the preceding Remainder to the Sum, for a second Sum; which he is to discover to you, and the Half of that Sum is the Number thought of. Thus 5 being thought of, and 3 taken from it, the Remainder is 2; and the same Number 3 added to 5 makes 8, and that, with the preceding Remainder, 10, the Half of which, 5, is the Number thought of.

PROBLEM XVI.

To find the Number thought of by another, without asking any Questions.

BID the other Person add to the Number thought of, its Half if it be even, or its greatest Half if it be odd; and to that Sum its Half or greatest Half, according as 'tis even or odd, for a second Sum, from which bid him subtract the Double of the Number thought of, and take the Half of the Remainder, or its least Half, if the Remainder be odd; and thus he is to continue to take Half after Half, till he comes to an Unit. In the mean time you are to observe how many Subdivisions he makes, retaining in your Mind for the first Division 2, for the second 4, for the third 8, and so on in a double Proportion, remembring still to add 1 every time he took the least Half; and that when he can make no Subdivision, you're to retain only 1. By this means you have the Number that

that he has halfed so often, and the Quadruple of that Number is the Number thought of, if so be he was not obliged to take the greatest Half at the beginning, which can only happen when the Number thought of is evenly even, or divisible by 4; in other Cases, if the greatest Half was taken at the first Division, you must subtract 3 from that Quadruple; if the greatest Half was taken only at the second Division, you subtract but 2; and if he took the greatest Half at each of the two Divisions, you are to subtract 5 from the Quadruple, and the Remainder is the Number thought of.

For Example, Let 4 be the Number thought of, which by the Addition of its Half, 2, becomes 6, and that, by the Addition of its Half, 3, is 9; from which, 8, the Double of the Number thought of, being subtracted, the Remainder is 1, that admits of no Division; and for this reason you retain only 1 in your Mind, the Quadruple of which, 4, is the Number thought of.

Again; let 7 be the Number thought of; this being odd, the greatest Half of it, 4, added to it makes 11, which is odd again; and so the greatest Half of 11 added to 11, makes 17, from which we take 14, the Double of the Number thought of, and so the Remainder is 3, the least Half of which is 1, that admits of no further Division. Here there being but one Sub-division, we retain 2, and to that add 1 for the least Half taken, so we have 3, the Quadruple of which is 12. But because the greatest Moiety was taken both in the first and second Division, we must subtract 5 from 12, and the Remainder 7 is the Number thought of.

PROBLEM XVII.

To find out Two Numbers thought of by any One.

HAving bid the Person that thinks add the two Numbers thought of (for Example, 3 and 5;) order him to multiply their Sum (8) by their Difference (2) and to add to the Product (16) the Square (9) of the least of the two Numbers (3) and tell you the Sum, 25, the Square Root of which, 5, is the greatest of the two Numbers thought of. Then for the least, bid him subtract the first Product (16) from the Square (25) of the greatest

greatest Number thought of (5) and tell you the Remainder, 9, of which the Square Root 3 is the least Number thought of.

An easier Way of doing it is this: Bid him add to the Sum of the two put together (8) their Difference (2) and tell you the last Sum, 10, for the Half of it, 5, is the greatest Number thought of. And as for the least, bid him subtract the Difference of the two Numbers thought of from their Sum, and ask him the Remainder, 6, the Half of which, 3, is the Number you look for.

This Problem may likewise be solv'd after the following manner: Bid him square the Sum of the two Numbers (*which is* 64 *in this Example*;) then bid him add to the least Number thought of (3) the Double (10) of the greatest (5) and multiply the Sum (13) by the least (3) and subtract the Product (39) from the foregoing Square (64) and discover the Remainder 25, the Square Root of which is the greatest Number thought of; and as for the least, order him to add to the greatest (5) the Double (6) of the least (3), and multiply the Sum (11) by the greatest (5) and subtract the Product 55, from the foregoing Square (64) and tell you the Remainder (9) the Square Root of which is 3, the least Number thought of.

Another, and a very easie Way, is this: Bid him multiply the two Numbers (5, 3,) together; and then multiply the Sum of the two Numbers (8) by the Number you want to find, whether the greater or lesser, and subtract the Product of the two Numbers (15) from that Product (which is 40, if you want the greater, and 24, if you look for the lesser Number) and tell you the Remainder, 25, or 9, the Square Roots of which satisfies the Demand.

Or else, bid him first take the Product of the two Numbers (15), then multiply their Difference (2) by the Number enquired for (3 or 5) and add to that Product the Product of the two Numbers (15) if you want the greatest, or subtract that Product from the Product of the two Numbers, if you look for the least. Then he telling you the Sum, or the Remainder, their Square Roots are the Numbers in question.

When the least of the two Numbers does not exceed 9, 'tis easie to find 'em out after this manner: Let 1 be added to the Triple of the greatest, and the two Numbers

bers thought of to the Triple of that Sum, and the Total Sum discover'd; from which you are to take off 3, and then the Right-hand Figure is the least, and the Left-hand Figure the greatest Number thought of. Thus 5 and 3 being thought of, 1 added to the Triple of 5, is 16, and the Triple of that (48) added to 8, the Sum of the two Numbers, makes 56, which loosing 3, is 53; 3 the Right-hand Figure being the least, and 5 on the Left the greatest Number thought of.

PROBLEM XVIII.

To find several Numbers thought on by another.

IF the Quantity of Numbers thought of is odd, ask for the Sums of the first and second, of the second and third, of the third and fourth, and so on till you have the Sum of the first and last; and having written all these Sums in order, so that the last Sum is that of the first and last; subtract all the Sums of the even Places from all those in the odd Places; and the Half of the Remainder is the first Number thought of, which being subtracted from the first Sum, leaves the second Number remaining, and that subtracted from the second, leaves the third Number remaining; and so on to the last. For Example, suppose these five Numbers thought of, 2, 4, 5, 7, 8, the Sums of the first and second, of the second and third; and so on to the Sum of the first and last, are 6, 9, 12, 15, 10; and 24 the Sum of the even Places, 9 and 15, being taken from 28, the Sum of the odd places, there remains 4, the Half of which 2 is the first Number thought of, and that being taken from the first Number 6, leaves 4 for the second Number, and 4 taken from the second, 9, leaves 5 for the third, and so on.

If the Quantity of Numbers thought upon is even, ask for the Sums of the first and second, of the second and third, of the third and fourth, and so on to the Sum of the second and the last; write them all in order, so that the Sum of the second and last may be last in order; take all the Sums in the odd Places (excepting the first) from those in the even, and the Half of the Remainder is the second Number thought of, and that taken from the first Sum, leaves the first Number, which taken from the third Sum,

Sum, leaves for a Remainder the third Number, and so on. Thus 2, 4, 5, 7, 8, 9, being the Numbers thought of, the Sums proposed, as above, are 6, 9, 12, 15, 17, 13. Then take 29 the Sum of 12 and 17 the odd Places (excepting the first) out of 37 the Sum of 9, 15, 13, the three even Stations, and the Remainder is 8, the Half of which, 4, is the second Number thought of; and that taken from 6, the first Sum, leaves 2 the first Number, as the same second Number 4, taken from the second Sum 9, leaves 5 for the third Number, which taken from the third Sum 12, leaves 7 for the fourth, and so on.

When each of the Numbers thought of consists only of one Figure, they are easily found in the following manner: Let the Person add 1 to the Double of the first Number thought of, and multiply the Sum by 5, then add to the Product the second Number thought of. If there's a third Number, add 1 to the Double of the preceding Sum, and after multiplying the whole by 5, add to the Product the third Number thought of. In like manner, if there's a fourth Number, bid him add 1 to the Double of the last preceding Sum, and after multiplying the whole by 5, add to the Product the fourth Number thought of, and so on, if there are more Numbers. This done, ask for the Sum arising from the Addition of the last Number thought of, and subtract from it 5 for two, 55 for three, and 555 for four Numbers thought of, and so on, if there are more; and then the first Left-hand Figure of the Remainder is the first Number thought of, the next (moving to the Right) is the second, the next to that the third, and so on till you come to the last Right-hand Figure, which is the last Number thought of.

For Example, Let 3, 4, 6, 9, be the Numbers thought of, and 1 added to 6, the Double of the first 3, and the Sum 7 multiplied by 5, the Product of which, 35, with the Addition of the second Number, 4, is 39; then 1 being added to 78, the Double of 39, and the Sum 79 multiplied by 5, the Product 395, with the Addition of the third Number 6, is 401; and the Double of that, with the Addition of an Unit is 803, which multiplied by 5 is 4015, and with the Addition of the fourth Number, 9, 4024. Now, if from this Sum 4024, we take 555, the Remainder is 3469, the four Figures of which are the four Numbers thought of.

But

But there's a Method for this purpose that's still easier, *viz*. Let 1 be subtracted from the Double of the first Number, and the Remainder multiplied by 5, to the Product of which Multiplication, let the second Number thought of be added. Then, if there be more Numbers than two, let him add 5 to the last Sum for a second Sum; let 1 be taken from the Double of this second Sum, and the Remainder multiplied by 5, and the third Number added to that Product; this done, if there are no more Numbers thought of (otherwise you must add 5, and go on again) ask for the last Sum, add 5 to it, and the Figures of the whole Sum will represent the Numbers thought of, as above.

For Instance, Let 3, 4, 6, 9, be thought of; take 1 from 6, the Double of the first 3, multiply the Remainder 5, by 5, add to the Product 25, the second Number 4; to the Sum 29 add 5, which gives you 34 for a second Sum; take 1 from 68, the Double of this second Sum, multiply the Remainder 27 by 5, and to the Product 335, add the third Number 6, which makes 341; add 5 to this last Sum, then it makes 346, the Double of which, wanting 1, is 691, and that multiplied by 5, 3455, which, with the Addition of the fourth Number 9, is 3464. Now adding 5 to this Sum, you have 3469, the four Figures of which represent the four Numbers thought of.

PROBLEM XIX.

A Person has in one Hand a certain even Number of Pistoles, and in the other an odd Number; 'tis required to find out in which Hand is the even or the odd Number.

LET the Number in the Right-hand be multiplied by any even Number you will, as 2, and the Number in the Left by such an uneven Number as you pitch upon, as 3; then order the Person to add together the two Products, and take the Half of their Sum, and if he can take an exact Half, so that the Sum is even, you'll know by that, that the Number in the Right-hand being multiplied by an even Number is odd, and consequently that in the Left multiplied by an odd Number is even. But on the contrary, if he can't take an exact Half, the Number in the Right is even, and that in the Left odd.

For

For Example: Suppose 9 Pistoles in the Right-hand, and 8 in the Left; multiply 9 by 2, and 8 by 3; the Sum of the two Products 42 being an even Number, shews that 9 the odd Number multiplied by the even 2. is in the Right-hand, and consequently 8 the even in the Left. This Problem directs us to the Solution of the following Question.

Question. *A Man having a piece of Gold in one Hand, and Silver in the other, 'tis ask'd what Hand the Gold or Silver is in?*

Fix a certain Value in an even Number, as 8, on the Gold, and an odd, as 5, upon the Silver. Direct the Person to multiply the Number answering to the Right-hand by any even Number, as 2, and that in the Left by a determin'd odd Number, as 3, and ask him whether the joynt Sum of the Products is even or odd; or bid him half it, and so you'll learn whether 'tis even or odd, without asking. If this Sum is odd, the Gold is in the Right-hand; if even, *è contra.*

PROBLEM XX.

To find two Numbers, the Ratio *and Difference of which is given.*

TO find two Numbers, the first of which, for Example, is to the second, as 5 to 2, and the Difference or Excess 12: Multiply the Difference 12 by 2, the *least* Term of the given *Ratio*, and divide the Product 24, by 3, the Difference of the two Terms 5, 2, and you'll find the Quotient 8, the least of the two Numbers look'd for, and that added to the Difference 12, *viz.* 20, the greatest.

If you will, you may multiply the given Difference by the *greatest* Term of the given *Ratio*, and after dividing the Product by the Difference of the two Terms of the *Ratio*, you'll find the Quotient the great Number, which, upon the subtraction of 12, leaves the lesser remaining. Or you may take this Way; Multiply each of the two Terms of the given *Ratio*, by the Difference given, and divide each of the Products by the Difference of the two Terms, and the Quotients are the Numbers demanded. This Problem furnishes an easie Solution to the following Question.

Question

Question. *If a Man has as many Pieces of Money in one Hand as in the other, how shall we know how much is in each Hand?*

Bid him put two out of the Left into the Right-hand, which by that means will have 4 more than the Left, and ask for the *Ratio* of Number of Pieces in the Right to that in the Left, which we shall here suppose to be as 5 to 3. Then multiply 4, the Difference of the two Hands, by 3, the least Term of the given *Ratio*, and divide the Product 12 by 2, the Difference of the two Terms of the *Ratio* 5, 3: The Quotient 6 is the Number of Pieces in the Left, to which if you add the Difference 4, you have 10 for the Right. These two put together make 16, and consequently at first the Man had 8 in each Hand.

PROBLEM XXI.

Two Persons having agreed to take at pleasure less Numbers than a Number propos'd, and to continue it alternately, till all the Numbers make together a determin'd Number greater than the Number propos'd; 'tis requir'd how to do it.

SUppose the first is to make up 100, and both he and the second are at liberty to take alternately any Number under 11; let the first take 11 from 100 as often as he can, and these Numbers will remain, 1, 12, 23, 34, 45, 56, 67, 78, 89, which he is to keep in mind; and first take 1, for then let the second take what Number he will (under 11) he can't hinder the first to come at the second Number 12; for if the second takes 3, for Example, which, with 1 makes 4, the first has nothing to do but to take 8, and so reach 12. After that, let the second Person take what Number he will, he can't hinder the first from coming at the third Number 23; for, if he takes 1, for Instance, which with 12 is 13, the first takes 10, and so makes 23. In like manner, the first can't be hindred to reach the fourth Number 34, then the fifth 45, then 56, then 67, then 78, then 89, and at last 100.

As for the second Person, he can never touch at 100, if the first understands the Way: Indeed if the first takes

2 at the beginning, his business is to take 10, and so clap in upon 12, with the same Advantage the first had above. But if the first is acquainted with the Artifice, he'll be sure to take 1, and so the second can never make 12, nor 23, &c. nor, in fine, 100.

If the first would be sure to win, he must take care that the lesser Number propos'd does not measure the greater; for if it does, he has no infallible Rule to go by. For Example, If, instead of 11, 10 were the Number propos'd; taking 10 continually from 100, you have these Numbers, 10, 20, 30, 40, 50, 60, 70, 80, 90; now the first being obliged to pitch under 10, can't hinder the other from making 10, and so 20, 30, &c. and in fine 100.

You need not be at the pains to make a continued Subtraction of the lesser Number from the greater, in order to know the Numbers the first is to run upon; for if you divide the greater by the lesser, the Remainder of the Division is the first Number you're to take. Thus divide 100 by 11, 1 is the Remainder for the first Number, add to that 11, it makes 12 for the second, and 12 with 11 makes 23 for the third, and so on to 100.

PROBLEM XXII.

To divide a given Number into Two Parts, the Ratio *of of which is equal to to that of Two Numbers given.*

SUppose 60 is to be divided into Two Numbers, the least of which must be to the greater as 1 to 2: Add together the two Terms of the given *Ratio* 1,2, and divide 60 by their Sum 3; the Quotient 20 is the least Number wanted, and that subtracted from 60 leaves 40 the greater. Or, multiply the two Terms 1, 2, separately, by 60, and divide each of the Products, 60, 120, by 3, the Sum of the Terms; and the two Quotients, 20, 40, are the Numbers you look for. This Problem gives an easie Solution to the following Question.

Question. *To divide the Value of a Crown into Two different Species or Denominations, the Number of which shall be equal.*

The Solution being demanded in Integers, 'tis impossible to solve this or the like Question, unless the Sum of of the two Terms of the Ratio of the different Species propos'd, does exactly divide the Crown when reduc'd to smaller Money. Thus 'tis impossible to divide an English Crown according to the tenour of the Question, into Shillings and Pence; because the *Ratio* of these Species or Denominations is 12, 1; and 13, the Sum of these two Terms, does not exactly divide 60 Pence, the Value of the Crown: But make the two Species Pence and Farthings 'twill do, since 4, 1, the Terms of their Ratio, make together 5, which exactly divides 240, the Value of the Crown in Farthings; and the Quotient 48, solves the Question, that is, 48 Pence, and 48 Farthings, make a Crown.

PROBLEM XXIII.

To find a Number, which being divided by given Numbers separately, leaves 1 the Remainder of each Division; and when divided by another Number given, leaves no Remainder.

TO find a Number which leaves 1 remaining, when divided by 5 and by 7, and Nothing when divided by 3: Multiply into one another the two first Numbers given, 5, 7; to their Product 35, add 1, which makes 36, the Number demanded. For, if you divide 36 by 5 and by 7, the Remainder is 1; and when you divide it by 3, there is, as it happens, no Remainder.

After finding this first and lowest Number of the propos'd Quality 36, you may find an infinite Quantity of greater Numbers of the same Quality, and that in the following manner: Add the first Number found 36, to 105, the Product of the three given Numbers 5, 7, 3; and the Sum 141 is a second Number of the same Quality; then add to 141 the Product abovemention'd 105, and you have 246 for a third; which, with the addition of 105, makes 351 for a fourth Number; and so on.

To find a Number that divided separately by 2, 3, 5, leaves 1 remaining, and no Remainder when divided by 11: If you take 30, the Product of the first three Numbers 2, 3, 5, and add 1 to it, you have the Number 31, which divided by each of the three first Numbers, 2, 3, 5, there should remain 11, and by 11, the fourth Number, Nothing: but so it is, that 31, when divided by 11, leaves 9 remaining, and therefore 31 is not the right Number; but in order to find out the right Number, take 30 the Product of the three Terms 2, 3, 5, and quadruple it, which makes 120, which with the addition of 1, is the Number required 121, and that added to 1320, the Product of the four Numbers given 2, 3, 5. 11, makes 1441 for a second Number of the same Quality; and so on, as above. In this Case, 30, the Product of 2, 3, 5, being divided by 11, left 8 remaining, and the Quadruple of that 8, 32, being but 1 short of 33, the Multiple or Triple of 11, we quadrupled the 30, and added to the Sum.

In like manner, to find a Number, that divided separately by 3, 5, 7, leaves 2 remaining, and no Remainder when divided by 8: Divide 105, the Product of the three first Numbers 3, 5, 7, by the fourth 8; and because there remains 1, multiply the Product 105 by 6, that the Product 630 divided by 8, may leave a Remainder of 6, which is less than 8 by 2, and then adding 2 to the last Product 630, you have 632 the Number required, which added to the Product of the four given Numbers, makes a second Number of the same Quality; and that, with the same Addition, a third, and so on.

To find a Number that divided separately by 3, 5, 7, leaves 2 remaining, and divided by 11 leaves no Remainder: Divide 105, the Product of the first three Numbers given 3, 5, 7, by the fourth 11; and in regard there remains 6, the Double of which, 12, surpasses the Divisor 11 by 1; multiply the Product 105 by 2, that 210 being divided by 11, there may remain 1; and since 'tis desired that 9 may be the Remainder, which is less than the Divisor 11 by 2, multiply the last Product 210 by 9, and then the Product 1890 being divided by 11, the Remainder will be 9; and therefore adding 2 to that last Product, you'll have a Number 1892, which leaves no Remainder, being divided by 11.

In like manner, to find a Number that being divided by 5, or 7, or 8, leaves 3 remaining, and nothing when divided by 11: Multiply by 9, 280 the Product of the first three Numbers given, 5, 7, 8, and the Product 2520 being divided by 11, there remains 1, upon which you may make the Remainder 8, which is less than 11 by the given Number 3, by multiplying the foregoing Product 2520 by 8, which makes 20163, and consequently that

Sum,

Sum, with the addition of 3, *viz.* 20163. is the Number sought for. This Problem directs us to solve the following Question.

Quest. *To find how many Pistoles were in a Purse that a Man has lost, but remembers, that, when he told them by Two's, or by Threes, or by Fives, there always remain'd an odd one; and when he counted 'em by Sevens, there remain'd none.*

Here we are to find a Number, that, when divided by either 2, or 3, or 5, still leaves 1 Remainder; and when divided by 7, leaves 0. Now there are several Numbers of that Quality, as appears from the foregoing Problem; and therefore to find the Number that really was in the Purse, it behoves us to be directed by the Bulk or Weight of the Purse, in order to determine that real Number.

Now to find the least of all these Numbers, let's first of all try for a Number that's exactly divisible by 2, by 3, and by 5, and likewise by 7 when 1 is added to it. If you multiply together the three first Numbers given, 2, 3, 5, their Product 30 will be divisible by each of these three Numbers; but when you have added 1 to it, the Sum 31 is not divisible by the fourth Number given, 7, for there remains 3; and since the Product 30, when divided by 7 leaves 2, its Double 60 will leave 4 upon the like Division, and by the same Consequence its Triple 90 will leave 6 remaining. Now 6 wanting but 1 of 7, add that 1 to this triple Number 90, and so 91 will be exactly divisible by 7, and consequently is the Number sought for.

To find the next larger Number that answers the Question, multiply together the four given Numbers 2,3,5,7, and to their Product 210 add the first and least Number found 91; the Sum 301 is the second Number sought for; and if you add to this second Number the foregoing Product 210, the Sum 511 will be the third Number that solves the Question; and so on *in infinitum.*

Thus, to resolve the Question, you may answer, that there might be in the Purse 91 Louis d'Ors, or 301, or 511; and the Bulk of the Purse will serve to direct you which of the Numbers was really in it.

PROBLEM XXIV.

Of several Numbers given to divide each into two parts, and to find two Numbers of such a Quality, that when the first part of each of the given Numbers is multiplied by the first Number given, and the second by the second, the Sum of the two Products is still the same.

SUppose, for Example, these three Numbers given, 10, 25, 30, and the Solution is requir'd in entire Numbers; Take any two Numbers for the two Numbers sought for, provided their Difference be 1, or such as may exactly divide the Product under the greatest of these two Numbers and the Difference of any two of the three given Numbers, and so, that the greatest of these two Numbers multiplied by the least given Number 10, may be greater than the least of these two Numbers multiplied by the greatest given Number 30; such are 2 and 7.

The two Numbers requir'd. 2 and 7, being thus found; the first part of the first given Number 10, may be taken at pleasure, provided 'tis less than 10, and than the Number arising from the Subtraction of the least found Number 2, multiplied by the greatest given Number 30, from the greatest found Number 7, multiplied by the least given Number 10; and than the Number that arises from the Division of the remainder 10 by 5 the Difference of the two Numbers found 2, 7; that is, less than 2, which is 1, which being subtracted from the first given Number 10, leaves the Remainder 9 for the other part; and that being multiplied by the second Number found 7, and the first part 1 being multiplied by the first Number found 2, the Sum of the two Products 63 and 2 is 65.

To find the first part of the second Number given, 25, multiply 15, the Difference of the first two Numbers given, 10, 25, by the greatest Number found 7; and divide the Product 105 by 5 the Difference of the two Numbers found 2, 7; then add the Quotient 21 to 1, the first part found of the first Number given 10; and the Sum 22 will be the first part of the second Number given 25, and consequently the other part will be 3, which being multiplied by the second Number found 7, and the first part 22, being multiplied by the first Number given 2, the Sum of their two Products 21, 44, makes likewise 65.

Last

Last of all, To find the first part of the third Number given 30, multiply 5, the Difference of the two last Numbers given 25, 30, by the greatest Number found 7, and divide the Product 35 by 5, the Difference of the two Numbers found 2, 7; then add the Quotient 7 to 22, the first part of the second Number given 30, and the Sum 29 will be the first part of the third Number given 30, and consequently the other part will be 1, which being multiplied by the second Number found 7, and the first part 29 being multiplied by the first Number found 2, the Sum of the two Products 7, 58, makes still 65.

Or else multiply 20, the Difference of the first and the third Number given, by the greatest Number found 7, and divide the Product 140 by 5, the Difference of the two Numbers found 2, 7; then add the Quotient 28 to 1, the first part of the first Number given 10, and you'll have 29, as above, for the first part of the third Number given 30.

If you take 1 and 6 for the two Numbers sought for, and 4 for the first part of the first Number given 10, in which Case the other part will be 6, which being multiplied by the second Number found, 6, and the first part 4 by the first Number found 1, the Sum of the two Products 36, and 4, is 40: Upon this Supposition, I say, the first part of the second Number given 25, will be 22, and consequently the other part 3, which being multiplied by the second Number found 6, and the first part 22 by the first found Number 1, the Sum of the two Products 18, 22, is likewise 40; and in fine, the first part of the third Number given 30, will be 28, and the other 2, which being multiplied by the second Number given 6, and the first 28 by the first 1, the Sum of the two Products is still 40. This Problem directs us to the Solution of the following Question.

Quest. *One Woman sold at Market 10 Apples at a certain rate apiece; another sold 25 at the same rate; and a third sold 30 still at the same Price; and yet each of them brought the same Sum of Mony home with them. The Question is, how this could be?*

'Tis manifest, That to save the Possibility of the Question, the Women must sell their Apples at two different Sales, and at two different Rates, seeing at each Sale or Division, they sell at the same Rate. Let the two different Rates be 2 and 7, which are the two Numbers that we found in the foregoing Problem; and we'll suppose

	Apples		*Farthings*	*Apples*		*Farth.*	
X.	1	at	2	9	at	7	
XXV.	22	at	2	3	at	7	65
XXX.	29	at	2	1	at	7	

that at the first Sale they sold at 2 Farthings an Apple, and that at this rate the first sells 1 Apple, the second 22, and the third 29; the three Numbers 1, 22, 29, being the first Parts of the three given Numbers X, XXV, XXX, which were found in the foregoing Problem; in this Case the first Woman will take 2 Farthings, the second 44, and the third 58. In the next place, if we suppose they sell the rest of their Apples at 7 Farth. then the first Woman will take 63 Farthings for the 9 Apples she had left, the second will take 21 Farthings for the 3 Apples she had left, and the third 7 Farthings for the 1 Apple she had left; and so each of 'em will take in all 65 Farthings.

Or, if you will, make the two different Rates 1 and 6, which were the two Numbers found in the last Problem; and suppose at the first Sale they sell at a Farthing an Apple, at which Price the first sells 4, the second 22,

	Apples		*Farth.*	*Apples*		*Farth.*	
X.	4	at	1	6	at	6	
XXV.	22	at	1	3	at	6	40
XXX.	28	at	1	2	at	6	

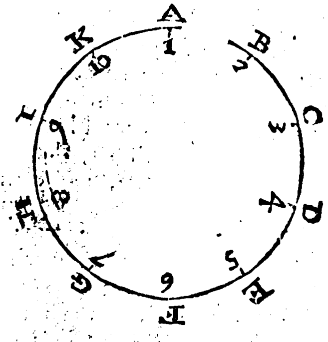
and the third 28; these three Numbers 4, 22, 28, being the first parts of the given Numbers X, XXV, XXX, which were found in the last Problem; the first Woman will take 4 Farthings, the second 22, and the third 28. Then suppose again, that they sell the rest of their Apples at 6 Farthings apiece, the first Woman will take 36 Farthings for the 6 Apples she had left, the second 18 for the 3 Apples she had left, and the third 12 Farthings for 2 Apples she had left. And thus every one of 'em will take in all 40 Farthings.

PROBL.

PROBLEM XXV.

Out of several Numbers given in Arithmetical Progression, and ranged in a Circular Order, the first of which is an Unit; to find that which one has thought of.

TO find the Number thought upon, of Ten Natural Numbers; for Instance, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, dispose 'em in a Circular Order, as you see in the annext Cut; which Numbers may represent Ten different Cards, the first of which corresponding to A, may be the Ace, and the last represented by K, may be the Ten.



Bid him who thinks of one, touch any one Number or Card, which he pleases; add to the Number of the touch'd Card the Number that expresses the Multitude of the Cards, which in this Instance is 10. Then make him who thinks of a Card, count that Sum backwards, or contrariwise to the Order of the Cards, beginning from the Card he touch'd, and ascribing to it the Number thought of: For, by counting in this Order, he'll just finish or make up the Sum at the very Card thought of.

For Example, Let the Number thought of be 3, represented by the Letter C; and the Number touch'd be *6*, corresponding to F; if you add 10 to *6*, the touch'd Number, it makes *16*; and reckoning *16* backwards from the touch'd Card F, by E, D, C, B, A, and so on in a Retrograde Order, so as to begin the Number 3 upon the touch'd Card F, 4 upon E, 5 upon D, *6* upon C, and so on to *16*, the *16* Number will fall upon C, which shews that 3, its respective Number, was the Number thought of.

PROBLEM XXVI.

Among Three Persons, to find how many Cards or Counters each of 'em has got.

LET the third Person take what number of Cards or Counters he pleases, provided it be evenly even, that is, divisible by 4; let the second take as many 7's as the other has taken 4's; and the first as many 13's. Then bid the first give to the other two as many of his Counters as each of 'em had before; and the second to give to the remaining two as many of his Counters as each of 'em; and in like manner, the third to give to each of the other the same Number that they have. By this means 'twill so fall out, that they will all have the same number of Counters, and each of 'em will have double the Number that the third had at first. And for this reason, if you ask one of the three how many Counters he has got, half his Number is the Number the third had at first; and if you take as many 7's, and as many 14's as there were 4's in the third Person's Number, you'll have the number of Cards or Counters that the second and first took.

For Example, If the third took 8 Cards, it behov'd the second to take 14, that is, twice 7, because there's twice 4 in 8; and the first must take 26, that is twice 13 by the same reason. If the first who has 26 Cards, gives to the second 14 that is, as many as he had at first; and to the third 8, that being his first Number, he will have only 4 left to himself; and the second will have 28; and the third 16. But if the second, who has 28 Cards, gives out of his Cards 4 to the first, who had just as many before; and 16 to the third, who had likewise as many; he will have 8 left to himself, and the first will have 8, and the third 32. In fine, if the third, who has got 32, gives 8 to each of the others, all the three will have 16, which is the Double of 8, the Number that the third took up at first.

1st.	2d.	3d.
26	14	8
4	28	16
8	8	32
16	16	16

PROBL.

PROBLEM XXVII.

Of Three unknown Cards, to find what Card each of Three Persons has taken up.

THE Number of each Card taken up must not exceed 9. Then, to find out that Number, bid the first subtract 1 from double the Number of the Points of his Card, and after multiplying the remainder by 5, add to the Product the Number of the Points of the second Person's Card. Then cause him to add to that Sum 5, in order to have a second Sum; and after he has taken 1 from the Double of that second Sum, make him to multiply the Remainder by 5, and add to the Product the Number of the Points of the third Person's Card. Then ask him the Sum arising from this last Addition; for if you add 5 to it, you'll have another Sum compos'd of three Figures, the first of which towards the Left is the number of the Points of the Card that the first Person took up; the middle Figure will be that of the second Person's Card; and the last towards the Right directs you to the third Person's Card.

For Example, If the first took a 3, the second a 4, and the third a 7; by taking 1 from 6, the Double of the first 3, and multiplying the Remainder 5 by 5, we have 25 Product, to which we add 4, the Number of the second Person's Card, which makes 29, and that, with the Addition of 5, makes the second Sum 34, the Double of which is 68, and taking 1 from that, there remains 67, which being multiplied by 5, makes 335, and this, by the Addition of 7, the Number of the third Person's Card, and 5 over and above, makes the last Sum 347, the three Figures of which severally represent the Number of each Card.

Or, if you will, you may bid the first add 1 to the Double of the Number of the Points of his Card, and multiply the Sum by 5, and add to the Product the Number of the second Person's Card. Then bid him add in like manner 1 to the Double of the preceding Sum, and multiply the whole by 5, and add to the Product the Number of the third Person's Card. Then ask him the Sum arising from the last Addition, and subtract

Another way of answering the Problem.

55 from it, that so there may remain a Number compos'd of three Figures, each of which represents, as above, the Number of each Card.

As in the foregoing Example, by adding 1 to 6 the Double of 3, the Number of the first Person's Card, and by multiplying the Sum 7 by 5, we have 35, which, with the Addition of 4, the Number of the second Person's Card, makes 39, the Double of which is 78, to which if we add 1, and multiply the Sum 79 by 5, we have 395; to that we add 7, the Number of the third Person's Card, and so have 402, from which if we subtract 55, the Remainder is 347, the three Figures of which severally represent the Number of each Card.

PROBLEM XXVIII.

Of Three Cards known, to find which and which is taken up by each of three Persons.

OF the three known Cards, we shall call one A, the other B, and the third C, and leave each of the three Persons to pitch upon one of the three, which may be done six different ways, as you see in the annext Scheme. Give the first Person the Number 12, the second 24, the third 36. Then direct the first Person to add together the half of the Number of that Person that has taken the Card A, the third part of the Number of the Person that takes the Card B, and the fourth part of the Number of the Person that takes the Card C; and then ask him the Sum, which you'll find to be either 23, or 24, or 25, or 27, or 28, or 29, as you see in the Table or Scheme, which shews, that if the Sum is, for *Example*, 25, the first will have taken the Card B, the second the Card A, and the third the Card C; and if the Sum is 28, the first has taken the Card B, the second the Card C, and the third the Card A; and so on in the other Cases.

1st.	2d.	3d.	*Sums.*
12	24	36	
A	B	C	23
A	C	B	24
B	A	C	25
C	A	B	27
B	C	A	28
C	B	A	29

PROBL.

PROBLEM XXIX.

To find out among several Cards, one that another has thought of.

HAving taken out of a Pack of Cards a certain Number of Cards at pleasure, and shewn them in order upon the Table, before the Person that is to think, beginning with the lowermost, and laying them cleverly one above another, with their Figures and Points upwards, and counting them readily, that you may find out the Number; which, for *Example*, we shall here suppose to be 12; Bid him keep in mind the Number that expresses the Order of the Card he has thought of, namely 1, if he has thought of the first, 2, if he has thought of the second, 3, if he has thought of the third, *&c.* Then lay your Cards, one after another, upon the rest of the Pack, in a contrary Situation, putting that upon the Pack first that was first shewn upon the Table, and that last that was last shewn. Then ask the Number of the Card thought of, which we shall here suppose to be 4, that is, the fourth Card in order of laying down, is the Card thought of. Lay your Cards, with their Faces up, upon the Table, one after another, beginning with the uppermost, which you're to reckon 4, the Number of the Card thought of; so the second next to it will be 5, and the third under that 6, and so on, till you come to 12, the Number of the Cards you first pitch'd upon to shew the Person; and you'll find the Card that the Number 12 falls to, to be the Card thought of.

PROBL.

PROBLEM XXX.

Several Parcels of Cards being propos'd or shewn, to as many different Persons, to the end that each Person may think upon one, and keep it in his mind; To guess the respective Card that each Person has thought of.

WE'll suppose there are 3 Persons, and 3 Cards shewn to the first Person, that he may think upon one of 'em, and these three Cards laid aside by themselves; Then 3 other Cards held before the second Person, for the same end, and laid apart; And at last, 3 different Cards again to the third Person, for the same end, and likewise laid apart. This done, turn up the 3 first Cards, laying them in three Stations; upon these three lay the next three other Cards that were shewn to the second Person; and above these again the three last Cards. Thus you have your Cards in three Parcels, each of which consists of 3 Cards. Then ask each Person in what Lift is the Card he thought of; after which 'twill be easie to distinguish it; for the first Person's Card will be the first of his Heap; and in like manner the second's will be the second in his; and the third Person's Card will be the third in his.

PROBLEM XXXI.

Several Cards being sorted into Three equal Heaps, to guess the Card that one thinks of.

'TIs evident that the Number of Cards must be divisible by 3, since the three Lifts are equal. Suppose then there are 36 Cards, by consequence there are 12 in each Lift; ask in what Lift is the Card thought upon; then put all the Heaps together, so as to put that which contain'd the Card thought upon between the other two; then deal off the 36 Cards again into three equal Hands, observing that order, of the first Card to the first, the second to the second, the third to the third, the fourth to the first again, and so round, dealing one Card

ard at a time, till the Cards are dealt off. Then ask again, what Hand or Heap is the Card thought upon, and after laying together the Cards, so as to put that Lift which contain'd the Card between the other two, deal off again, as you did before, into three equal Lifts. Thus done, ask once more, what Lift the Card is in, and you'll easily distinguish which is it, for it lies in the middle of the Lift to which it belongs; that is, in this Example, 'tis the sixth Card; or, if you will, to cover the Artifice the better, you may lay them all together, as before, and the Card will be in the middle of the whole, that is, the Eighteenth.

PROBLEM XXXII.

To guess the Number of a Card drawn out of a compleat Stock.

AFter one hath drawn what Card he pleases out of a compleat Stock of 52 Cards, for Instance, such as we play at *Ombre* with, you may know how many Points are in the Card thus drawn, by reckoning every fac'd Card 10, and the rest according to the Number of their Points; Then looking upon the rest of the Cards one after another, add the Points of the first Card to the Points of the second, and the Sum to the Points of the third, and so on, till you come to the last Card, taking care all along to cast out 10, when the Number exceeds it; upon which account you see 'tis needless to reckon in the 10's or the faced Cards, since they are to be cast out however. Then if you subtract your last Sum from 10, the Remainder is the Number of the Drops of the Card drawn.

'Tis easie to know, that when Nothing remains, the Card drawn is either a 10 or a faced Card; and that in this Case, if it be a faced Card, one can't distinguish whether it be King, Queen, or Knave: Now, in order to be Master of that Distinction, the best way is, to make use of a Stock of 36 Cards only, such as we formerly us'd for *Piquet*, and reckon a Knave 2, a Queen 3, and a King 4.

If you make use of a Stock of 32 Cards only, such as is now used for *Piquet*, you're to follow the same Course

Course as is above prescrib'd, only, you must always add 4 to the last Sum, in order to have another Sum, which being subtracted from 10 if it be less, or from 20 if it surpasses 10, the Remainder will be the Number of the Card drawn; so that if 2 remains 'tis a Knave, if 3 a Queen, if 4 a King, &c.

If the Stock is not full, you must take notice what Cards are wanting, and add to the last Sum the Number of all the Cards that are wanting, after subtracting from that Number as many 10's as are to be had; upon which, the Sum arising from this Addition, is to be subtracted, as above, from 10 or from 20, according as 'tis above or under 10. This done, 'tis evident by casting your Eye once more upon the Cards, you may tell what Card was drawn.

PROBLEM XXXIII.

To guess the Number of the Points or Drops of Two Cards drawn out of a compleat Stock of Cards.

LET a Man draw at pleasure Two Cards out of a Stock of 52 Cards; bid him add to each of the Cards drawn as many other Cards as his Number is under 25, which is the half of all the Cards, wanting 1, fixing upon each faced Card what Number he pleases; as if the first Card be 10, add to it 15 Cards; and if the second Card be 7, add to it 18 Cards; so that in this Example there will remain but 17 Cards in the Stock, the whole Number taken out amounting to 35. Then taking the remainder of the Pack into your hands, and finding they are but 17, conclude that 17 is the joint Number of all the Points of the two Cards drawn.

To cover the Artifice the better, you need not touch the Cards, but order the Drawer to subtract the Number of the Points of each of the two drawn Cards from 26, which is half the Number of all the Cards, and direct him to add together the two Remainders, and acquaint you with the Sum, to the end you may subtract it from the Number of the whole Stock, *i. e.* 52; for the Remainder of that Subtraction is what you look for.

For *Example*, Suppose a 10 and a 7 are the Cards drawn; take 10 from 26, there remains 16; and taking

7 from

7 from 26, the Remainder is 19: the Addition of the two Remainders 16, 19, makes a Sum of 35, which ſubtracted from 52, leaves 17 for the Number of the Drops of the two Cards drawn.

The ſame is the Management in a Stock of 36 or 32 Cards; only to colour the Trick the better, inſtead of 26, the half of the Cards, when they make 52, take another leſſer Number, but greater than 10, as 24, from which taking 10 and 7, there remains 14 and 17, the Sum of which, 31, being ſubtracted from 52, the Sum of all the Cards, leaves 21 the Remainder; from which ſubtract again 4, which is the Double of the Exceſs of 26 above 24, and ſo the Remainder is 17, the Number of the Points of the two Cards drawn, *viz.* 10 and 7.

If you make uſe of a *Piquet* Stock, conſiſting of 36 Cards, inſtead of 18, the Half of 36, the Number of all the Cards, take in like manner a leſſer Number, ſuch as 16, from which take 10 and 7, and there remains 6 and 9, the Sum of which, 15, being ſubtracted from 36, the Number of all the Cards, leaves 21 remaining; from which ſubtract again 4, the Double of the Exceſs of 18 above 16, and ſo the 17 remaining is the Number of the Points of the two Cards drawn.

In like manner, if this *Piquet*-Stock conſiſts only of 32 Cards, inſtead of 16, the Half of 32, the Number of the whole Stock, take any leſſer Number you will, provided it be greater than 10, ſuch as 14, from which take 10 and 7, and the Remainders are 4 and 7, the Sum of which, 11, being taken from 32, leaves 21, and taking from that 4, the Double of the Exceſs of 16 above 14, you have 17 remaining, the Number of the Prints of the 10 and the 7 drawn.

PROBLEM XXXIV.

To gueſs the Number of all the Drops of Three Cards drawn at pleaſure out of a compleat Stock of Cards.

TO ſolve this Problem as the former, after the ſhorteſt way, the Number of Cards contain'd in the Stock muſt be diviſible by 3; ſo that neither a Stock of 52, nor one of 32, are proper; but one of 36 is, in regard 36, the Number of all the Cards, has 12 for its third

third part, which will assist us in the Solution of the Question, as follows:

Let a Man draw at pleasure Three Cards out of a *Piquet*-Stock of 36 Cards, bid him add to each of these Cards as many other Cards as the Number of their Points falls short of 11, which is the third part of the Number of all the Cards, wanting one, allotting, as in the foregoing Problem, to each faced Card what Number he pleases: As if the first Card is 9, he adds to it 2 Cards; if the second is 7, he adds to it 4; and if the third is 6, he adds 5, which make in all 14 Cards; so that in this Example, the Remainder of the whole Stock is 22 Cards, which denotes the Number of all the Points of the Three Cards drawn.

The better to colour the Artifice, you need not touch a Card, but bid him subtract the Number of the Points of each of the three drawn Cards, from 12, the third part of 36, the Number of the whole Stock, and add together the three Remainders, and tell you the Additional Sum, which you're to subtract from 36, and the Remainder of that Subtraction is what you look for.

As in this Example; Suppose he drew a Nine, a Seven, and a Six; take 9 from 12, there remains 3; take 7 from 12, there remains 5; and take 6 from 12, there remains 6; add the three Remainders, 3, 5, 6, the Sum is 14, which taken from 36 leaves 22 for the Number of the Drops of the three Cards drawn.

To colour the Trick the better, and to apply the Rule to a Stock that consists of fewer or more than 36 Cards, such as one of 52 Cards, make use of a Number greater than 10, and lesser than 17, the third part of 52, for Instance 15: Bid him who drew the three Cards, add to each of his drawn Cards as many other Cards as the Number of their respective Points is under 15: For Example, if the first Card be 9, he adds to it 6 Cards; if the second is 7, he adds 8; if the third is 6, he adds 9, which makes in all 26 Cards; so that in this Example there will remain in the main Stock 26 Cards. Taking the main Stock into your hands, and finding you have 26 Cards, subtract from 26 the Number 4, which is the Excess of 52, the Number of the whole Stock, above the Triple of 15, + 3, *i.e.* 48; and the Remainder 22, is the Number of all the Points of the three Cards drawn.

Or

Or else you need not touch the Cards, but bid the Person that draws subtract the Number of the Drops of each of the three Cards drawn, from 16, which is 1 more than the first Number 15, and add together all the Remainders, and acquaint you with the Sum; then do you subtract that Sum from the Number above-mention'd, 48, and you'll find the Remainder to be the Number of all the Points of the three Cards drawn.

For Example, Suppose he drew a 9, a 7, and a 6; take 9 from 16 there remains 7; take 7 from 16 there remains 9; take 6 from 16 there remains 10; add these three Remainders, 7, 9, 10, the Sum is 26, which subtracted from 48, leaves 22 for the Number of the Points of the three Cards drawn.

In like manner, in a Pack of 36 Cards, take a larger Number than 10, for Instance 15; and taking notice of the Additional Cards, which amount to 26, as you saw but now, subtract that Number, 26, from 36, the Number of the whole Pack, and to the Remainder 10 add 12, which is the Excess of the Triple of 15, + 3, *i.e.* 48, above 36, the Number of the whole; and you'll find the Sum 22 to be the Number of Points enquired after. In a *Piquet* Pack of 32 Cards, instead of 12 you must add 16, by reason that 16 is the remainder of 32 subtracted from 48.

In imitation of this and the foregoing Problem, 'twill be easie to solve the Question upon four, or more, Cards drawn.

PROBLEM XXXV.

Of the Game of the Ring.

THIS is an agreeable Game in a Company of several Persons, not exceeding 9, (unless you have a mind to it) in order to the easier Application of the 18*th Problem, viz.* by reckoning the first Person 1, the second 2, the third 3, and so on; and in like manner, reckoning the Right-hand 1, the Left-hand 2; the Thumb of the Hand 1, the Fore-finger 2, the third Finger 3, the fourth 4, and the little one 5; the first Joynt 1, the second 2, and the third 3. For, if you put the Ring to one in the Company, for Instance, the fifth Person, and that upon the first Joynt of the fourth Finger of the Left-hand; 'tis

 evident,

evident, that in order to guess who has the Ring, and upon which Hand, which Finger, and which Joynt, one has only these four Numbers to guess, 5, 1, 4, 2, the first Number 5 representing the fifth Person; the second 1, the first Joynt; the third 4, the fourth Finger; and the last 2, the Left-hand. Now this is perform'd by observing the last Method of *Problem* 18. foregoing, as appears from the following Operation.

Taking 1 from 10, the Double of the first Number 5, and multiplying 9, the Remainder, by 5, you have 45; adding to that the second Number 1, you have 46, to which if you add 5, you have 51 for a second Sum: The Double of this second Sum is 102, from which take 1, there remains 101, which being multiplied by 5, makes 505, and that with the Addition of 4, the third Number, makes 509, to which if you add 5, you have this second Sum 514, the Double of this 1028 lessned by 1, and the Remainder multiplied by 5, makes 5135, to which adding the fourth Number 2, you have this Sum 5137, and that augmented by 5, gives this second Sum 5142, the four Figures of which represent the four Numbers inquired for, and by consequence denote, that the Ring is upon the first Joynt of the fourth Finger of the Left-hand of the fifth Person,

PROBLEM XXXVI.

After filling one Vessel with Eight Pints of any Liquor, to put just one half of that Quantity into another Vessel that holds Five Pints, by means of a third Vessel that will hold three Pints.

THIS Question is commonly put after the following manner: A certain Person having a Bottle fill'd with 8 Pints of excellent Wine, has a mind to make a Present of the Half of it, or 4 Pints to one of his Friends; but he has nothing to measure it out with but two other Bottles, one of which contains 5, and the other 3 Pints. *Quære*, how he shall do to accomplish it?

To

To answer this Question; let's call the Bottle of 8 Pints A, the 5 Pint-Bottle B, and the 3 Pint-Bottle C. We suppose there are 8 Pints of Wine in the Bottle A, and the other two, B and C, are empty, as you see in D. Having fill'd the Bottle B with Wine out of the Bottle A, in which there will then remain but 3 Pints, as you see at E; fill the Bottle C with Wine out of the Bottle B, in which, by consequence, there will then remain but 2 Pints, as you see at F. This done, pour the Wine of the Bottle C into the Bottle A, where there will then be 6 Pints, as you see in G; and pour the 2 Pints of the Bottle B into the Bottle C, which will then have 2 Pints, as you see at H; then fill the Bottle B with Wine out of the Bottle A, by which means there will remain but 1 Pint in it, as you see at I; and conclude the Operation by filling the Bottle C with Wine out of the Bottle B, in which there will then remain just 4 Pints, as you see at K; and so the Question is solv'd.

	8	5	3
	A	B	C
D	8	0	0
E	3	5	0
F	3	2	3
G	6	2	0
H	6	0	2
I	1	5	2
K	1	4	3

If, instead of the Bottle B, you would have the 4 Pints to remain in A, which we supposed to be fill'd with 8 Pints; fill the Bottle C with Wine out of the Bottle A, and so there will remain but 5 Pints in it, as you see at D; pour the three Pints of the Bottle C into the Bottle B, which will then have 3 Pints of Wine, as you see at E; and having again fill'd the Bottle C with Wine out of the Bottle A, where there will then remain but 2 Pints, as you see at F; fill up the Bottle B with Wine out of C, where there will then remain but 1 Pint, as you see at G; at last, having pour'd the Wine of the Bottle B into the Bottle A, where there will then be 7 Pints, as you see at H; pour the Pint of Wine that is in C into the Bottle B, which by consequence will have only 1 Pint, as you see at I; fill the Bottle C with Wine out of the Bottle A, where there will then remain just 4 Pints, pursuant to the Demand of the Question, as you see at K. Remark.

	8	5	3
	A	B	C
	8.	0.	0.
D	5	0	3
E	5	3	0
F	2	3	3
G	2	5	1
H	7	0	1
I	7	1	0
K	4	1	3

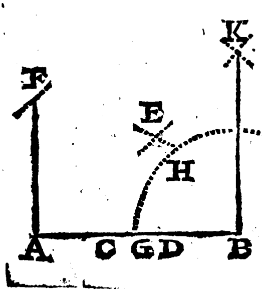
PROBLEMS OF GEOMETRY.

GEOMETRY is not less fertile than *Arithmetick*, but 'tis not so easily understood, and consequently not equally agreeable, by reason that without Demonstration it does not lay open the Proof of its Operations so exactly as Arithmetick; upon this Consideration, I shall here take in only such Problems as seem to be the plainest and most entertaining.

PROBLEM I.

To raise a Perpendicular on one of the Extremities of a Line given.

IN order to draw a Line perpendicular to the given Line A B, at its Extremity A, take at pleasure three equall parts of it, extending the Line to B, so as to make the last part terminate in B. These equal parts being A C, C D, and D B, describe at the Interval CB, from the Points B and C, two Arches of a Circle that cut one another at the point E; and



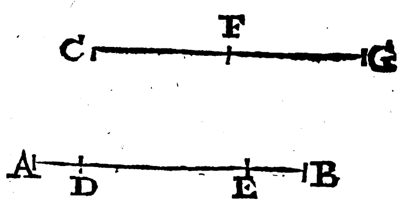
and from the two points E and C, describe with the same Extent of the Compass two other Arches of a Circle that cut one another at the point F, to which from the given End A, draw the streight Line A F which is perpendicular to the given Line A B.

If you have a mind to draw another Line equal and perpendicular to A B, upon B, the other end of the given Line A B, divide the given Line into three equal parts at the points C and D, and after finding the point F, as above directed, draw, with the Interval A F, upon the Extremity B, the Arch of a Circle G H I, and set off the same Aperture of the Compass twice upon the same Arch, *viz.* from G to H, and from H to I. Then keeping still the same Aperture, describe from the two points H and I, two Arches of a Circle that cut one another at the point K, and draw the streight Line A K, which is equal and perpendicular to the Line given A B.

PROBLEM II.

To draw from a point given, a Line parallel to a Line given.

LET the point given be C, and the Line given be A B; take at pleasure two points upon the given Line near the two Extremities A and B, such as D and E; with the distance D E, describe an Arch of a Circle from the point given C; then describe from the point E, with the Aperture C D, another Arch of a Circle, that meets the first at the point F, from which to the point C draw the streight Line C F; 'twill be parallel to the Line given A B.

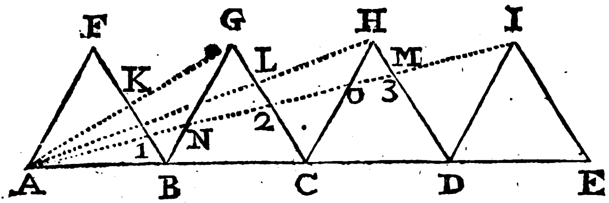


If you would have the parallel Line equal to the Line given A B, instead of making use of the two poins D and E, pitch at A and B, that is, describe from the given point C with the distance of the Line given A B, an Arch of a Circle; and another from the Extremity B at the distance A C; these two Arches will meet at G, to which from the point given C, draw the streight Line C G equal and parallel to the Line given A B.

PROBLEM III.

To divide, with the same Aperture of the Compass, a given Line, into as many equal parts as you will.

If you would divide the given Line A B, into four equal parts, for Instance; prolong the same Line, and run out upon it the four equal parts AB, BC, CD, DE;



and continuing the same Aperture of the Compass, raise upon these equal parts the four Equilateral Triangles ABF, BCG, CDH, DEI; lastly, draw the Right-Lines A G, A H, A I, and then the Line H M will represent one of the four equal parts of the given Line A B; the Line D M will consequently represent the remaining $\frac{3}{4}$, and the Line F K, or B K will represent two of 'em.

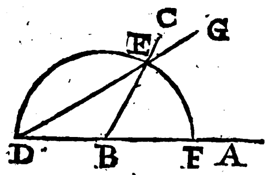
But the Line A I alone is sufficient for the Operation; for it cuts off the Line B 1 equal to the fourth part of the Line A B, the Line C 2 equal to the half of A B, and the Line D 3 equal to $\frac{3}{4}$ of A B. The Line A H divides the given Line A B into three equal parts, of which the Line G L represents one, and by consequence C L represents two: But the Line A I gives likewise the Division of A B into three equal parts; for the Line B N represents one, C O two, and by consequence H O also represents $\frac{1}{3}$.

PROBL.

PROBLEM IV.

To make an Angle equal to the Half, or to the Double of an Angle given.

TO make an Angle equal to the half of the given Angle A B C, describe upon its point B what Semi-Circle you will, as D E F, and draw the right Line D E, which will form at the point D, the Angle A D G equal to the half of the given Angle ABC.

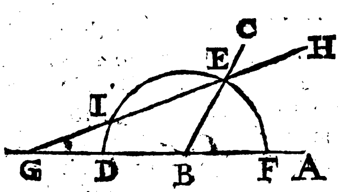


For an Angle equal to the double of the given Angle A D G, fix the point B upon any part of the Line A D, and from thence at the Distance D describe the Semicircle D E F, and joyn the Line B E, which will form at B the Angle A B C, equal to the double of the Angle given A D G.

PROBLEM V.

To make an Angle equal to the third part, or to the Triple of an Angle given.

FIRST, for an Angle equal to the third part of the Angle given A B C, describe at pleasure from its point B the Semicircle D E F, and apply a streight Ruler to E, in such a manner, that its part G I, terminated by the Circumference D E F, and by the Line A D prolong'd, may be equal to the Semidiameter B D or B E; then draw the right Line G E, which will form at the point G the Angle A G H, equal to $\frac{1}{3}$ of the Angle A B C; and consequently the Arch I D will likewise be equal to a third



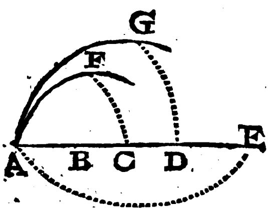
part of the Arch E F, which is the Measure of the given Angle A B C.

In the second place, for an Angle equal to the triple of the Angle given A G H, take the point I at discretion upon the Line G H, upon which Point I, set one Foot of your Compasses, and with the Distance I G, make an Arch which will cut the Line A G in the Point B, upon which, with the same Distance, describe the Semicircle D E F, which will pass through the point I, and give upon the Line G H the point E, to which, from the point B, draw the right Line B E, which will form the Angle A B C, the triple of the given Angle A G H.

PROBLEM VI.

To find a third Proportional to two Lines given, and as many other Proportionals as you will.

LET the two Lines given be A B, A C, to find a third Proportional to 'em, describe from B, the end of the first Line, at the distance A the other end, the Arch of a Circle A F; upon that Arch take the Length of the second Line AC, from A to F; then set off from F the same Length upon the Line A C prolong'd as far as you have occasion, which will reach to D, and A D will be a third Proportional to the two Lines given A B, A C.



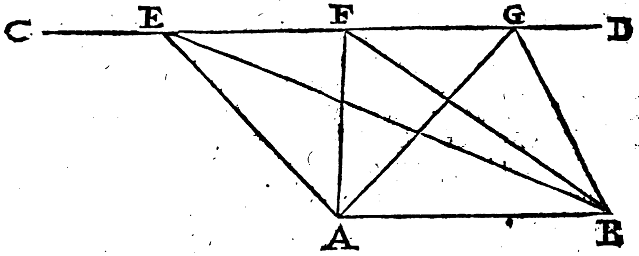
In like manner, to find a fourth Proportional to the three Lines A B, A C, A D, (which is the same thing as a third Proportional to the two Lines A C, A D) describe from C the end of the first Line A C with the Compasses open'd to A the Arch of a Circle AG, upon which set off the Length of the other Line A D stretching from A to G; and upon the Line A D, being prolong'd, set off the same Distance from G, which will reach to E, and the Line A E will be the Line you want; and so of the other Proportionals.

PROBL.

PROBLEM VII.

To describe upon a Line given as many different Triangles as you please with equal Area's.

IF the Line given be AB, draw at pleasure the Parallel CD, upon which mark, at discretion, as many different points as you would have equal Triangles, as

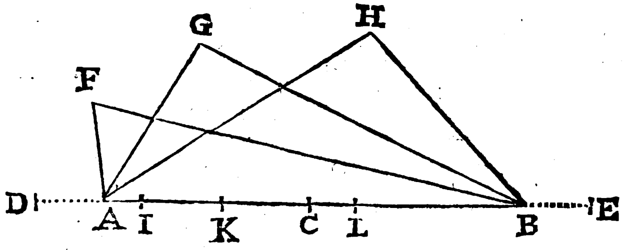


E, F, and G, for three Triangles. Draw from these three points right Lines to A and B, the Extremities of the given Base AB, and then you have three equal Triangles AEB, AFB, AGB, upon the same Base AB.

PROBLEM VIII.

To describe upon a given Line any demanded Number of different Triangles, the Circumferences of which are equal.

IF the Base given is AB, divide it equally into two at the point C, and lengthen it on each hand, at plea-



sure,

ſure, to D and E, for Inſtance, making the two Lines C D and C E equal, and taking the whole Line D E for the Sum of the two ſides of each Triangle, that's to be deſcrib'd on the given Baſis A B, after this manner :

From the point A deſcribe, with the Compaſſes a little more opened than A D, the Arch of a Circle, and apply the ſame Aperture to the Line D E, ſtretching from D to I ; then with the Aperture or Diſtance I E, deſcribe from the Centre B another Arch of a Circle, which here cuts the firſt at F, and that ſhall be the top of the firſt Triangle A B F.

In like manner, draw from the point A, with an Aperture ſomewhat larger than A F, an Arch of a Circle, and ſetting off the ſame Diſtance upon the Line D E from D to K, deſcribe from the point B at the Diſtance K E, another Arch of a Circle, that cuts the former at G, which will be the top of the ſecond Triangle A G B, the Circumference of which will be equal to that of the firſt A F B.

If you deſire a third Triangle, draw from the point A, an Arch of a Circle, with the Compaſſes open'd a little more than the length of A G, and having ſet off the ſame Diſtance, as above, upon the Line D E, from D to L ; deſcribe from the point B with the Interval L E another Arch of a Circle that here cuts the former at H, which will be the top of the third Triangle A H B, the Circumference of which is the ſame with that of the two preceding Triangles. And ſo of the reſt.

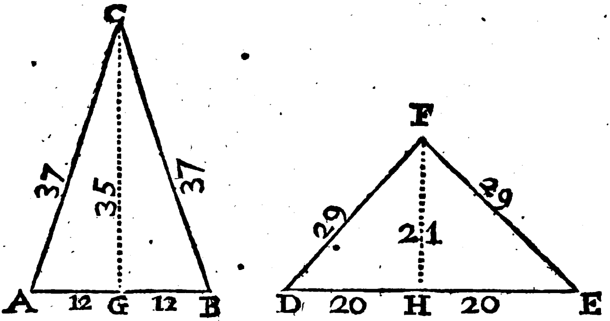
Remark. F, G, and H, the tops of all theſe Triangles fall upon the Circumference of an *Ellypſis*, the great *Axis* of which is D E, and the two *Forus*'s A, B.

PROBL.

PROBLEM IX.

To describe two different Isosceles Triangles, of the same Area, and the same Circumference.

HAving prepared a Scale of equal parts of what Length you please, take upon the Base A B, the two parts or Segments G A and G B, each of which is equal to 12 parts



upon the Scale. From the point G upon the Base A B raise the Perpendicular G C equal to 35 of the same parts, and joyn the two equal Lines A C, B C, and so you'll have the first Isosceles Triangle A B C, in which each of the two equal Sides A C, B C, will be found 37 parts, as will appear by adding 144 the Square of the Segment A G, to 1225 the Square of the Perpendicular C G, and by taking the Square-Root of the Sum 1369.

Now, to have a Triangle of the same Area and Circumference with that now describ'd, take upon the Base D E, the two Segments H D, H E, of 20 parts each; and having rais'd from the point H upon the Base D E, the Perpendicular H F of 21 parts, joyn the equal Lines E F, D F, each of which will be 29 parts, as will appear by adding 400 the Square of the Segment D H, to 441, the Square of the Perpendicular H F, and extracting the Square-Root of the Sum 841.

Thus you'll have the Isosceles-Triangle D E F, the Circumference of which, 98, is equal to the Circumference, that is, the Sum of the three Sides of the first Isoscelesſceles-Triangle ABC; and of which the Area or Content 420, is equal to that of the ſame firſt Triangle, as appears by multiplying DH by FH, or 20 by 21; becauſe the Product 420 ariſing from thence, for the Area of the Triangle DEF, is the ſame with that ariſing from the Multiplication of AG by CG, or 12 by 35, for the Area of the Triangle ABC.

You may deſcribe as many Couples as you will of Iſoſceles-Triangles with the ſame Area and Circumference, by finding their Numeral Quantities; and that is done by finding the two Generative Numbers of the two Halfs AGC, DHF, which are two equal Rectangle-Triangles, that may then, by the means of their Generative Numbers, be expreſs'd in Numbers, as was ſhewn above, *Probl.* VI. *Arithm.* Now, theſe two Generative Numbers will be found by this General Rule, which is demonſtrable:

If you divide the Difference of two Cubes by the Difference of their Sides, and multiply that Difference of the Sides by the Sum of the ſame Sides, you'll have the two Generative Numbers of the firſt Rectangle-Triangle AGC; *and if you divide the Difference of the ſame two Cubes by the Difference of their Sides, as above, and multiply the Sum of the leſſer Side and the Double of the larger, by the leſſer Side, you have the two Generative Numbers of the ſecond Rectangle-Triangle* DHF.

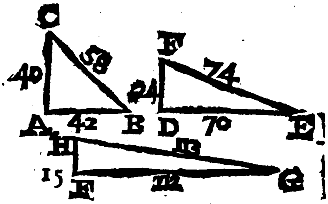
You may find to Infinity the two ſame Rectangle-Triangles, by this other *Canon*: *If, of two Numbers, the greateſt of which is leſs than the Quintuple of the leaſt, you multiply the Sum by the Difference; and if you multiply the Sum of the greater, and of the Septuple of the leaſt, by the Double of the leſſer, you have the two Generative Numbers of the firſt Rectangle-Triangle* AGC; *and if from the Square of the Sum of the greateſt, and of the Double of the leaſt, you ſubtract the Square of the leaſt, and multiply the Exceſs of the Quintuple of the leaſt above the greateſt, by the Double of the leaſt, you'll have the two Generative Numbers of the ſecond Rightangled Triangle* DHF.

PROBL.

PROBLEM X.

To describe three different Rectangle-Triangles, with equal Area's.

FROM a Scale of equal parts take the Base A B of 42 parts, and the Altitude or the Perpendicular A C of 40 parts; and then B C the Hypothenuse of the first Rightangled Triangle A B C will be found of 58 Parts, as appears by adding 1764, the Square of the Base A B, unto 1600, the Square of the Perpendicular A C, and extracting the Square Root of the Sum 3364.



Then lay down D E, the Base of the second Rightangled-Triangle, of 70 parts, and the Altitude D F of 24, and the Hypothenuse will be found to be 74, as appears by adding together 4900, the Square of the Base D E, to 576 the Square of the Altitude D F, and extracting the Square Root of the joynt Sum 5476. Thus the Area of this second Rightangled-Triangle DEF will be equal to that of the first, each being 840, as appears by multiplying the Base by the Height, and halving the Product.

At last take F G, the Base of the third Rightangled-Triangle F G H of 112 parts, the Altitude F H of 15, and the Hypothenuse B C will be 113, as appears by adding 12544 the Square of the Base F G, to 225 the Square of the Altitude F H, and extracting the Square Root of the Sum 12769. Thus the Area of this third Triangle is likewise 840.

These three Triangles have thus been found in Integers, by the Rule drawn from Algebra, which shews, that in order to find three equal Rightangled Triangles in entire Numbers, we must first find three Numbers that will serve for Generative Numbers, and that after this manner:

If you add the Product of any two Numbers, to the Sum of their Squares, you have the first; The Difference of their Squares is the second; and the Sum of their Pro-

duct and of the Square of the least is the third Generative Number.

If of the three Numbers thus found you form three Rightangled Triangles, viz. one of the two first, another of the two Extremes, and a third of the first and the Sum of the other two, these three Rightangled-Triangles will be equal one to another.

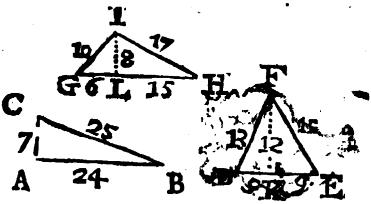
You may find in Fractional Numbers as many Rightangled Triangles as you will, whose Area's are equal to one another, and equal to one of the three foregoing, by finding from this Rightangled Triangle another Rightangled-Triangle equal, after the following manner:

From another Rectangle-Triangle of the Hypothenuse of the Rectangle-Triangle propos'd, and the Quadruple of its Area. Divide the Triangle thus form'd by the Double of the Product arising from the Multiplication of the Hypothenuse of the Rectangle Triangle propos'd, by the Difference of the Squares of the two other Sides of the same Rectangle-Triangle. Thus you'll have a Rightangled Triangle equal to the propos'd Triangle.

PROBLEM XI.

To describe three equal Triangles, the first of which shall be Rightangled, the second an Oxygonium, and the third an Amblygonium.

FROM a Scale of equal parts which may represent Feet, Fathoms, or what you will, take AB. the Base of the Right-angled Triangle ABC of 24 parts, and the Altitude AC of 7, and then the Hypothenuse BC will be 25, as appears by adding 576, the Square of the Base AB to 49 the Square of the Altitude AC, and extracting the Square Root of the Sum 625.



Then upon DE the Base of the Acute-angled Triangle DEF, take the Segment KD of 5 parts, and the Segment KE of 9; and from the point K, upon the Base DE, raise the Perpendicular KF of 12 parts, and then

then the side DF will be found 13, as appears by adding 25, the Square of the Segment DK, to 144 the Square of the Altitude FK, and extracting the Square Root of the Sum 169: And the other side will be found 15, by adding the Square 81 of the Segment KE, to 144 the Square of the Perpendicular KF, and extracting the Square Root of the Sum 225.

At last, upon GH the Base of the Obtuse-angled Triangle GHI, take the Segment LG of six parts, the Segment LH of 15, and from the point L upon the Base GH raise the Perpendicular LI of 8 parts; and the side GI will be found 10, by adding and extracting as before; as the side HI will be 17 by the like Operation.

Now we know the Triangle ABC is right-angled at A, because 625 the Sum of the Squares of the two sides AG, AB, is equal to the Square of the third side BC. We know that the Triangle DEF is acute-angled, because the Sum of the Squares of any two sides is larger than the Square of the third. And in fine, That the Triangle GHI is an Amblygonium, and the Angle I is the obtuse; because 441 the Square of its opposite Side GH is greater than 389, the Sum of the Squares of the two other sides GI and HI.

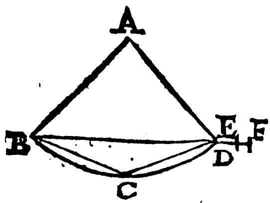
In fine, We know that these three Triangles ABC, DEF, GHI, are equal, that is, their Area's are equal among themselves; because, in multiplying the Base AB by the Altitude AC, we have the same Product as in multiplying the Base DE by the Altitude FK, or the Base GH by the Altitude LI; *viz.* 168 the Double of the Area of each Triangle, which by consequence is 84. The three sides of the Oxygonium DEF, and the Perpendicular FK, are in a continual Arithmetical Proportion.

PROBL.

PROBLEM XII.

To find a Right Line equal to the Arch of a Circle given.

LET the given Arch be BCD, the Centre of the Circle A, and AB or AD the Radius or Semidiameter; divide this Arch into two equal parts at the point C, and draw the Chords BC, CD, BD. Extend the Chord BD to E, so that the Line BE may be the Double of one of the two equal Chords BC, CD; *i.e.* may be equal to the Sum of these two Chords. Prolong the Line BE to F, so that the Line EF may be equal to the third part of the Line DE, and the Line BF shall be *almost* equal to the Curve BCD. I said *almost*, because the Line BF is a very little less than the Arch BCD; but when the Arch does not exceed 30 Degrees, the Difference is so small, that, of a Hundred Thousand parts that may be given to the Radius AB or AD, the Difference will not amount to One.



Remark. Those who understand Trigonometry, will find that if the Arch BCD is precisely 30 Degrees, or the 12th part of the Circumference of the whole Circle; and if the Radius AB be 50000 parts, and consequently the Diameter 100000, each of the two Chords BC, CD, will be 13053, and consequently their Sum, or the Line BE, will be 26106; from which, if you subtract the Chord BD, which will be found 25882, there will remain 224 for the Line DE, the third part of which is 74 for the Line EF; and that Line EF being added to the Line BE or 26106, their joynt Sum will be 26180 for the Line BF, or for the Arch BCD, which multiplied by 12, gives 314160 for the Circumference of the Circle. And thus we know, that when the Diameter of a Circle consists of 100000 parts, the Circumference is about 314160 such like parts, and consequently the Diameter of a Circle is to the Circumference, very near, as 100000 is to 314160, or as 10000 to 31416.

This

This puts us in a way to find the Circumference of a Circle, the Diameter of which is known, by multiplying the Diameter by 31416, and dividing the Product by 10000; for if we cut off from the Product the four Right-hand Figures, the Figures to the Left will give the Circumference of the Circle, and the Figures cut off will be the Numerator of a Fraction, the Denominator of which is 10000.

To find, for Instance, the Circumference of a round Vase of a Fountain, the Diameter of which is 64 Foot, we multiply 64 by 31416, and from the Product 2010624 cut off four Figures to the Right-hand, which leaves us 201 Foot and $\frac{624}{10000}$ for the Circumference demanded.

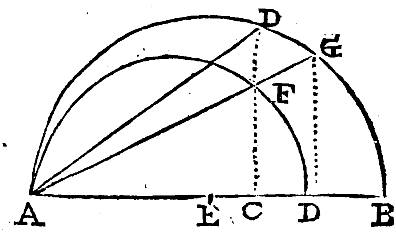
If we want to know the Diameter of a Circle or Ball by the Circumference given, we must reverse the Operation; that is, multiply the Circumference by 10000, which is done by adding to it four Cyphers to the Right, and dividing the Product by 31416.

Thus to know the Diameter of a round Tower, the external Circuit of which is by a long Rope found to be 154 Foot, we add four Noughts to the Right of 154, and divide 1540000 by 31416, which gives 49 Foot for the Diameter we look for.

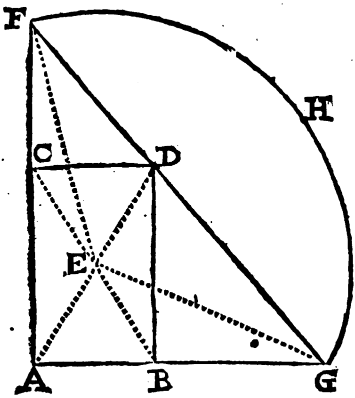
PROBLEM XIII.

To find One, Two, or Three mean Proportionals to two Lines given.

TO find in the first place one mean Proportional between the two Lines given AB, AC, we describe round the greatest AB the Semicircle ADB, and from C the end of the least AC raise the Perpendicular CD, and draw the right Line AD, which is a mean Proportional between the two Lines AB, AC.



To find two Means continually proportional between the two given Lines AB, AC; we make of these two Lines the Rectangle Parallelogram ABDC, and from its Centre E describe the quarter of a Circle GHF, of such a bigness, that the Right Line FG drawn through the two Points where the Curve cuts the two given Lines AB, AC prolong'd, passes by the Right Angle D; for then the two Lines CF, BG, will be the mean Proportionals enquired for, and the four Lines AB, CF, BG, AC, will be continually proportional.



In fine, To find three Means continually Proportional between the two Lines given AB, AC, we first find one mean Proportional AD, as was above directed; and then pursue the same Method in finding AE, (*See the last Fig. but one*) another mean Proportional between AD, and AC the first given Line, and at last AG yet another mean Proportional between AD and AB; and thus the three Lines AF, AD, AG, will be the Mean Proportionals demanded; so that the five Lines AC, AF, AD, AG, AB, will be in continual proportion.

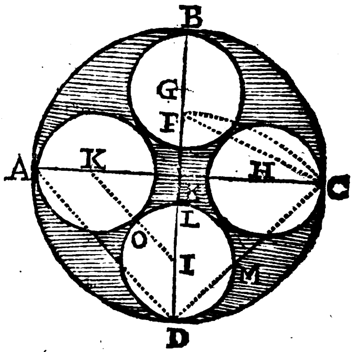
Remark. If the two Lines AB, AC, are given in Numbers, as if AB were 32, and AC 2, we may express in Numbers the three Means AF, AD, AG, by multiplying together 32 and 2 the two Numbers of the two given Lines, and taking the Square Root of the Product 64, *viz.* 8 for the Mean AD; which being multiplied by AC the first, and AB the last, separately, the Square Roots of the two Products 16 and 256, make 4 for AF, and 16 for AG.

But to find in Numbers only two Means proportional between the two given Lines AB, AC, such as CF, and BG (*See the last Fig.*) supposing AB the least to be be 2 Foot, and AC the greatest 16, multiply 4 the Square of the first AB, by the last AC, and take the Cube Root of the Product 64; thus you have 4 for the first Mean Proportional CF, which follows in proportion the first of the given Lines. Then multiply in like manner 256 the Square of the last given Line AC, by the first AB, and extract the Cube-Root of the Product 512, which brings you 8 for the other Mean Proportional BG.

PROBLEM XIV.

To describe in a given Circle four equal Circles that mutually touch one another, and likewise the Circumference of the given Circle.

THE Circle given being ABCD, the Centre of which is E, divide it into four equal parts by the two perpendicular Diameters AC, BD, upon the Diameter BD take the Line DF equal to the Line CD, which is the Subtender or Chord of the quarter of the Circle, and the Line EF will give the Length of the Radius of each of the equal Circles demanded. So if you set off the Length of EF upon the perpendicular Diameters AC and BD, as from A to K, from B to G, from C to H, from D to I, and upon the Centres K, G, H, I, describe through the Points A, B, C, D, four circular Circumferences, they will both touch one another, and touch the Circumference of the Circle given ABCD.



If you joyn any two Centres, as I, K, with the Right Line IK, this Right Line will be parallel to its corresponding Chord DA, and will pass through the point of the Contact O; and consequently 'twill make at I, half a Right

Remark.

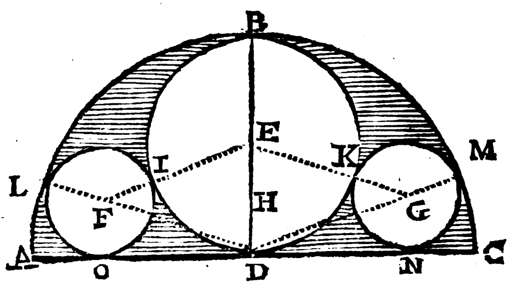
G 2

a Right Angle, or an Angle of 45 Degrees with the Diameter BD; and so the Arch LO will be likewise 45 Degrees, as well as the Arch MO, the whole Arch LM being a quarter of a Circle. From whence it follows, that if you draw the Right Line CF the Angle ECF will be 22 Degrees 30 Minutes, which affords another Construction for the Resolution of the Problem.

PROBLEM XV.

To describe in a given Semicircle three Circles that touch the Circumference and Diameter of the given Semicircle; and of which, that in the middle, being the biggest, touches the two others that are equal.

FROM the Centre D of the Semicircle given ABC upon the Diameter AC raise the Perpendicular DB



and divide it equally at the Point E, which will be the Centre of the greatest of the three Circles demanded, *viz.* BIDK. For the other Circles, which are equal one to another, divide the Semidiameter DE into two equal Halves at the point H; and with the Interval BH describe on each side of the two Points ED, two Arches of a Circle which here cut one another at the Points FG for the Centres of the two equal Circles; which may easily be describ'd, in regard the Radius of each of 'em is equal to the Line DH, or the fourth part of the Diameter BD, or, which is the same thing, to the eighth part of the great Diameter AC.

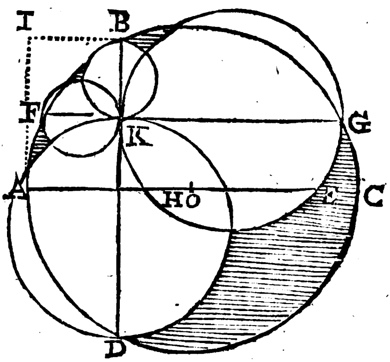
Ti

'Tis evident that the Semicircle ABC is the Double of the Circle BIDK, since the Diameter AC is the Double of the Diameter BD; and in like manner, that the Semicircle BID is the Double of the Circle ILO, since the Radius DE is double the Radius FI or FL. From thence 'tis easie to conclude, that the Mixtilineal-Triangle ABID is equal to the Semicircle BDI, and consequently, that the Semicircle ABC is divided into four equal parts by the Diameter BD, and the Circumference BIDK. Remark.

PROBLEM XVI.

To describe Four proportional Circles, in such a manner, that their Sum shall be equal to a given Circle, and that the Sum of their Radius's be equal to a Line given.

LET the given Circle be ABCD, the Centre of which is O, and one Diameter AC; and let the



Line given be AE greater than the Radius AO, and less than the Diameter AC, if the four Circles demanded are required to be unequal. The Diameters of these four Circles will fall thus:

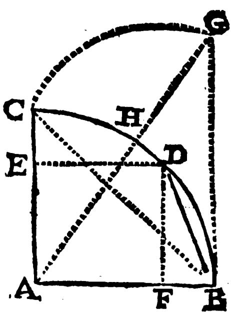
Having drawn at pleasure in the Circle given ABCD the Line FG parallel to the Diameter AC; and having cut off from the Line given AE the part EH equal to the Half of the Line F G, draw from A the Extremity of the Diameter A C, the Line A I equal to the Line A H, and perpendicular to the Diameter A C; and from the point I draw I B parallel to the same Diameter A C; which Parallel Line here meets the Circumference of the given Circle at B: from that point B draw B D perpendicular to the Line F G, and the four Lines K F, K B, K G, K D, will be the Diameters of the four Circles sought for.

Remark. It may so fall out, that the two smaller Circles K F, K B, shall be equal, as well as the two larger K G, K D; namely, when the Line F G is equal to the Line given A E. And consequently when you would have all the four Circles unequal, it behoves you to draw the Line F G either greater or lesser than the Line given A E, and in that case the Circle K F will be the least of 'em all, and the Circle K D the greatest.

PROBLEM XVII.

Upon the Circumference of a Circle given, to find an Arch the Sinus of which is equal to the Chord of the Complement of that Arch.

LET the Quadrant of a Circle be given A B C, the Centre of which is A; from B the Extremity of the Radius A B raise the Perpendicular B G equal to B C the Chord of the Quadrant; then from the Centre A to the point G draw the Right Line A G, and having taken upon the Radius A B, the part A F equal to the part G H, raise from the point F upon the Line A B the Perpendicular F D, which will determin the Arch demanded, *viz.* C D, the Sinus of which is equal to B D the Chord of the Complement of that Arch.



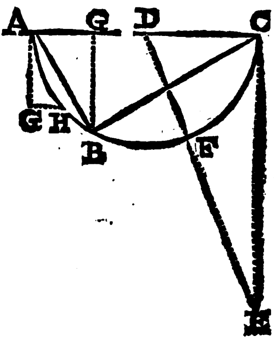
Remark. The Secant A G of the Arch B H is equal to the Tangent of an Arch of 60 Degrees; that is to say, the Radius A H, being 100000 parts, the Line A G, contains

contains 173205, from which if you take AH or 100000, the remainder 73205 is the part GH or AF; that is, the Sinus ED of the Arch CD, which will be 47. 3'. 31". and by consequence its Complement BD is 42. 56'. 29". Thus we know that the Sinus of an Arch of 47. 3'. 31". is equal to the Chord of an Arch of 42. 56'. 29". which is its Complement.

PROBLEM XVIII.

To describe a Rectangle-Triangle, the three sides of which are in Geometrical Proportion.

HAving drawn at pleasure the Semicircle ABC; the Centre of which is D; and of which the Diameter AC shall be taken for the Hypothenuse of the Rectangle-Triangle desir'd; draw from C, the Extremity of the Diameter AC, the Line CE equal and perpendicular to the Diameter it self AC, and joyn the Right-Line DE, which is here cut by the Circumference of the Semicircle ABC at the Point F. Take the Length of the part EF upon the Circumference ABC, extending from A to B, and joyn the Right-Lines AB, BC, which at the Point B will form a Right-Angle, and the Rectangle-Triangle ABC will be the Triangle enquired for; and so there will be the same *Ratio* between the Side AB and the Side BC, as there is between BC and the Hypothenuse AC.



If from the Right-Angle B you draw the Line BG perpendicular to the Hypothenuse AC, the greater Segment CG will be equal to the least Side opposite AB, or to the part EF; from whence we draw another Construction for the Resolution of this Problem, namely, by taking upon the Diameter AC the part CG equal to the part EF, and letting fall from the Point G the Perpendicular GB, &c. Remark.

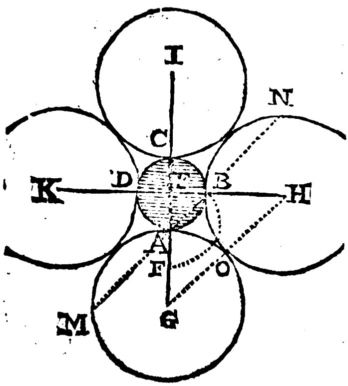
A third Construction may be obtain'd, if we consider that the Hypothenuse AC is cut at the Point G by its Perpendicular BG in the mean and extream *Ratio*; that is, the Hypothenuse AC is to its greatest Segment CG as the same greatest Segment CG is to the lesser AG.

If you desire a fourth Construction, let fall from the Extremity A, the Line AG perpendicular to the Diameter AC, and equal to the third part of the same Diameter AC; and from the Point G draw the Line GH parallel to the Diameter AC; this Parallel GH will be equal to the third part of the lesser Segment AG, &c.

PROBLEM XIX.

To describe Four equal Circles which mutually touch one another, and on the outside touch the Circumference of a Circle given.

HAving divided the given Circle ABCD into four equal parts by the two Diameters AC, BD, which cut one another at Right-Angles at the Centre E; take upon the Diameter AC prolong'd, the Line AF equal to the Line AB, or to the Chord of the Quadrant of the Circle; and the Line EF will give the Length of the Radius of each of the four equal Circles demanded. So run the Length of EF upon each of the two Diameters prolong'd, AC, BD, from the Circumference of the Circle given ABCD to the Points G, H, I, K; and from these Points or Centres describe by the Points A, B, C, D, as many equal Circles, which will mutually touch one another, and likewise the Circumference of the Circle given ABCD.



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If you joyn any two Centers, as G, H, by the Right-Line G H, this Line G H will be parallel to the corresponding Chord A B, and will pass through the Point of Contact O; and by consequence will form at the Points G, H, half Right-Angles, or Angles of 45 degrees; so that each of the Arches, A O, B O, will be likewise 45 degrees. *Remark.*

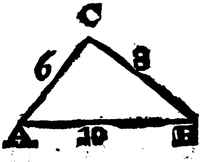
PROBLEM XX.

To describe a Rectangle-Triangle, the Three Sides of which are in Arithmetical Proportion.

TAKE the Indefinite Line A B, and mark upon it five equal parts of what length you will, from A to B; and let this determin'd Line A B be the Hypothenuse of the Rectangle-Triangle demanded. From the Extremity A, at the Interval of three of the parts describe an Arch of a Circle, and from the other Extremity B, at the distance of four parts describe another Arch, which will cut the first at a Point, as at C; and from this Point C if you draw to the two Extremities of the Hypothenuse A B, the Right-Lines A C, B C, you have a Rectangle-Triangle A B C, the three Sides of which, A B, B C, A C, are in Arithmetical Proportion, that is, they equally rise one above another in length, the Side A B containing 5 parts, the Side B C 4, and the Side A C 3.

These Rectangle-Triangles, the Sides of which are Arithmetically Proportional, have this peculiar Property, That the Sum of their Cubes in Numbers is a perfect Cube: For, A B being 5, its Cube is 125; B C being 4, its Cube is 64; and A C being 3, its Cube is 27; and 216 the Sum of the three Cubes, 125, 64, 27, has 6 for its Cube-Root, which in this Rectangle-Triangle is equal to its Area. *Remark.*

If you double all the Sides of the Triangle A B C, and so make the Side A B to contain 10 parts, the Side B C 8, and the Side A C 6, you'll have another Rectangle-Triangle similar to the former; so

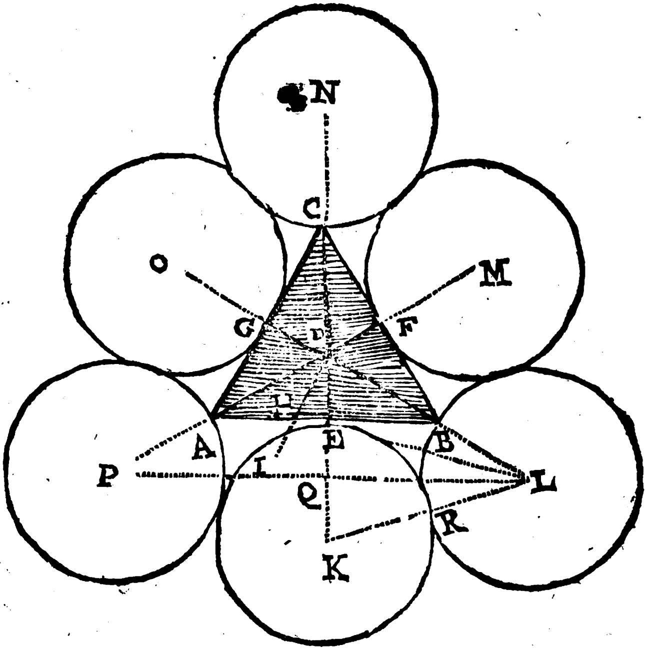


so that its three Sides are still in Arithmetical-Proportion, and the Sum of their Cubes is a perfect Cube, *viz.* 1728, the Cube-Root of which is 12. Besides the Area and the Circumference of this second Rectangle-Triangle are equal, each of 'em being 24. *See* Probl. XXIII.

PROBLEM XXI.

To describe Six equal Circles which mutually touch one another, and likewise the Three Sides, and Three Angles of an Equilateral-Triangle given.

LET the Equilateral-Triangle given be A B C, and its Center D. From the Center D draw by the Three Angles A, B, C, and by E, F, G, the middles of the three Sides, as many Right Lines; in order to mark upon



'em

'em K, L, M, N, O, P, the Centers of the Six Circles demanded, and that in the following manner.

Upon the Side A B take the part E H equal to the half of the Perpendicular D E; and having joyn'd the Right Line D H, prolong it to I, so as to make the part H I equal to the part H E, the whole Line D I will give the Length of the Radius of each of the six equal Circles to be describ'd, the Centers of which will be found by running the length of D I from E to K, from B to L, *&c.*

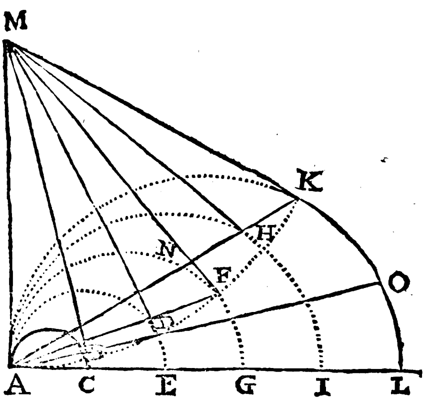
Remark. If you joyn the two Centers P, L, by the Right-Line P L, this Line P L will be parallel to the Side A B, and by consequence will divide the Radius E K at Right-Angles, and into two equal Halfs. Hence it follows, that if you draw the Right-Line E L, and the Right-Line K L, which will pass through the point of Contact R, the Triangle E L K will be an Isosceles-Triangle, each of the two equal Sides, E L, K L, being double the Base E K; and the Arch E K will be 75. 31'. 20". as the Arch B R will be 44. 28'. 40". So that these two Arches will make together just 120 Degrees, that is, as much as the Angle P D L.

PROBL.

PROBLEM XXII.

Several Semicircles being given which touch one another at the Right-Angle of two perpendicular Lines, and have their Centers upon one of these two Lines; to find the Points where these Semicircles may be touch'd by straight Lines drawn from these Points to a Point given upon an other perpendicular Line.

LET the given Semicircles ABC, ADE, AFG, AHI, AKL, the Centers of which are upon the Line AL, perpendicular to the Line AM, touch one another at the



Right-Angle A. And let it be requir'd to find the Points at which all the Semicircles may be touch'd by a Right-Line for each drawn from the Point M.

From the Point given M, as a Center, and through the Point of Contact A, describe the Arch of the Circle A K, which will cut the Circumferences of the given Semicircles at Points, as here, at B, D, F, H, K; and these will be the Points of Contact requir'd.

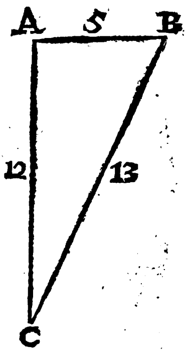
Remark. When the Divisions of the Line A L are equal, you may make use of these Semicircles to divide a Line given into equal parts, *viz.* by applying that Line, suppose A K or A O, from the Point A to the Circumference of the fifth Semicircle; when you have a mind to divide it into five equal parts, for the Circumferences of the other

other Semicircles will mark upon it so many Divisions. By the like Method any Line may be divided into any other number of Parts.

PROBLEM XXIII.

To describe a Rectangle-Triangle, the Area of which in Numbers is equal to its Circumference.

DRAW the two Perpendicular Lines A B, A C, making the first, A B, to contain 5 parts, taken by a Scale of equal Parts, and the other 12 from the same Scale; then draw the Hypothenuse B C, which will contain 13 equal parts, as is easily found out by adding 25, 144, the Squares of the two Sides A B, A C, and extracting the Square-Root of their Sum. The Area of this Rectangle-Triangle will be equal to its Circumference, or to the Sum of its three Sides, *viz*. 30. The same is the Quality of a Rectangle-Triangle made of 6, 8, 10, in Numbers, the Area and Circumference being either of 'em, 24.



No Rectangle-Triangles, in entire Numbers, enjoy this Quality, but the Two now mention'd, *viz*. 6, 8, 10, and 5, 12, 13. But in the Fractional-Numbers we may find an Infinity of this sort, and that by following this General Rule, which is grounded on Demonstration. *Remark.*

Form a Rectangle-Triangle from any square Number, and the same Square augmented by the Addition of 2; then divide this Triangle by the Square Number, in order to have a second Rectangle-Triangle, the Area of which is equal to its Circumference. For *Example*, Take 9 and 11, and form this Rectangle-Triangle 40, 198, 202, and divide it by 9; you have another Rectangle-Triangle $\frac{40,\ 198,\ 202,}{9}$ the Area and Circumference of which are equal, each of them being $\frac{440.}{9}$ In like manner, if from 16 and 18 *How to find Rectangle-Triangles, the Area's and Circumferences of which are equal.*

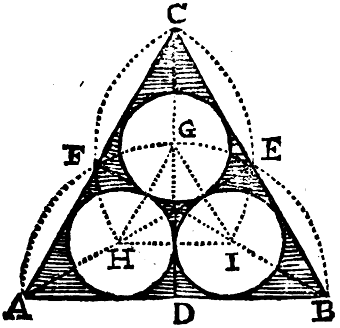
you

you form the Rectangle-Triangle 68, 576, 580, and divide it by 16, you have this other Triangle $\frac{17, 144, 145}{4}$, the Circumference and Area of which are equally $\frac{135}{2}$. And so on.

PROBLEM XXIV.

To describe within an Equilateral-Triangle Three equal Circles which touch one another, and likewise the Three Sides of the Equilateral-Triangle.

LET the Equilateral-Triangle be ABC; divide each of its Sides into two equal parts at the Points D, E, F, and through these Points draw to the opposite Angles as many straight Lines, upon which you are to take the Centers G, H, I, of the three Circles demanded, by setting off upon each Perpendicular Line, half the side of the Equilateral - Triangle from the respective middle Point, namely, from D to G, from E to H, from F to I, &c.



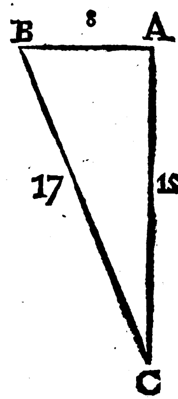
Remark. If you joyn the Three Centers, G, H, I, by the straight Lines which pass through the Points of Contact, you have the Equilateral-Triangle GHI, whose Sides will be parallel to those of the given Triangle ABC, and three equal Trapezia AHIB, BHGC, CGIA, each of which hath Three Sides equal to those of the Equilateral-Triangle GHI, and the Area's of which, are, each of 'em, equal to the eighth part of the Square of AB, the Side of the Triangle given ABC.

PROBL.

PROBLEM XXV.

To describe a Rectangular-Triangle, the Area of which, in Numbers, is one and an half of the Circumference.

DRAW two Perpendicular-Lines AB, AC, the first of which contains 8 parts, taken from a Scale of equal parts, and the other 15; joyn the Two Extremities with the Hypothenuse B C, which will contain 17 parts, as is easily perceiv'd, by adding 64, 225, the Squares of the two Sides AB, AC, and extracting the Square-Root of the Sum 289. Here 60, the Area of the Right-Angled-Triangle ABC, is to the Circumference 40, as 3 is to 2. The same is the Quality of this other Rightangled-Triangle 7, 24, 25; the Circumference 56 being two Thirds of the Area 84.



Remark.

Besides the two Rightangled-Triangles now mention'd, *viz.* 7, 24, 25, and 8, 15, 17; we have no other in entire Numbers that possess this Quality; but many in Fractional-Numbers, which are found by the following General-Rule taken from Algebra. *Form a Rightangled-Triangle of any square Number, and the same Number, with the Addition of* 3; *and divide the Triangle by the same square Number; you have a second Rightangled-Triangle, the Area of which leaves a Sesquialteral Proportion to the Circumference.* Thus, if from 4 and 7 you form the Rightangled-Triangle, 33, 56, 65, and divide it by 4, you have this other Rectangle-Triangle $\frac{33, 56, 65,}{4}$ the Area of which $\frac{231}{4}$ is to the Circumference $\frac{77}{2}$ as 3 is to 2. In like manner, if from 16 and 19 you form the Rectangle-Triangle, 105, 608, 617, and divide it by 16, you have this other Rightangled-Triangle $\frac{105, 608, 617,}{16}$ the Area of which $\frac{1995}{16}$ is to its Circumference $\frac{665}{8}$ as 3 is to 2. And so of the rest.

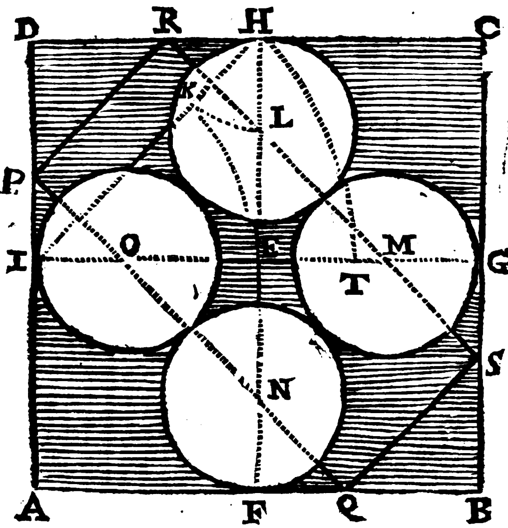
How to find Rightangled-Triangles, the Area's and Circumferences of which are in Sesquialteral Proportion.

PROBL.

PROBLEM XXVI.

To inscribe in a Square given four equal Circles which touch one another, and likewise the Sides of the Square.

LET the Square given be ABCD, divide each of its Sides into equal Parts at the Points F, G, H, I, and draw the Right Lines FH, GI, which will cut one ano-



ther at Right-Angles into two equal parts at E the Center of the Square. Upon these two Lines FH, GI, you are to mark out the Points L, M, N, O, for the Centers of the Four Circles required, and that in the following manner.

Joyn with a straight Line H and I, and cut off from that Line the part IK equal to IE or GE, the halves of the Line IG, or of the side of the given Square; and the Remainder, HK, will be the Radius of each of the Four Circles you would draw. And so if you take the Length of HK upon the Lines FH, GI, from their Extremities F, G, H, I, to the Points N, M, L, O, the Problem is resolv'd.

An

An easier Method is this: From the Line I G cut off the Part I T equal to the Line I H; and make the Lines E L, E M, E N, E O, each of 'em equal to the Remainder T G, in order to have as before, the Centers L, M, N, O, of the four Circles to be described, which are found by making the Lines F N, G M, H L, I O, equal, each of 'em, to the part E T.

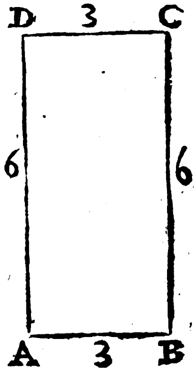
Or else, make the Four Lines A P, A Q, C R, C S, equal, each of 'em, to the Line I H, and draw the Right Lines P Q, R S, which will give you upon the two Lines F H, G I, the Centers L, M, N, O, for thr Four Circles required.

'Tis evident that each of the two Lines P Q, R S, is equal to the Side A B of the Square given A B C D; and each of the two Lines P R, Q S, is equal to the Diameter of each of the equal Circles, which mutually touch. 'Tis likewise evident, that each of the two Isosceles Right-Angled Triangles A P Q, C R S, is equal to the Square D I E H, or to the fourth of the proposed Square ABCD; and that the Isosceles Right-Angled Triangle O E N is equal to the Square of the Radius O I. Remark.

PROBLEM XXVII.

To describe a Rectangle-Parallelogram, the Area of which in Numbers is equal to its Circumference.

DRaw the Two Perpendicular Lines A B, A D, so as to make the first contain 3 Parts taken from a Scale of equal Parts, and the other 6. From the Point D, with the Aperture of the Compass A B describe an Arch of a Circle; and from the Point B, with the Distance A D describe another Arch of a Circle, which here meets the first at the Point C, from which you are to draw the two Lines B C, C D, to perfect the Rectangle A B C D, the Area of which is equal to the Circumference, each of 'em being 18.



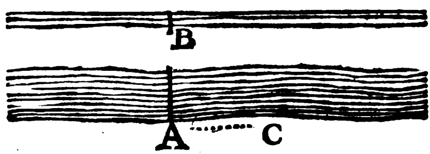
Remark. In Integers we have only this Rectangle, and the Square of 4 that admit of this Quality of having their Area equal to the Circumference; but in Fractions there are many, the Length and Breadth of which is thus determin'd.

How to find Rectangles with the Area's equal to the Circumferences. Fix upon the side A D what Number you please, only it must be larger than 2; suppose then 8; divide its Double 16 by the same side wanting 2, *i. e.* by 6, and the Quotient $\frac{8}{3}$ is the other Side A B. Thus you have in Numbers a Rectangle Parallelogram, which has for its Length 8, for its Breadth $\frac{8}{3}$; and for either its Circumference or its Area $\frac{64}{3}$, or $21\frac{1}{3}$.

PROBLEM XXVIII.

To measure with a Hat, a Line upon the Ground accessible at one of its Extremities.

THE Line to be measured must not be extravagantly long, otherwise 'twill be hard to measure it exactly



with one's Hat; for the least Failure of a just Aim, or departure from an upright Position, would make very sensible Errors in the Measure of a very long Line, especially if the Ground is somewhat uneaven.

To measure then with the Hat the Line A B accessible at the Extremity A, suppose the Breadth of a small River, he who pretends to measure, must stand very straight at the Extremity A, and support his Chin with a little Stick, resting upon one of the Buttons of his Coat, so as to keep his Head steddy in one Position. Thus posited, he must pull his Hat down upon his Forehead, till the Brim of his Hat cover from his View the inaccessible Extremity B of the Line to be measured A B; then he must turn himself to a level uniform piece of Ground, and with the same Position of his Hat observe the Point

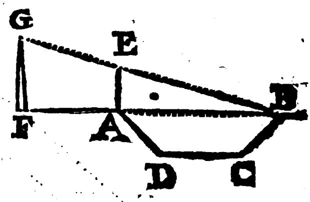
of

of the Ground where his View terminates, as C; then measuring with a Line or Chain the Distance A C, he has the Length of the Line propos'd, A B.

PROBLEM XXIX.

To measure with two unequal Sticks a Horizontal Line accessible at one of its Extremities.

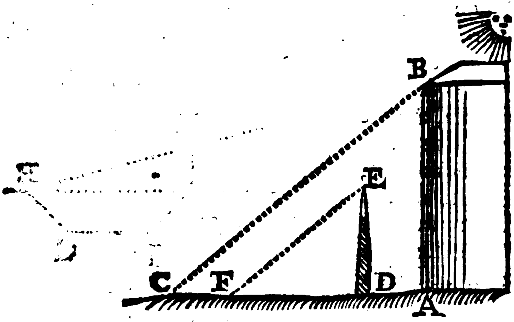
TO know the Length of the Horizontal-Line A B, which represents the Breadth of the Ditch ABCD, and is accessible at its Extremity A; set up, perpendicularly, at that Extremity A, the least of the two Sticks A E; and the greater of the two FG, upon a streight Line with the Line to be measured, at such a Distance from the first, A E, that you may just perceive the inaccessible Extremity B, over the two Ends E G of the two Sticks thus fixed. Then take an exact Measure of the Distance A F, which we here suppose to be 12 Foot; and of the Length of the two Sticks A E, F G, of which we here suppose the least, A E, to be 3 Foot; and the greatest, F G, 5; so that by this Supposition, the Excess of the greater Stick above the lesser is 2 Foot. Now, let this Excess 2 be the first Term of an Operation of the Rule of Three Direct; the second being 12, or the Distance A F; and the third 3, or the least Stick A E; and the fourth the Line A B enquir'd after, which is thus found to be 18 Foot; for if you multiply the second Term 12 (the Distance A F) by the third 3 (the least Stick A E) and divide the Product 36 by 2 (the Excess of the greater Stick beyond the lesser) you have 18 Foot for the Length of the Line proposed A B.



PROBLEM XXX.

To measure an accessible Height by its Shadow.

TO measure the accessible Height A B by its Shadow A C, terminated by the Ray of the Sun B C. Set up perpendicularly a Stick D E, of what Length you



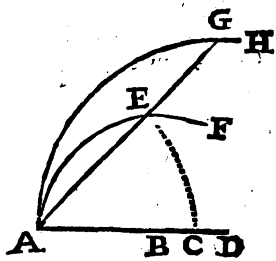
will, suppose 8 Foot; and measure the Extent of its Shadow D F, which we shall here suppose to be 12 Foot. At the same time measure the Shadow A C, which we here suppose to be 36 Foot; I say, *at the same time*, for otherwise, the Ray varying either by the Motion of the Sun, or that of the Earth, the Rays B C, E F, would no longer be parallel, and so would prevent the Operation of the Rule of Three Direct, which runs thus; If 12 Foot of Shadow arise from the Height D E of 8 Foot, from what Height must the Shadow A C of 36 Foot proceed? Here you'll find the Height A B, in question, to be 24; for multiplying the third Term 36 by the second 8, and dividing the Product 288 by the first 12, you have the Quotient 24 for a fourth Proportional Term, *i. e.* the proposed Height A B.

PROBL.

PROBLEM XXXI.

To find a Fourth Line proportional to three Lines given.

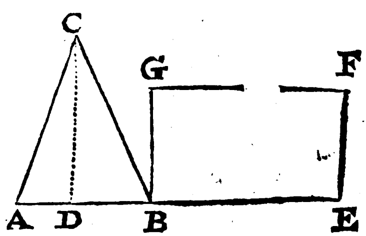
THREE Lines being given AB, AC, AD, to find a fourth Proportional: Upon the two Extremities B, D, of the first and the third Line given, describe, from the common Extremity A, the two Arches of a Circle AEF, AGH, and having apply'd to the first Arch AEF, the Line AE equal to the second Line given AC, prolong the Line AE till it meets the second Arch AGH in some Point, as in G, and the whole Line AG will be the fourth Proportional demanded.



PROBLEM XXXII.

Upon a Line given to describe a Rectangle-Parallelogram, the Area of which is the Double of that of a Triangle given.

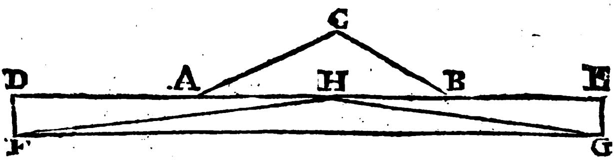
LET the Triangle given be ABC, and the Line given BE; draw EF perpendicular to it, and a fourth proportional to the Base given BE, the Base AB of the Triangle given ABC; and the Height CD; then finish the Rectangle BEFG, which solves the Problem. This Problem is placed here only as subservient to that which follows.



PROBLEM XXXIII.

To change a Triangle given into another Triangle, each side of which is greater than each side of the Triangle given.

If the Triangle given be A B C, prolong its Base A B on both Sides to D and E, so, that the Line A D may be equal to the Side A C, and the Line B E to the



Side B C; and by the Direction of the foregoing Problem, describe upon the Line D E the Rectangle Parallelogram D E G F, the Double of the Triangle given A B C. This done, take upon the Line D E, between the Points A, B, a Point at discretion, such as H, from which draw to the two Extremities F, G, the Right Lines F H, G H. Thus you have the Triangle F G H, equal to the propos'd Triangle A B C, each of 'em being the half of the Rectangle F G E D, and each of the Sides of the one being greater than each of those of the other, which was to be done.

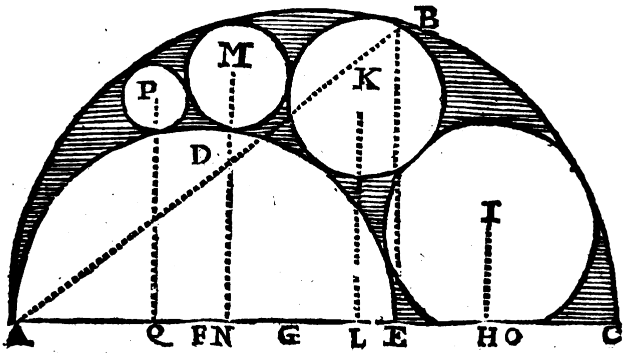
Remark. You may have a Triangle less than the propos'd Triangle, with all its Sides longer than those of the Triangle A B C, *viz* by taking H the top of the Triangle F G H under the Base A B,

PROBL.

PROBLEM XXXIV.

Two Semicircles upon one Right Line being given, which touch one another on the inside; to describe a Circle that touches both the Right Line and the Circumferences of the two Semicircles given.

I Suppose the two Semicircles A B C, A D E, are placed upon the Right Line A C, and touch one another at the Point A. To describe a Circle that



touches the two Circumferences A B C, A D E, and the part E C of the Right Line A C; lay the Length of the Semidiameter A G of the great Semicircle A B C, from F the Center of the lesser Semicircle A D E to O, in order to have the Line A O equal to the Sum of the Semidiameters A F, A G, of the two Semicircles given, A B C, A D E. From the Point E upon A C raise the Perpendicular E B, and joyn A and B. Then to the two Lines A O, A B, find a third Proportional A H, and so you have in H the Point of Contact between the Circle to be described and the Right-Line E C. From this Point H raise upon E C the Perpendicular H I, a fourth Proportional to the three Lines given, A O, A H, F G; and so I gives you the Center of the Circle you want to describe, the Circumference of which must pass through the Point H.

Remark. If within the space terminated by the two Circumferences A B C, A D E, you describe a second Circle that touches the first describ'd from the Center I, and the two Circumferences A B C, A D E; and if from the Center K of this second Circle, you let fall the Right Line K L perpendicular to the Diameter A C, that Perpendicular K L, will be the Triple of the Radius of the Circle describ'd upon the Center K: And if within the same space you draw a Circle that touches both the second drawn from the Center K, and the Circumferences A B C, A D E, the Perpendicular drawn from the Center M of that third Circle to the Diameter A C, will be the Quintuple of the Radius of the same Circle: And in like manner, if within the same space you describe a fourth Circle, that touches both the third drawn upon the Center M, and the Circumferences of the two Semicircles, the Perpendicular let fall from P, the Center of that fourth Circle, upon the Diameter A C, will be the Septuple of the Radius of the same Circle; and so on in the Progression of the uneven Numbers 3, 5, 7, 9, &c,

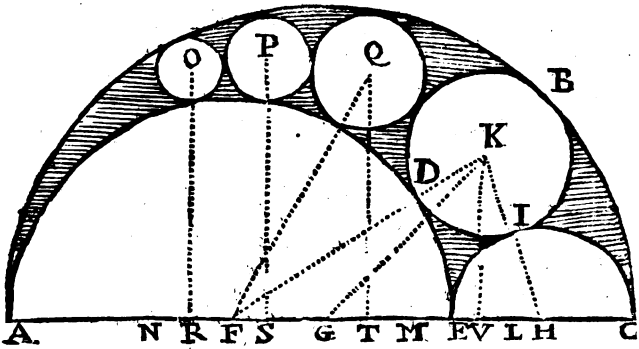
Here we shall take notice by the bye, for the sake of the Learned, that all the infinite Circles that can touch the two Circumferences A B C, A D E, have their Centers in the Circumference of an Ellypsis, the Axis of which is O, which has the Line A H for its Parameter.

PROBL.

PROBLEM XXXV.

Three Semicircles upon one Right Line being given, which touch within, to describe a Circle that touches the Circumferences of the Three Semicircles.

IF the three Semicircles are ABC, ADE, EIC, the Centers of which F, G, H, are upon the Right Line AC; having found to the Line FG and the Radius



AF a third Proportional AL, find a fourth Proportional to the Sum of the two Lines AL, AG, the Radius AG, and the Radius AF. This fourth Proportional will be the Length of KI, the Radius of the Circle to be described, and that Length must be taken upon the Line AC, from G to M, and from F to N, in order to describe upon the Center N, and with the Distance NE an Arch of a Circle, and from the Center H, with the Interval MF another Arch of a Circle, which might likewise be drawn from the Center G, with the Aperture MC. Here K, the common Intersection of these two Arches, gives you the Center of the Circle to be described; which is readily done, now the Radius is known, *viz.* GM, or FN.

If you joyn the Center K with the Centers F, G, H, of the three Semicircles given, by the straight Lines FK, GK, HK, you'll have two Triangles, FKG, GKH, of the same Circumference; the Circumference *Remark.*

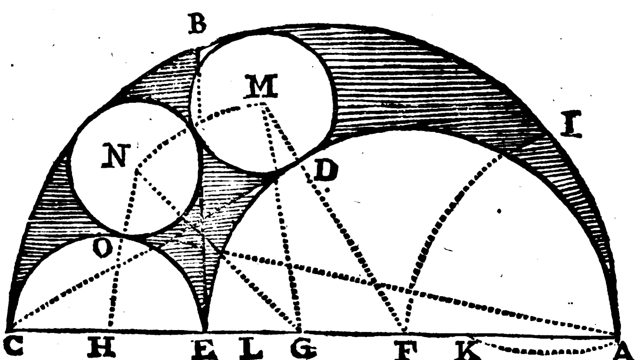
rence of each being equal to the Diameter AC of the great Semicircle given ABC, by reason of the two equal Lines AF, GH.

If between the two Circumferences ABC, ADE, you describe, as in the foregoing Problem, as many Circles as you will that touch one another, and the two Circumferences ABC, ADE; and if from their Centers O, P, Q, K, you let fall upon the Diameter AC as many Perpendiculars, the Perpendicular KV will be equal to the Diameter of its Circle, the Perpendicular QT will be the Double of the Diameter of its Circle, the Perpendicular PS will be the Triple of the Diameter of its Circle, the Perpendicular OR will be the Quadruple of the Diameter of its Circle, and so on, according to the Series of the natural Numbers 1, 2, 3, 4, 5, 6, &c.

PROBLEM XXXVI.

Three Semicircles upon one straight Line, which touch on the inside, being given, with another Right Line drawn from the Point of Contact of the two interiour Circles perpendicular to the first Right Line given: To describe two equal Circles which touch that Perpendicular and the circumferences of the two Semicircles.

LET the three Semicircles given be ABC, ADE, EOC; of which the Centers F, G, H, are plac'd upon the Right Line AC, which is cut at Right-Angles



at

at the Point E by the Right Line BE: The common Radius of the two equal Circles which must touch the Perpendicular BE, and the Circumferences of the two Semicircles, will be found by describing from the Point A through the Center F the Arch of a Circle FI, and from the Point I by the Point A, the Arch AK; for the Line KF is the Length of the Radius of the two equal Circles, the Centers of which, M and N, are found out as follows:

Having drawn the Line GL equal to the Line KF, describe from the Center G with the Aperture LC the Arch MN, and from the Center F with the Aperture KE, another Arch of a Circle, which will cut the first Arch at M; this M is the Center of a Circle that shall touch the Circumferences of the two Semicircles ABC, ADE, and the Perpendicular EB. Describe likewise from the Center H with the Aperture FL another Arch of a Circle that shall cut the first MN at N, the Center of the other Circle that shall touch the Perpendicular BE, and the two Circumferences ABC, EOC.

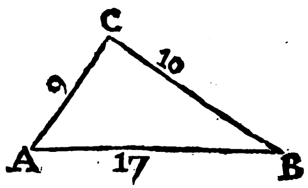
If you joyn the two Centers MN with the three Centers F, G, H, by straight Lines, you'll have the two Triangles FMG, GNH, of equal Circumferences, the Circuit of each being equal to the Diameter AC of the greatest Semicircle given ABC, by reason of the two equal Sides GM, GN, of the Base GH equal to the Radius AF, and of the Base FG equal to the Radius EH. Besides, MN or NO, is a fourth Proportional to the three Lines AG, AF, FG. In fine, if you draw the Right Lines AO and CD, they'll be perpendicular to their Radius's, *that is*, the Line AO will be perpendicular to the Radius HO or NO, and by consequence will touch the Circumferences of these two Radius's at the Point O; and the Line CD will be perpendicular to each of the two Radius's FD, MD, and by consequence will touch the Circumferences of these two Radius's at the Point D. From hence we may draw another Construction for the Resolution of the Problem. *Remark.*

PROBL.

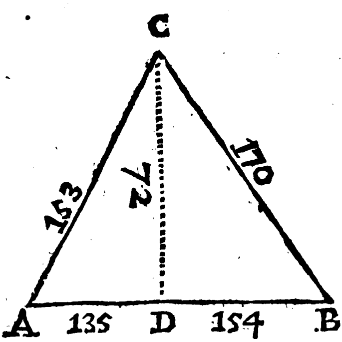
PROBLEM XXXVII.

To describe a Triangle, the Area and Circumference of which are one square Number.

TAKE from a Scale of equal Parts 17 for the Base A B; from the Extremity of which, A, with the Aperture of 9 Parts describe an Arch of a Circle, and from the Extremity B, with the Interval of 10 Parts, describe another Arch of a Circle, which will cut the first at a Point, which we here suppose to be C. Then draw the straight Lines AC, BC, and the Triangle A B C is the Triangle you want, its Area and Circumference being, either of them, 36, the Square-Root of which is 6.



Remark. This Triangle has been found in Numbers by the means of these two Numeral Right-Angled Triangles of the same height, 72, 135, 153, and 72, 154, 170; the Generating Numbers of which are 12, 3, and 11, 7. It has been found, I say, by joyning together these two Right-Angled Triangles, in order to have the Oblique-Angled Triangle ABC, the Height of which C D, is 72; the Base A B, being 289; and by dividing each Side by the Square-Root 17 of that Base 289, &c.

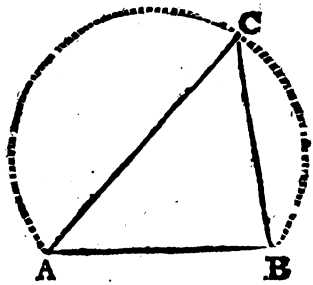


PROBL.

PROBLEM XXXVIII.

To make the Circumference of a Circle pass through three Points given without knowing the Center.

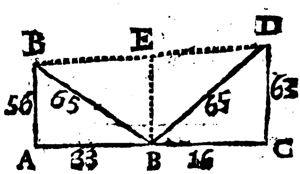
TO draw an Arch of a Circle through three Points given; for instance, the three Angles of the Triangle A B C, without knowing its Center, make an Angle equal to C of some solid Matter, such as Pastboard, and apply several ways one side of this Angle to the Point A, so that the other side may fall on the Point B, and then the Point of the same Angle will mark out the Points of the Arch demanded; which is easily drawn out by joyning all its diverse Points, which may be found *in infinitum*, by a curve Line, &c.



PROBLEM XXXIX.

Two Lines being given perpendicular to one Line drawn through their Extremities, to find upon that Line a Point equally remov'd from each of the two other Extremities.

GIVE the two Lines A B, C D, perpendicular to the Line A C, which passes through their Extremities A, C, you'll find upon that Line A C, the Point F equally remov'd from the two other Extremities B, D; you'll find it, I say, by joyning these two Extremities with the Right-Line B D, and drawing to



the

the middle Point of that Line E, the Perpendicular E F, which will mark out upon A C the Point F required, the two Lines F B, F D being equal.

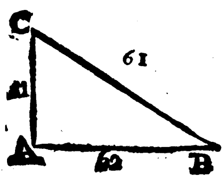
Remark. This Problem is commonly propos'd after the following manner: *The Heights* A B, C D, *being given, with their Distance* A C, *to find upon the Ground* A C, *a Point* F, *from which the Ropes extended to the tops* B *and* D *shall be equal.*

When the Heights A B, C D, and their Distance A C are known in Numbers; as if the Height A B were 56 Foot, the Height C D 63, and the Distance A C 49; the Part A F is found by taking from the Sum of the two Squares A C, B D, the Square A B, and dividing the Remainder by the Double of A C; and in like manner, the Part C F is found by subtracting from the Sum of the Squares A C, A B, the Square C D, and dividing the Remainder by the Double of A C. Thus the Part A F will be found 33 Foot; the other Part C F 16 Foot; and each of the two equal Chords F C, F D, 65 Foot, as is easily computed, by adding the two Squares A B, A F, or the two C D, C F, and extracting the Square-Root of the Sum 4225, &c.

PROBLEM XL.

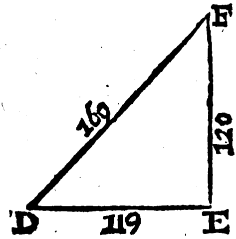
To describe two Right-Angled Triangles, the Lines of which have this Quality, That the Difference of the two smallest Lines of the first is equal to the Difference of the two greatest of the second; and Reciprocally the Difference of the two smallest of the second is equal to that of the two greatest of the first.

DRAW first the two perpendicular Lines A B, A C, of such a size that the first A B contains 60 Parts of a Scale of equal Parts, and the second A C 11; in which Case the Hypothenuse B C will be 61, as appears by adding the Squares of A B, A C, and extracting the Square-Root of the Sum 3721.



Then

Then draw the two perpendicular Lines DE, EF, making the first DE 119 Parts, and the second 120; in which Case the Hypothenuse will be 169, as appears by adding the Squares DE, EF, and extracting the Square-Root of 28561. This done, the two Right-Angled Triangles ABC, DEF, will resolve the Problem; for the Difference, 49, of the two smallest Lines AB, AC in the first Triangle ABC, is equal to the Difference of the two greatest DE, EF, in the second Triangle DEF; and Reciprocally, the Difference 1 of the two smallest DE, EF in the second is equal to that of the greatest AB, BC in the first.



Remark.

These two Differences 49, 1, happen here to be square Numbers, and will always be such in all Right-Angled Triangles calculated according to the following General Rule taken from *Algebra*. *The Double of the Product arising from the greatest of any two Numbers, and their Sum, and the Sum of the Squares of the same two Numbers, are the two Generative Numbers of one of the Right-Angled Triangles to be describ'd; and the Double of the Product arising from the least of the same two Numbers and their Sum, and the Sum of the same Squares, are the two Generating Numbers of the other Right-Angled Triangle demanded.*

A General Rule for Right-Angled Triangles, the Reciprocal Difference of whose Sides are equal.

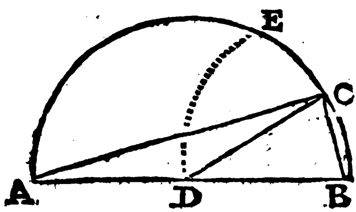
Of these three Generating Numbers, that which is common to two Right-Angled Triangles, is the Hypothenuse of a third Right-Angled Triangle; and of the other two, one is the Circumference of that third Triangle; and the other is the same Circumference, only the greatest Generating Number of that third Triangle is then changed into the least.

PROBL.

PROBLEM XLI.

To divide the Circumference of a Semicircle given into two unequal Arches, in such a manner, that the Semi-Diameter may be a Mean Proportional Between the Chords of these two Arches.

IF the Semicircle given is ABC, the Center of which is D, describe through the Center D from B the Ex-



tremity of the Diameter AB, the Arch of a Circle DE, and having divided the Arch BE into two equal parts at C, draw the two Chords AC, BC, between which the Semi-Diameter AD, or CD, is a Mean-Proportional.

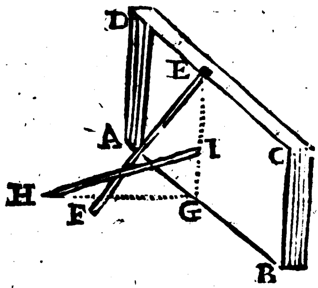
Remark. 'Tis evident, that the Arch BE contains 60 Degrees, and consequently, its Half BC or CE is 30, and the other Arch AEC is an Arch of 150 Degrees. From whence we may readily conclude, that since the Sinus of an Arch is the Half of the Chord of a double Arch, and the Half of the Radius or Sinus Total, is the Sinus of an Arch of 30 Degrees; this Sinus of an Arch of 30 Degrees is a Mean Proportional between the Sinus of an Arch of 15, and the Sinus of its Complement, or the Sinus of an Arch of 75 Degrees.

PROBL.

PROBLEM XLII.

A Ladder of a known Length being set, so as to rest upon a Wall, at a certain Distance from the Wall; to find how far 'twill descend when mov'd a little farther from the Foot of the Wall.

WE'll suppose the Ladder E F standing against the Wall A B C D, to be 25 Foot long, and at the distance of 7 Foot from the Foot of the Wall, and consequently F G perpendicular to the Wall to be just 7 Foot. Suppose again, that the Ladder is mov'd 8 Foot from F to H, so that the Situation being as H I, the part FH must be 8, and by consequence the whole Line G H 15 Foot in which Case the Ladder will have descended from E to I, which is found thus:



Multiply E F, the Length of the Ladder, by it self, *i. e.* 25 by 25, and so you have its Square 625; multiply likewise the Distance F G by it self, *i. e.* 7 by 7, and so you have its Square 49 to be subtracted from the foregoing Square 625, and the Remainder 576 is the Square of the Height E G; because G is the Right-Angle of the Triangle E F G; so that 24 the Square Root of the Remainder 576 is the Height E G.

In like manner, multiply the Distance H I by it self, *i.e.* 25 by 25, so you have 625 for its Square; then multiply the Distance H G by it self, or 15 by 15, and its Square is 225; which subtracted from the other Square 625, leaves for a Remainder 400, the Square of the Height I G; and so 20 the Square-Root of 400, (*i.e.* the Height I G) subtracted from 24 the Height E G found above, leaves 4 the Length of E I, which answers the Problem.

PROBLEM XLIII.

To measure an accessible Line upon the Ground by means of the Flash and the Report of a Canon.

WITH a Musquet-Ball make a Pendulum 11 Inches and 4 Lines long, calculating the Length from the Center of the Motion to the Center of the Ball; and the very moment that you perceive the Flash of the Canon (which must be at the very place, the Distance of which, from the place where you are, is inquir'd after) put the Pendulum in motion, so as that the Arches of the Vibrations do not excede 30 Degrees; multiply by 200, the Number of the Vibrations from the moment you perceiv'd the Flash to the moment in which you hear the Report, and reckon as many * *Paris*-Toises for the Distance of the place where the Gun was fired, from the place where you stood.

* A *Toise* is six Foot.

Remark. Much after the same manner you may measure the Height of a Cloud, when 'tis near the Zenith, and at a time of Thunder and Lightning. But this way of measuring Distances is very uncertain, and I only mention'd it here as a Recreation.

A surer way is that of measuring a tolerable Distance upon the Ground, the Extremities of which can't be seen one from another; but then, in this Case, instead of a Cannon, 'twould do better to make use of an Arquebuse, the Report of which goes 230 Toises in one Second of Time. And so to measure such a Distance, you must have a Pendulum-Clock, and count the Seconds of Time running from the Flash of the Gun let off at one of the Extremities of the Line proposed, and the Perception of the Report in the Ears of another Person placed at the other Extremity of the same Line. Thus the Multiplication of the Seconds of Time by 230, gives you the Length of the proposed Line or Distance in *Toises*.

Father *Schot* says, That 'tis known by several Experiments, that a large Cannon-Ball Horizontally directed, will fly a *German* League of 4000 Geometrical Paces in two Seconds of Time; so that this may serve for the Mensuration of Distances upon the Ground, if it be true, that the Velocity of the Sound is equal to that of the Ball; for then we may compute, That the Distance in Geometrical Paces is to the Number of the Seconds of Time (run between the Flash and the Perception of the Report) as 4000 is to 2, or 2000 to 1, &c.

PROB-

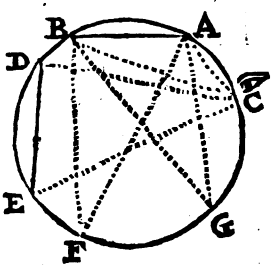
PROBLEMS OF THE OPTICKS.

THE Opticks, according to the Etymology, is a Science of Vision, which is perform'd three different ways. The first is by direct Rays or Rays sent directly from the Object to the Eye; and this makes what we call *Perspective*, which deceives the imagination very agreeably by representing in a Picture which it supposes Transparent, all sorts of Objects, not as they really are, but as they act upon the Eye, and appear in the Picture. The second way of Vision is by Reflex Rays, that is, by Rays that rebound when they strike upon any Body that they can't penetrate; and this is the Object of what we call *Catoptrice*, which supposes the Angle of Reflexion to be equal to the Angle of Incidence. The third way is perform'd by Refracted Rays, or Rays that break in passing through Transparent Bodies. About this the *Dioptrice* is imployed, which supposes that when a Ray passes from one *Medium*, which it penetrates easily, to another that's more difficult to penetrate, it breaks off approaching to a Perpendicular; and on the contrary, when it passes from a difficult to an easie Medium, it Refracts, departing from the Perpendicular. The Opticks supposes likewise, that the Objects seen under the smallest Angles, appear smallest, which ordinarily happens, when they are most Remote. Upon these Suppositions we shall now resolve several useful and agreeable Problems.

PROBLEM I.

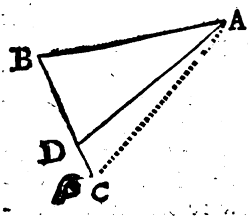
To make an Object to appear still of the same Magnitude, when seen at a distance or nearer.

TO make the Line AB appear to the Eye posited at C always of the same Magnitude, place it in what part you will of the Circumference of a Circle that passes by the Eye C. For if you place it as D E at the remotest part from the Eye, its apparent Magnitude will still be the same, because the Eye continuing still at C, sees these two equal Lines AB, DE, under the equal Angles ACB, DCE.



Remark. 'Tis evident that the Line proposed AB, will always be seen under the same Angle, and consequently will always have the same apparent Magnitude, at any distance from the Eye, provided it never departs from the Circumference of the Circle that passes thro' the two Extremities A, B; and consequently, That without altering the situation of the Line A B, you may change that of the Eye, by placing it in what point you will of the Circumference of any Circle that passes thro' the two extremities of the Line or Body propos'd AB, as in F or in G, the visual Angles AFB, AGB, ACB, being still equal.

'Tis likewise evident that the same Line AB, will retain the same apparent Magnitude when brought nearer to the Eye, without being placed in the Circumference of the Circle, provided its two extremities continue in the same Visual Rays, A C, B C; as it happens in the situation A D, for in that situation 'tis beheld under the same Visual Angle ACB, and so its apparent Magnitude is not alter'd,



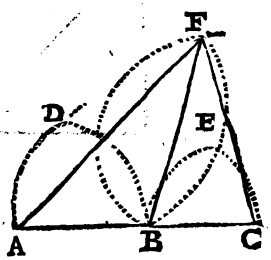
ter'd, notwithstanding that 'tis brought nearer to the Eye.

'Tis by this equality of the Visual Angles that one may write upon a Wall Characters, which tho' very unequal, shall appear equal when seen from a certain Point; and that one may place upon a Pinacle or some high Frontispiece, a Statue of such a length and such a thickness, that when 'tis seen from below, it appears of a bigness proportional to the heighth of the Place, without any necessity of polishing the Figure much, and far less of touching up the muscles of the Body on the plaits of the Drapery, which they would be obliged to do if 'twere to undergo a nearer view.

PROBLEM II.

To find a Point, from which the two unequal parts of a Right Line shall appear equal.

THere's an infinite number of different Points, from which if the two unequal parts AB, BC, of the Right Line AC be view'd, they will appear equal, as being in the Circumference of a Circle: But without insisting upon the Theory, I shall here subjoyn a very short method for finding one of these Points.



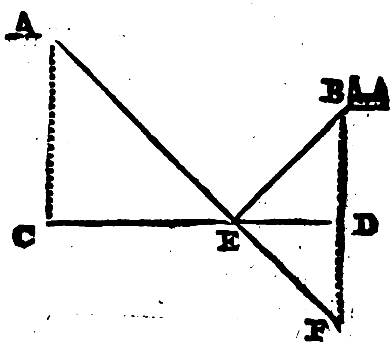
From the two extremities A, B, with the aperture or distance A B, describe two Arches of a Circle, which here cut one another at the point D; and from that point D, draw another Arch of a Circle with the same aperture of the Compasses. In like manner from the two Extremities B, C, with the aperture B C, describe two Arches of a Circle, which here cut one another at E; and from that point E, with the same distance describe another Arch which here cuts that described from D at F. Now F thus found is the point from which if the two propos'd Lines AB, BC, be seen, they will appear equal by reason of the equality of the two Visual Angles AFB, BFC.

Remark. The same is the Construction when the two Extremities of the two Lines propos'd AB, BC, are not to joyn.

PROBLEM III.

The point of any Object being given, and the place of the Eye, to find the point of Reflexion upon the surface of a flat Looking-Glass.

If the point of the Object be B, and the place of the Eye A, and if the surface of the Glass be represented by the Right Line CD; the point of Reflexion will be found by drawing from the two Points A and B, the two Lines AC, BD perpendicular to the Plain CD, and finding a fourth Proportional to the Sum of the two Perpendiculars AC, BD, their di-



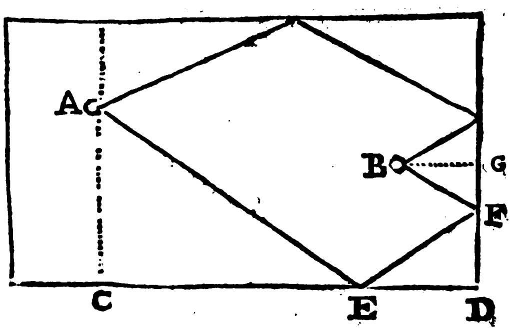
stance CD, and the Perpendicular AC. The length of this fourth Proportional being taken upon CD, from the point C, terminates in E, the point of Reflexion sought for. So if you draw the two Lines AE, BE, the Angle of Incidence AEC, will be found equal to the Angle of Reflexion BED; as 'twere easie to demonstrate.

The Problem applyed to the Billiards. In my Mathematical Dictionary, you'll find this Problem applyed to a Spherical Looking-Glass; but it might easily be applied to the Billiards. For the purpose; if the Line CD represent the side of the Billiardliard-Table, and at the Points A and B there were two Balls, of which the one A could not be made to ſtrike directly upon the other B, by reaſon of the Intervention of the Port, the Player's buſineſs is to find out the Point E by the foregoing Directions, againſt which Point when his Ball ſtrikes, 'twill by a back-ſtroke hit the other Ball at B. But in Practice there's a way of doing it eaſier, as follows.

Let CD be the ſide of the Billiard-Table, and ſuppoſe the Gameſter has a mind with one Ball at A, to hit the other Ball at B by Reflexion. To find the Point E of the ſide, from which the due Reflexion muſt be, let him prolong in his mind the Perpendicular BD to F, ſo that DF may he equal to BD, and after a viſible mark plac'd at F, let him ſtrike his Ball A in the full direction of the Line AF, and then the Ball meeting with the ſide of the Table at E will reflect, and of neceſſity hit the Ball at B, eſpecially if 'twas ſtruck with ſuch force as to conquer the defects of the Table.

But in regard 'tis not always allowed at this Game to put a viſible mark at F, becauſe the oppoſite party may remove it if he pleaſes; the Gameſter muſt content himſelf with taking the aim of his Ball from the Point F, and by the Viſual Ray AF, obſerve the Point E upon the ſide of the Table, where his Ball muſt reflect to B.

If you want to find the point of Reflexion E, with intent to make the Ball A hit B by two back-ſtrokes;

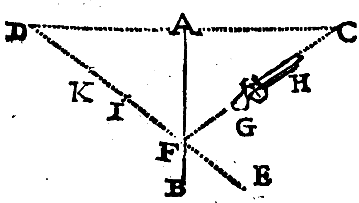


draw from the Point A, the Line AC parallel to the Line DG, and from the Point B, the Line GB parallel to the Line CD; then see to find a fourth Proportional to the sum of the two Parallel Lines AC, DG, to the Line AC, and to the sum of the two Parallel Lines, CD, BG; and taking the length of that fourth Proportional upon CD, fix your point of Reflexion where it terminates, *viz.* at E.

PROBLEM IV.

To shoot a Pistol behind one's Back as true as if the Person took his aim with his face to the Object.

MAke use of a plain Looking-Glass here represented by the straight Line AB, a Perpendicular to which is the Line CD drawn from the Point C, which represents the Butt to be shot at; the Image or Representation of which in the Glass is supposed to be D, at an equal distance from the Glass with the Point C, with respect to the Eye placed at E, from whence the Person that is to shoot sees by Reflexion the Point C, by the Ray of Reflexion EFD, the Ray of Incidence being the Line CF, according to which the Pistol GH must be placed and turn'd till its reflexive appearance IK, agrees with the Line of Reflexion EFD, and covers D the appearance or representation of the Point C; and then 'twill hit the Mark.

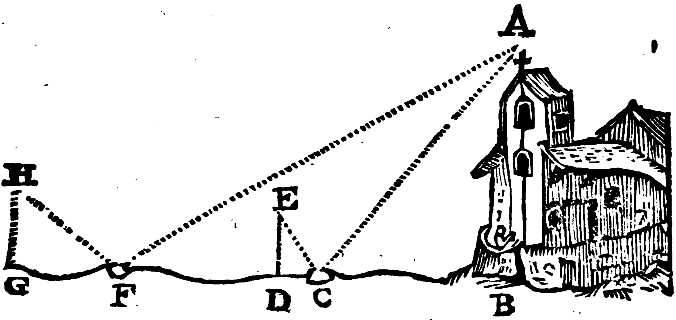


PRO-

PROBLEM V.

To measure a height by Reflexion.

FIrst, if the Eminence is acceſſible, as AB acceſſible at B, ſo as to give one an opportunity of knowing how far they are from it upon an Horizontal Plain, level with the Baſe of the Eminence: Make



upon this Horizontal Plain at a known diſtance from the Point B, a ſmall Cavity or Hole, which fill with Water, that ſo you may ſee the top A of the Eminence to be meaſured AB, by the Ray of Reflexion CE which paſſes to the Eye ſuppoſed to be at E; then meaſure exactly the height of your Eye ED, and the diſtance CD from the Point of Reflexion C. We'll ſuppoſe the height of the Eye ED to be 4 Foot, the diſtance CD 3 Foot, and the diſtance BC 48 Foot. Now, ſay, by the Rule of three direct; If the Diſtance CD of 3 Foot gives 4 Foot for the height ED, how much will be given by the Diſtance BC of 48 Foot? And you'll find the Eminence 64 Foot high, which is the Solution of the Problem; for if you multiply the two laſt Terms, 4 and 48, and divide their Product by the firſt Term, 3, you have 64 for your fourth Proportional.

But if the Eminence is inacceſſible, ſo that you can't actually meaſure the Diſtance BC, dig another Hole in the ſame Plain in a ſtraight Line, and at a known distance from the first Point C, as at F, and fill it also with Water, that you may see the same top A by the Ray of Reflexion FH, reaching the Eye supposed to be at H. Here take notice that the Person who sees this Reflexion at H, must be the same that saw it at E, that the height of the Eye from the Ground or Plain may be the same, which we supposed to be 4 Foot. As we supposed the Distance CD to be 3 Foot, we shall now suppose the Distance CF to be 32 Foot, and the Distance FG 5 Foot; so we multiply the Line ED into the Line CF, *i. e.* 4 by 32, and divide the Product 128 by 2, the excess of the Distance FG above the Distance CD, and the Quotient gives 64 the height of the Eminence.

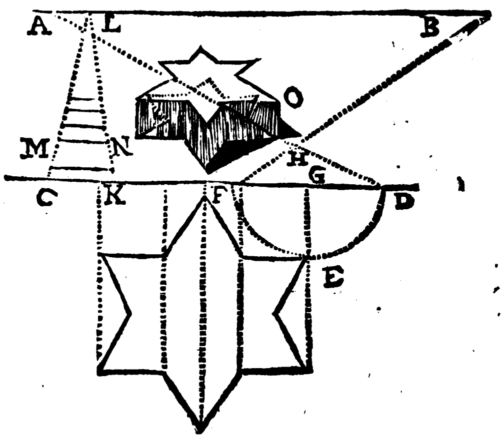
Remark. How to know the Distances.

If you would know the distance BC without knowing the height AB, multiply the distance CD by the distance CF, *i. e.* 3 by 32, and divide the Product 96 by 2, the excess of the distance FG above the distance CD, and the Quotient gives you 64 for the distance BC.

PROBLEM VI.

To represent any thing in Perspective, without making use of the point of Sight.

TO find in the Picture the appearance of any Point of a Geometrical Plan, of E for instance, draw



from

from that Point E, the Line EG Perpendicular to the Ground-Line CD, and take the length of the Perpendicular EG, on each side the Point G in the Ground-Line CD, extending it from G to F and to D. Fix at pleasure two Points of distance upon the Horizontal Line AB, for instance A and B; then draw from these Points A and B to the Points D and F, the Right Lines AD, BF, which by their Intersection will give the appearance H of the Point proposed E. By the same method one may find the appearance of any other Point of a Geometrical Plan, and by consequence the Representation of the Base of any Body whatsoever, which by another Consequence may easily be represented in Perspective, by drawing from all the Points of its posture or perspective Plan, Perpendicular Lines to the Ground-Line CD, and those equal in appearance to the height of the propos'd Body; which is done after the following manner.

Having laid down the natural height of the proposed Body upon the Ground Line CD, from C for instance to K, draw from these two Points, C, K, to the Point L taken at discretion upon the Horizontal Line AB, the Right Lines LC, LK, which will determine the apparent heights of all the Points of the propos'd Body, by Lines drawn from these Points parallel to the Earth or Ground-Line CD; as to find the height of the Point H, the Perpendicular HO is rais'd equal to the part MN, *&c.*

PROBLEM VII.

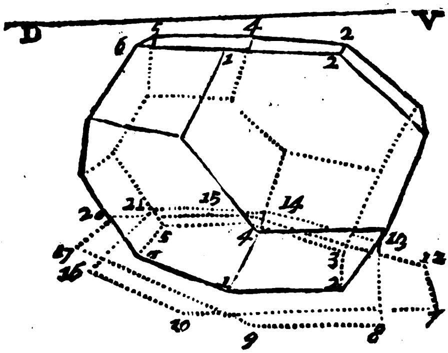
To Represent in Perspective an Equilateral Polyedron, terminated by six equal Squares, and by eight regular and mutually equal Hexagons.

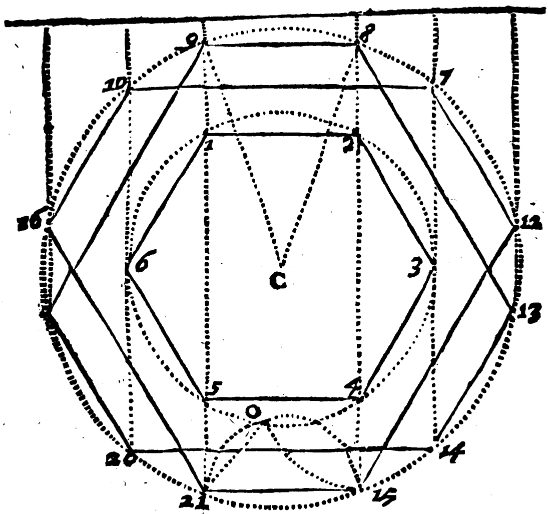
THose who understand Perspective will readily represent this Body in the Picture, in which the Point of Sight is V, and one of the two Points of distance is D, mark'd upon the Horizontal Line DV, which is parallel to the Ground-Line AB; they'll readily do it, I say, if they know how to draw a Plan and a Profil; which is done after the following manner.

In

How to draw a Plan.

In the first place, if you would have the Body to rest upon one of its eight Hexagons, as 1, 2, 3, 4, 5, 6. describe from its Center C, a Circle, the Radius or Semidiameter of which, C 8, or C, 9, is such, that its Square is to that of the Hexagon, as 7 is to 3; so that if the Radius or side 1,2, of the Hexagon is 65465



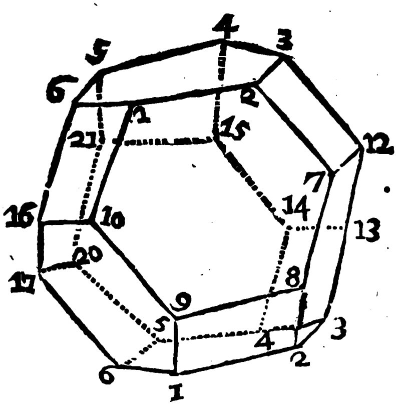


equal Parts, the Radius C 8 or C 9 of the great Circle, is 100000.

Having thus drawn the great Circle, divide it unequally, as you see it done in the Figure, so as to make the least side, 8, 9, and the other five, equal each of 'em 'em to the side of the Hexagon; and the greatest, 7, 10, and the other five, double, each of 'em, of the smallest side; in which case, the least side will subtend an Arch of 38. 12'. and the greatest (*i. e.* the double of the least) an Arch of 81. 48'. But without this trouble 'twere an easie matter to describe this by the sole inspection of the Figure.

For the Profil, describe round the smallest side 21, 15, the Semi-Circle 21, O, 15; and after describing from the Point 4 thro' the Point 15 the Arch of a Circle 15, O; draw the Right Line 21, O, and this shall be the heighth of the Points 9, 8, 14, 13, 20, 17; the heighth of the Points, 7, 12, 15, 21, 16, 10, being equal to double the Line 21, O; and the heighth of the Points, 1, 2, 3, 4, 5, 6, being equal to the triple of the same Line 21, O. How to draw a Profil.

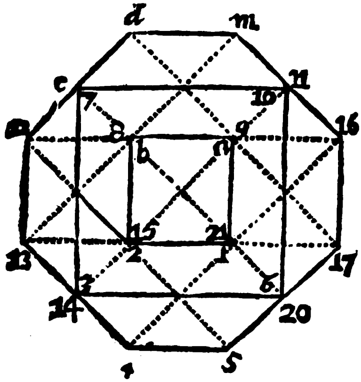
Now if you put the Plan thus describ'd in Perspective, and from all its Angles raise Perpendiculars to the Ground-Line, for laying down the hegihts suitable to those of the Profil, you have nothing more to do but to joyn the sides as in the foregoing Figure,



and yet more distinctly in this here annex'd, which we have made larger for the distincter apprehension of the sides that are to be joyn'd; of which those mark'd with black Lines, are the sides that appear to the Eye; and the others mark'd with Points are those which are not seen.

In

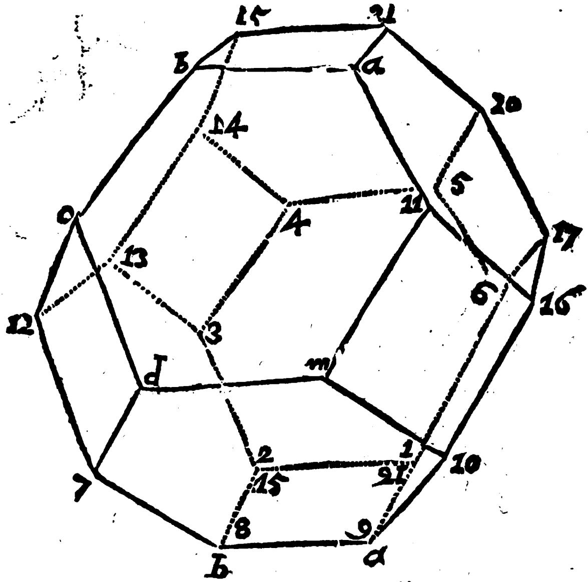
In a second place if you would have the Body to rest upon one of its six square surfaces, as upon the square *a*, *b*, 15, 21, the Plan or Posture of the Polyedron will be changed into that represented in this Figure, which any one may apprehend by the bare inspection, especially when they know, that the great side of the Irregular Octagon, d 12, is equal to the Diagonal, a 15 or b 21, of the inner square that serves for the basis of the Polyedron.



The Profil likewise changes; for the height of the Points, 3, 7, 6, 10, is equal to c d the half of d 12 the great side of the irregular Octagon; the height of the Points, 4, 5, 17, 6, m, d, is equal to the whole side d 12; the height of the Points 14, 20, n, c, is equal to the same side d 12, and its half c d; and in fine the heighth of the Points a, b, 15, 21, is the double of the same side d 12, the square of which is to the square of the Radius of the Irregular Octagon, as 4 is to 5; and consequently if the Radius be 100000 equal parts, the great side d 12 is 89442, and subtends an Arch of 53. 8′; and the little side d m is 63245 parts and subtends an Arch of 36. 52′.

By the means of this Plan and Profil we have put the Polyedron in Perspective, as you see it in the annexed Cut.

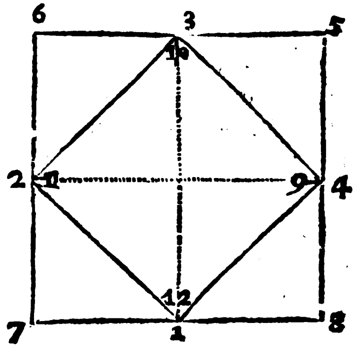
PRO-



PROBLEM VIII.

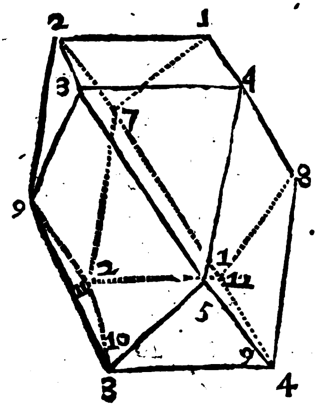
To represent in Perspective an Equilateral Polyedron, terminated by six equal squares and by eight equilateral and mutually equal Triangles.

IF you would have the Polyedron rest upon one of its six equal Squares, as 9, 10, 11, 12, you have nothing to do but to Circumscribe another Square about it, and then your Plan's finish'd, the Profil of which is as followeth.



The height of the Points, 5, 6, 7, 8, is equal to 3, 5, the half of the side 6, 5, of the circumscribed square; and the height

height of the Points, 1, 2, 3, 4, is equal to the whole side 6, 5, or the Diagonal 11, 9. or 10, 12, of the inscribed square, which serves for a basis to the Polyedron.



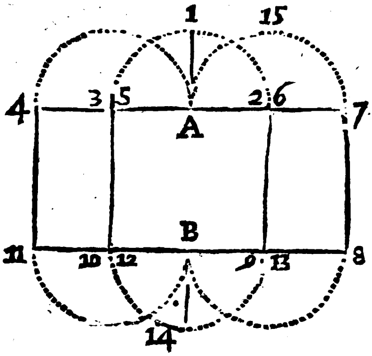
By the means of this Plan and this Profil we have put this Polyedron in Perspective, as you see it in the annexed Figure, where you have a distinct view of the sides you are to joyn, when once you have found in the Picture the appearance of the Points that limit the Extremities.

PROBLEM IX.

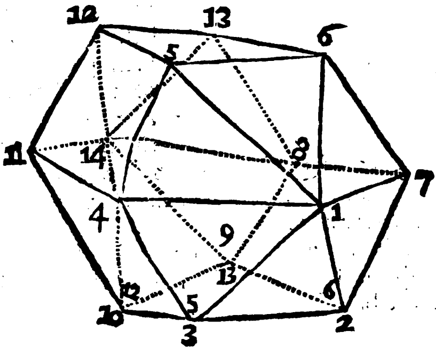
To represent in Perspective an Equilateral Polyedron terminated by six equal squares, and by twelve Isosceles and equal Triangles, the heighth of which is equal to the base.

IN the first place, if you would have the Polyedron to insist upon one of its six equal squares, as 3, 6, 9, 12, its position will be such as you see in this Figure, in which the Plan is made plain by the semicircles describ'd from the four Right Angles of the base, 3, 6, 9, 12, and from the middle Points A, B, of the two opposite sides 5, 2, and 12, 9.

As



As for the Profil; the height of the Points, 4, 11, 7, 8, 1, 14, is equal to the Tangent 7, 15; and the height of the Points, 5, 6, 13, 12, is double to the Tangent 7, 15. There remains nothing further, but to look upon the two annex'd Figures, for understanding the manner of representing this Polyedron in Perspective; which you have all over shaded in the one, and after another manner in the other.



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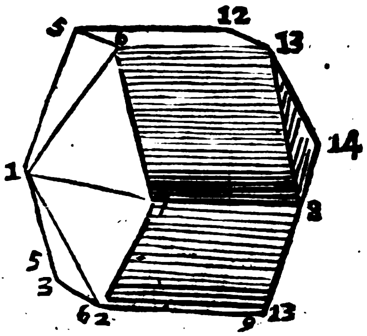
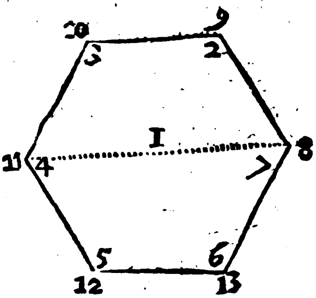


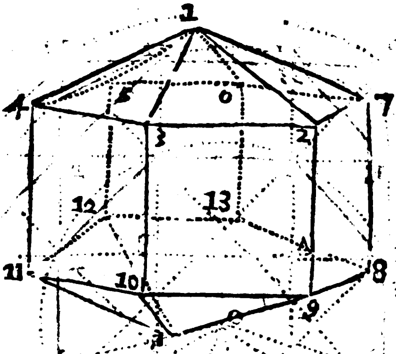
In the second place, if you have the Polyedron raised upon one of its solid Angles, as 1, in this case its posture will be the single Regular Hexagon, 2, 3, 4, 5, 6, 7, the Center of which will be the Point 1, and the Profil such as followeth.



The height of the Points, 8, 9, 10, 11, 12, 13, is equal to half the side of the Hexagon; the height of the Points, 2, 3, 4, 5, 6, 7, is equal to the triple of that, *i. e.* three half sides of the Hexagon; and the heighth of the Point 1 is double the side of the Hexagon, or equal to the Diameter, 4, 7.

Without insisting further upon the Perspective of this Polyedron, I shall content my self with leaving with you the bare Figure of it.

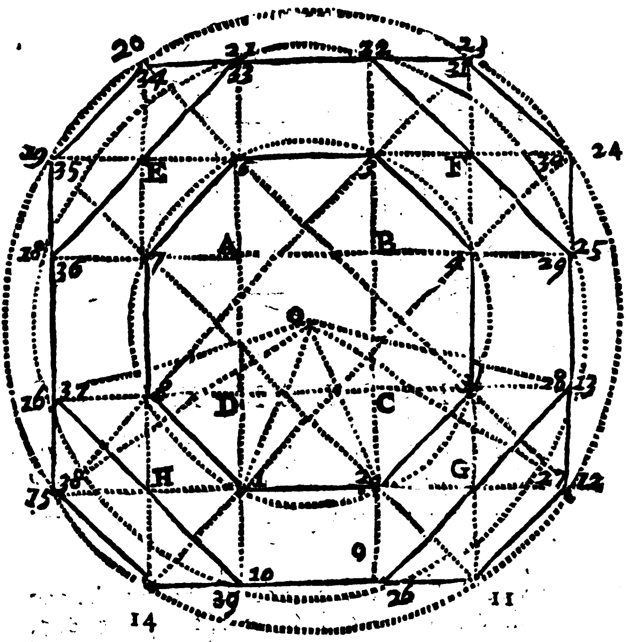
PRO-



PROBLEM X.

To Represent in Perspective an Equilateral Polyedron, limited by twelve equal squares, by eight Regular and equal Hexagons, and by six Regular and equal Octagons.

IF you would have the base of this body to be one of its six Octagons, for instance 1, 2, 3, 4, 5, 6, 7, 8, the Center of which is O; joyn the extremities of the two opposite and parallel sides by Right Lines parallel to one another, which by their mutual intersections will form a square, such as ABCD. Prolong the two opposite and parallel sides, 1, 2, and 5, 6; and likewise the two opposite and parallel sides, 3, 4, and 7, 8; which meeting with the two former will form another larger square EFGH. This done, 'twill be an easy matter to finish the Plan, namely, by making the Line E20 equal to the part E7, &c.



For a more exact description of this Plan, let's consider, That in supposing the Radius O1 or O2 to contain 1000 equal parts, the Radius O13 or O16 of the mean Circle must contain 1514 of those parts, and the Radius O12 or O15 of the greatest must be 1731: That the smallest side subtends in the greatest Circle an Arch (11, 12, or 14, 15) of 25, 32′; in the mean Circle an Arch (1, 2) of 29, 16′; and in the least Circle an Arch, 1, 2, of 45 Degrees: And that the greatest side subtends in the greatest Circle, an Arch, 14, 11, of 64. 28′. and in the mean Circle an Arch, 10, 13, or 9, 16, of 60. 44′. the Chord of which is double the least side, 9. 10.

For the Profil; we'll allow the whole Line 15, 12, for the heighth of the Points, 1, 2, 3, 4, 5, 6, 7, 8, the Posture of which is the Interior Octagon, or the least Regular Octagon. We'll allot the part, 15, G, for the heighth of the Points 9, 10, 13, 25, 22, 21, 18, 16, 16, the form of which is the mean Octagon: we'll allot the part 15, 2, for the heighth of the Points 14, 11, 12, 24, 23, 20, 19, 15, the position of which makes the greatest Octagon. We'll allow the part 15, 1, for the heighth of the Points 26, 27, 30, 31, 34, 35, 38, 39, the Posture of which is the greatest Octagon: And the part 15, H for the heighth of the Points 40, 41, 28, 29, 32, 33, 36, 37, the Posture of which is the mean Octagon.

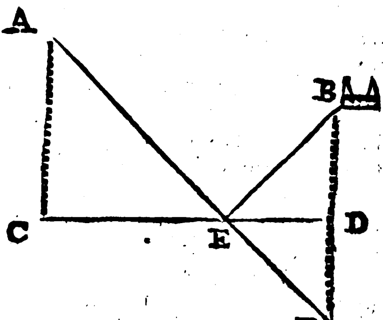
The heighth 15, 12, will be 2930 parts, the Radius Or of the least Octagon 1000; The heighth 15, G, 89, the heighth 15, 2, 1848; the heighth 15, 1, 1082; and in fine the heighth 15, H, will be 541.

When the Plan of this Polyedron terminated by twenty six faces is put into Perspective, and the position of the solid Angles determin'd according to the different heighths pointed to in the foregoing Profile; you must joyn the solid Angles by Right Lines, which will be the equal sides of the Polyedron.

PROBLEM XI.

The Points of the Eye and of some Object being given, together with the point of Reflexion upon the surface of a plain Looking-glass; to determine the place in the Glass of the Image of the Object proposed.

LET the Eye be A, the Object B, and the point of Reflexion E upon the surface CD of a plain Looking-glass; draw from the Object B the Line BF perpendicular to that surface; and prolong the Ray of Reflexion AE till it meets that Perpendicular in a Point, as at F; or, which is the same thing, make DF equal to DB, and the Point F is the



place of the Image of the Object B, *that is*, the Point where the Object B will be seen by the Eye in the plain Looking-glass CD, according to the Principles of Opticks, from which we learn that the Image of an Object is made at the concurse of the Ray of Reflexion, and a Right Line drawn from the Object Perpendicular to the surface of the Glass, whether Plain or Spherical. From hence we may readily conclude by the equality of the Angles of Reflexion and Incidence, that, when the Glass is plain, as we here suppose it to be, the Object ought to appear as deep sunk in the Glass as 'tis distant from it; and for that Reason we ordered the Line DF to be made equal to the Perpendicular DB.

Another Consequence, is, That the distance AF of the Image F of the Object B, to the Eye at A, is equal to the Ray of Incidence BE and the Ray of Reflexion AE, the Ray of Incidence BE being equal to the Line EF, by reason of the equality of the two Right Angled Triangles EDB, EDF.

A Third Inference, is, That if the Eye moves any certain space nearer or further from the Point of Reflexion E in the same Ray of Reflexion AE, the Image F of the Object B will make exactly the same approaches or departure with respect to the Eye, because the distance EF continuing still the same, the distance AF will increase or decrease as the distance AE do's.

We may infer further, that when the plain Looking-glass is parallel to the Horizon, as CD, a magnitude perpendicular to the Horizon must appear inverted; and when the plain Glass is perpendicular to the Horizon, the right of the Person seems to be on the left of his Image, and *è contra*.

The last Inference I shall here make, is, that the distance of the Eye from the Image of the Object seen in the last Glass by vertue of several reflexions from several plain Glasses, is equal to the sum of all the Rays of Incidence and Reflexion; and that an Object may sometimes be multiplied in a plain Looking-glass, or reflecting surface, when 'tis made of Glass.

Thus 'tis that we sometimes see a lighted Flambeau appear double in a Looking-glass that's somewhat thick, by reason of the double Reflexion there made; namely, one upon the external surface of the Glass;

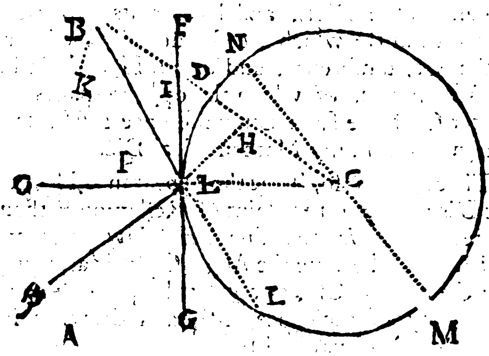
and

and another in the bottom or inner part of the Glaſs; for the light can't be all reflected upon the external ſurface of the Glaſs; but it penetrates the icy ſubſtance of the Glaſs (I ſpeak only of thoſe made of Glaſs) till it meets that pewter leaf that's done over the back of the Glaſs to hinder the paſſing of the Rays, where by conſequence it ſuffers a ſecond Reflexion, and the Eye falling in with a concourſe of two Rays of Reflexion that can't be parallel, 'tis no wonder the Object ſeems to be double, or appears in two different places of the Glaſs. 'Tis manifeſt that the various irregularity of the Glaſs and the divers Reflexions, may multiply the Object yet more, eſpecially when 'tis ſeen a little ſideways.

PROBLEM XII.

The Points of the Eye and of ſome Object being given, together with the Point of Reflexion upon the Convex ſurface of a ſpherical Looking-glaſs to determine the Image or Repreſentation of the propos'd Object either within or out of the Glaſs.

LET the Eye be A, and the Object B, and the Point of Reflexion E upon the Convex ſurface DEL of a ſpherical Looking-glaſs, the Center of which is



C; draw from the Center C to the Object B, the Right Line BC Perpendicular to the ſurface of the ſpherical Looking-glaſs, in which by conſequence will be the

the Image of the Object B, *viz.* H, which is found by prolonging the Ray of Reflexion AE which here meets within the Glass the Cathete of Incidence BC at the Point H, but might have met it at the Point D of the surface of the Glass, and even out of the Glass when the Angle of Incidence BEF, or the Angle of Reflexion AEG is very small: So that the Object B may be seen either within the spherical Glass, as here, or upon its surface or out of it.

Scholium. The Tangent FG which passes by the Point of Reflexion, determines as you see the Angles of Incidence and Reflexion, and cuts the Cathete of Incidence BC in I, and that in such a manner, that the four Lines, BC, CD, BI, DI, are proportional, and consequently the Line BC is cut at the Points I, D, in the mean and extreme proportional Ratio, that is, the Rectangle of the whole Line BC and its mean part DI is equal to a Rectangle of the two other extreme parts BI, CD; as is easily demonstrated by drawing from the Point B the Line BK parallel to the Radius of Reflexion AE.

'Tis evident from the property of the focus's of an Ellypsis, that the two Points A, B, are the two focus's of an Ellypsis, which touches the spherical Glass at the Point of Reflexion, E; and which has for its great Axis the sum of the two Rays, AE, BE, of Reflexion and Incidence; So that, to find the Point of Reflexion E, one needs only to describe an Ellypsis that touches the Circumference DEL, and has for its two focus's the Points A and B; which is easily done by the intersection of the Circumference, and of an Hyperbola between its Asymptotes, of which the opposite passes thro' the Center C of the same Circumference DEL, as I have demonstrated in my Mathematical Dictionary.

'Tis evident also, That the appearance H of the Object B is nearer to the Point of Reflexion E than to the Center C, *that is*, the Line CH is always greater than the Line EH, because the Angle CEH is always greater than the Angle ECH, as appears by prolonging towards L the Ray of Incidence, BE, and drawing from the Center C MN parallel to it.

'Tis further evident that the same appearance H of the Object B is likewise nearer to the Point of Reflexion E, or the Point D of the surface of the Glass, [illegible] than

than the Object B, *that is*, That the Line EH is less than the Ray of Incidence BE, and that the Line DH is less than the Cathete of Incidence BD.

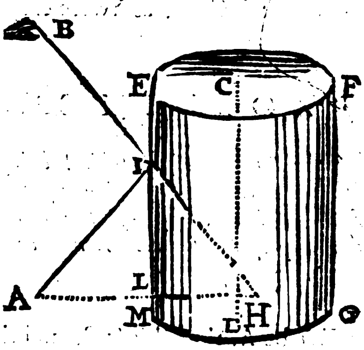
'Tis evident once more, that if the magnitude OE is perpendicular to the surface of the spherical Glass DEL, so that being prolong'd it passes thro' the Center C, the Point P nearest the Glass must appear less sunk or deep in the Glass than the remoter Point O; and that the magnitude OE must appear inverted and less.

A Consequence of this, is, that in a spherical Convex Glass a Magnitude must appear still greater as it approaches nearer to the Glass parallel wise to it self, for then it appears less sunk or less deep in the Glass, and consequently is nearer the Eye, and inclosed in a larger Angle. The same thing will happen if the Object continues unmoved, and the Eye approaches nearer to the Glass, and that for the Reasons mention'd but now.

PROBLEM XIII.

To determine the place of an Object seen by Reflexion upon the surface of a Cylindrical Looking-glass.

THIS Problem is none of the easiest, by reason that a Cylindrical Looking-glass taken lengthways may be consider'd as a plain Glass; and taken



precisely according to its roundness, it may be consider'd as a spherical; and again, when taken in another sense, it partakes of the properties both of a plain and of a spherical Glass.

For this Reason, if the Point of an Object and the Eye are in a plain that passes thro' the Axis of a Cylindrical Glass, that Point will be seen by Reflexion in the Cylindrical Glass as in a plain Glass, that is, as deep in the Glass as 'tis distant from it.

Thus, if we suppose a Point A of an Object, and the Eye B, in a plain that passes thro' the Axis CD of the Cylindrical Glass EFGM, that Point A will be seen in H by the Ray of Reflexion BIH, *i. e.* at the concurse of this Ray of Reflexion and the Line ALH perpendicular to the common section EM of the Glass and the plain which passes thro' the Eye and the Point of the Object A: And in this case, 'tis evident that the Object A appears as deep in the Glass as 'tis remote from it, *that is*, AL, LH, are equal, by reason of the two equal Rightangled Triangles, ALI, HLI.

But if the Eye and the Point of the Object are in a plain parallel to the base of the Cylindrical Glass, the section of that plain and the Glass being a Circle, the Object must appear in the Cylindrical Glass as in a spherical one. The Consequence of which is, that the magnitudes parallel to the base of a Cylindrical Glass, appear there much contracted, whereas those which are parallel to the axis of the same Glass appear a'most of the same magnitude as in a plain Glass. This holds likewise in a Conical Glass, as 'twere easy to demonstrate.

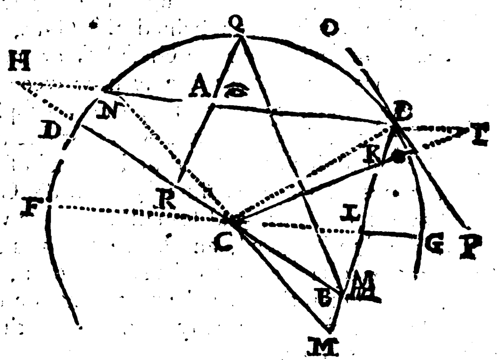
PROBLEM XIV.

The Points of the Eye and of an Object being given, together with the Point of Reflexion upon the Concave surface of a spherical Looking-glass; to determine the Image of the propos'd Object within or without the Glass.

LET the Eye be A, the Object B, and the Point of Reflexion E upon the concave surface FEG of a spherical Glass, the Center of which is C; draw from

the

the Center C to the Object B, the Right Line BC; which being prolong'd meets here the Ray of Reflexion



AE likewise prolong'd, at the Point H; which must be the Image or representation of the propos'd Object B, because that Point H is the concurse of the Ray of Reflexion AE, and the Cathete of Incidence CD drawn from the Center C thro' the Object B.

If the Object had been nearer the Glass, as at K, its appearance I had been on the other side, *viz.* at the concurse of the Ray of Reflexion AE and the Cathete of Incidence CI drawn from the Center C thro' the Object K: And if the Object had been at L, it had not appear'd at all in the Glass, because in that case the Cathete of Incidence FG drawn from the Center C thro' the Object L, being parallel to the Line of Reflexion AE would never meet it: And in fine, if the Object were at M, its appearance N would be without the Glass, at the Concurse of the Ray of Reflexion AE and the Cathete of Incidence CN drawn from the Center C thro' the Object M. Remark.

Here we see the Reason of what Experience shews us every Day, *viz.* That an Object may be seen by Reflexion in a Concave Glass, as well as in a Convex Glass, both out of the surface of the Glass, as here at N which is the representation of the Object M; and within the Glass, as at H, which is the representation of the Object B, and at I which represents the Object K, these two Images H and I appearing sunk in the Glass, but never so deep as in a plain Glass; which is owing

owing to the different concurses of the Rays of Reflexion, and the Cathetes of Incidence, which can make Objects appear, sometimes upon the surface of a Glass, sometimes within or behind the Glass, and sometimes without or before the Glass less or more; so that sometimes the Images are seen between the Object and the Glass, sometimes at the very place where the Object is, (and thus it comes that one may handle the Image of his own Hand or Face off of the Glass,) sometimes at a greater distance from the Glass than the Object really is, and sometimes at the very spot where the Eye is placed, and hence it comes that those who are unacquainted with the Reason of it, are affraid and retire when they see the representation of a Sword or Dagger, that some body holds behind them, advance out of the Glass.

'Tis evident that the Tangent OP which passes thro' the Point E of Reflexion, determines the Angle of Incidence BEP, and, which is equal to it, the Angle of Reflexion AEO; and that the Line CE which is Perpendicular to the Tangent OP, divides into two equal Parts, the Angle AEB made by the Rays of Incidence and Reflexion. The Consequence of which, is, that if you divide that Angle by a straight Line into two equal Parts, that Right Line will pass thro' the Center C of the spherical Glass, by reason of its being perpendicular to the Tangent.

We may easily apprehend, That the Object B may be seen by Reflexion in two different parts, when the Eye is placed at a certain point; for if you draw the Ray of any Incidence BE, with its Ray of Reflexion AE, and another Ray of Incidence BQ with its Ray of Reflexion QR, which will cut the former at A, where the Eye being placed will see the Object B thro' the two Rays of Reflexion AE, AQ, and consequently in two different places, *viz.* At the points H and R within and without the Glass.

We may with equal facility conceive, That if the Object is placed at the Center C of the Glass, its Image will Reflect back upon it self, because in that case the Angle of Incidence is Right. And therefore he who places his Eye at the Center C of the Glass, will see nothing but himself.

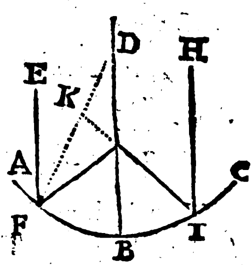
PRO-

PROBLEM XV.

Of Burning Glasses.

IN the foregoing Probleme we saw, that two Rays of Reflexion belonging to one Object, as AE, AQ, retaining to the Object B, will meet and unite in the point A, before a Glass. Now this can't be in plain Glasses, in which the Rays of Reflexion fall off from one another, and far less in Convex Glasses, in which the Rays of Reflexion run much farther asunder, and reunite behind the Glass. Hence it appears that by the means of these we can't produce Fire, as we do with the help of a Concave Glass which is call'd a Burning-glass, and may be Parabolick and spherical. The spherical ones are easily made because the Turn or Turning-wheel may easily serve for making a model for them, and they are easily polished: But when the Glasses are Parabolick or of any other Figure, the Turn can't be so easily made use of for making a model for them; and hence it comes that they are very scarce, and indeed are not so good as the spherical, tho' according to the Theory they ought to be better. And upon this consideration we shall here confine our selves to the spherical Burning Glasses.

Let the Concave surface of a spherical well polish'd Glass be ABC, the Center of which is D. Let a Ray of Light EF be Parallel to the semidiameter BD, which reflecting by the Ray of Reflexion FG shall cut the semidiameter BD in a point, *viz.* G, nearer the surface than the Center of the spherical Glass; *that is*, the Line BG will be always lesser than the Line DG, as appears by drawing the semidiameter DF, which makes the Isosceles Triangle FGD, &c.



We may readily apprehend, that, on the other hand if there's a Ray of Light parallel to the same semidiameter

ameter BD, and equally distant from it with the Ray EF, as HI, so that the Arches BF, BI are equal, this Ray HI will reflect by the Ray IG, which will pass thro' the same point G; and that if the Ray of Light were more or less distant from the semidiameter BD its Ray of Reflection would not cut the Semidiameter BD at the same point G; but where-ever it cuts it, the point of concurse will always be remoter from the Center than from the surface of the Glass. Now since we can conceive an infinite number of different Rays parallel one to another, and to the semidiameter BD, 'tis evident that all these Rays must reflect in one point, as G, which is call'd the *focus*; and at which one may by the Rays of the Sun light a Wax-Candle or a Flambeau, and melt in a small space of time any Metal whatsoever, and vitrify Stone if the Glass is pretty large.

Trigonometry will readily lay open to us the distance of the *focus* from the surface of the Glass, the distance of the Ray of Incidence or Light being once known in degrees, and the semidiameter of the Glass in Feet or Inches. For instance; If the Ray of Incidence EF is distant from the semidiameter BD 5 degrees, so that the Arch BF or the Angle BDF is 5 degrees, and if the semidiameter DB or DF be 100000 parts, we may find the distance DG in the same parts, by drawing from the focus G the Line GK perpendicular to the semidiameter DF, which will then be equally divided at the point K, and consequently its half DK will be 50000 parts, and in the Triangle DKG the Analogy will run thus,

As the whole sine	100000
To the secant of the Angle D	100382
So is the Line DK	50000
To the Line DG	50191

Now the Line DG being substracted from the semidiameter DB or 100000, there remains 49809 for the Line GB or the distance of the *focus* from the Concave surface of the Glass.

'Twas by this method that we calculated the following Table, in which we see the *focus* G still approaches nearer to the Concave surface of a spherical Glass

Glaſs, as the Rays of Incidence inlarge their diſtance from the Center; ſo that when the Rays are 60 degrees diſtant, the *focus* G is exactly at the point B of the Concave ſurface of the Glaſs.

1	49992	16	47985	31	41668	46	28022
2	49970	17	47715	32	41041	47	26686
3	49932	18	47427	33	40382	48	25276
4	49878	19	47269	34	39689	49	23787
5	49809	20	46791	35	38961	50	22214
6	49725	21	46443	36	38197	51	2054
7	49627	22	46073	37	37393	52	18787
8	49509	23	45682	38	36549	53	16918
9	49377	24	45268	39	35662	54	14935
10	49299	25	44831	40	34730	55	12828
11	49064	26	44370	41	33749	56	10586
12	48883	27	43884	42	32798	57	8196
13	48685	28	43372	43	31634	58	5646
14	48468	29	42832	44	30492	59	2920
15	48236	30	42265	45	29282	60	0000

We may likewiſe obſerve in this Table that the Rays of Incidence from 1 to 15 degrees of diſtance, unite by Reflexion almoſt in the ſame point, becauſe the diſtance of the *focus* G does not decreaſe ſenſibly. And hence 'tis, that ſuch a quantity of Rays darted from the Sun upon the Concave ſurface of a ſpherical Glaſs, which may paſs for parallel conſidering the great diſtance of the Sun from the Earth; hence 'tis, I ſay, that ſuch a quantity of Rays is reflected almoſt in the ſame point, and conſequently all the Rays of Reflexion comprehended in a Concave part of the ſphere of about 30 degrees, may by their union produce fire, as experience ſhews.

We obſerve further in the foregoing Table that the *focus* G is diſtant from the Concave ſurface of the Glaſs about the fourth part of the Diameter, or half the ſemidiameter DB, and by conſequence that a Concave ſpherical Glaſs will burn at ſo much the greater diſtance as its Diameter is greater. But after all we muſt not imagine 'twill burn at a vaſt unreaſonable diſtance, for beſides

besides the difficulty of making one so large, those Rays of Reflexion which unite in the same point in a little Glass, from 1 to 15 degrees distance, keep the *focus* G from any sensible change, and would not unite so perfectly in a great Glass, the consequence of which is a sensible change in the distance, and a diminution of the force of the Rays. So that what is written of Archimedes can't be credited, *viz.* That by the means of a Concave Glass he burnt with the Rays of the Sun the Naval force of the *Romans*, at a distance of 375 Geometrical paces, which amount to 1875 Feet.

COROLLARY.

From what has been said in this and the foregoing Problem, we infer, that if one puts a Luminous body, as a Candle, to the *focus* G, its Rays will be reflected in Lines very near parallel to one another and to the semidiameter DB; and if one puts the same Candle to the Center D its Rays will reflect upon themselves, as being then perpendicular to the surface of the Glass.

By such a Glass and the advantage of the Rays of the Sun, one may represent what Characters they will upon a dark Wall at a moderate distance from the Glass, *viz.* By writing upon the Concave surface of the Glass with Wax or otherwise, the Letters revers'd of a pretty large Character, and holding the Glass directly opposite to the Sun, for then the Letters will appear by Reflexion in their usual position upon the proposed Wall.

With the help of the same Glass one may increase the light in a large Room, by applying a lighted Candle to the *focus* of the Glass, for then the Rays of the Candle will reflect all over the Room, and shine so bright that one may easily read against a Wall.

In fine this Glass may be made use of for giving light in the Night-time, and for seeing what passes at a distance; it may be of use to those who mean to preserve their sight by using a Lamp set to the *focus* of the Glass, which ought to be placed a little high and aside, that it may conveniently convey the light of the Lamp to the Table where the Person Reads or Writes.

Remark. The Burning Glasses are usually made of Mettal, for the greater facility of Reflexion, and that it may be be more ſpeedy and vigorous; tho' there may be made of Glaſs ſuch as will make a very handſom Reflexion, provided the Glaſs is very clean and ſomewhat thin, and that its cover is good to hinder the Rays of Incidence to traverſe and refract.

You may eaſily find the *focus* of a Concave Glaſs, when oppos'd to the Rays of the Sun, if you take a piece of Wood or any other ſolid matter, and move it to or from the Glaſs, till the diſcus of light that appears by Reflexion againſt the piece appears as ſmall as poſſible, for then the piece is at the *focus*. Or elſe, put hot water near the Glaſs on that ſide of its concavity that points directly to the Sun, for the ſmoak that riſes from the hot water, will give you the pleaſant ſhew of the Cone of Reflexion, the top of which is the *focus*. Another way is this. Throw ſome duſt before the concavity of the Glaſs that lies directly to the Sun, for in that duſt as well as in ſmoak you'll obſerve the Cone of Light Reflected, and conſequently its point which is the *focus* you look for: Nay in Winter when the Air is thick and condenſed by cold you may obſerve the *focus* and the whole Cone of Reflexion, without the help either of duſt or ſmoak.

Tho' one would think that fire can't be produced by a Concave Glaſs, without it be illuminated with the Beams of the Sun in order to Reflexion, yet 'tis poſſible to produce fire in a dark place, namely by conveying the Rays of the Sun to the Concave Glaſs by the means of a plain Glaſs, which ought to be ſomewhat large, that ſo the greater number of Rays uniting at the *focus* may burn more forcibly.

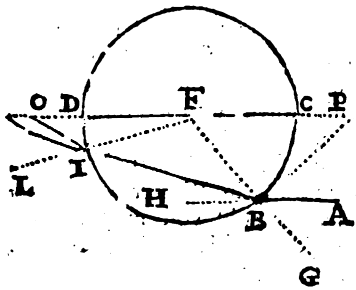
PROBLEM XVI.

Of the ſpheres of Glaſs, proper to produce Fire by the Rays of the Sun.

WE may likewiſe produce fire by the Sun Beams with a ſphere of Glaſs or Cryſtal, or of any other matter that's readily penetrable by light, as water in a very round Bottle, or with a ſphere of Ice: Not by means of Reflexion, but by Refraction, which can alſo gather into one point ſeveral parallel Rays of Light;

light; for when they enter the ſphere they bend or break off approaching to a Perpendicular, and in flowing out of the ſphere they refract again departing from the Perpendicular, which makes them approach to the Diameter of the ſphere to which the Rays of Incidence are parallel, and to meet it without in a point which is the *focus*; but the effect of this is neither ſo quick nor ſo vigorous as in a Burning Glaſs.

Let the ſphere or Ball of Glaſs be BCD, the Center of which is F, and the Diameter CD. Let AB be a Ray of Light or of Incidence which meeting the ſurface of the Ball of Glaſs at B, penetrates and enters it, but inſtead of going on in the ſtraight Line ABH, (which 'twould do if it met with no reſiſtance) it breaks off in the point B, which is therefore call'd *the point of Refraction*, and approaching to the Perpendicular GBF towards the Center, continues in the Line BI, which being prolong'd meets the Diameter CD likewiſe prolong'd to E, which would be the *focus* if the Refracted Ray did not refract a-freſh at the point I, into the Line IO, which moving from the Perpendicular IL meets the Diameter CD at the point O, which is the *focus*.



Before I ſhew you how to find this *focus* O, or its diſtance DO from the ſurface of the Ball of Glaſs, I ſhall explain ſome terms and properties of broken Angles and Angles of Refraction in a Glaſs, which are not the ſame in the other Diaphanous Bodies, as Experience ſhews.

If then the Line AB is a *Ray of Incidence*, the Line BI is call'd *the Ray of Refraction*, and the Angle HBI *the Angle of Refraction*. The Right Line BG, which is perpendicular to the ſurface of the Ball, and by conſequence paſſes thro' its Center F, is call'd the *Axis of Incidence*, and being prolong'd within the Ball, is call'd *the Axis of Refraction*.

The

The Plan imagin'd to be form'd by the Ray of Incidence AB, and the Ray of Refraction BF, is call'd *the Plan of Refraction*, which is always perpendicular to the surface of the Ball, which is call'd *the breaking surface*, because the Ray of Incidence breaks when it arrives there. 'Tis evident that the Plan of Refraction passes thro' the Axis's of Incidence and of Refraction, and that it contains the Angle of Refraction HBI, and the Angle IBF which is call'd *the Broken Angle*, and likewise the Angle ABG, which is call'd *the Angle of Inclination*, and which is always equal to the Complement of the *Angle of Incidence* ABP.

The broken Angle increases and decreases as the Angle of Inclination is greater or lesser, so that when one of these two Angles is sunk, the other is likewise sunk. Thus if the Perpendicular BG is a Ray of Incidence, there will be no Angle of Inclination, and the Ray of Incidence GB will not break in penetrating the Glass, but continue in a straight Line towards the Center F, and so there's no broken Angle neither. Thus you see that when the Ray of Incidence is perpendicular to the breaking Surface, it makes no Refraction, because there's nothing to determine the Refraction more to one side than another.

Tho the broken Angle increases in proportion with the Angle of Inclination, yet it does not increase after the same manner, *that is*, if the Angle of Inclination increases a Degree (for Example) the broken Angle will not also increase a Degree, but its augmentation is such, That the Sinus's of the Angles of Inclination in the same Medium are proportional to the Sinus's of their broken Angles in another that's easier or harder to be penetrated; so that the Sinus of the Angle of Inclination is to the Sinus of the broken Angle, as the Sinus of another Angle of Inclination is to the Sinus of its broken Angle. And hence it comes, that if once one knows by experience one broken Angle for any one Angle of Inclination, he may easily know by computation the broken Angles for all the other Angles of Inclination.

In regard the two Lines AH, CD, are parallel, the Angle E is equal to the Angle of Refraction HBE; and forasmuch as in all Rectilineal Triangles the Sinus's of Angles are proportional to their opposite sides, we know that the Sinus of the broken Angle EBF is to its opposite side EF, as the Sinus of the Angle BFC or of the Angle of Inclination ABG, to the Ray of Refraction BE : And since we know by Experience, that when the Ball BCD is of Glass, the Sinus of the broken Angle EBF is to the Sinus of the Angle of Inclination ABG or BFC, as 2 is to 3, it follows from thence that if the Line EF is 200 parts, the Ray of Refraction BE is 300, and so by Trigonometry one may easily find the Angle E, or the Angle of Refraction HBE, the broken Angle EBF, and the Semidiameter BF, having once discover'd the Angle of Inclination ABG, or its equal BFC in the Amblygonium BFE, where three things are known, namely, the Side BE of 300 parts, the Side EF of 200, with the Angle BFE, which is the remaining Part or the Complement of 80 degrees from the Angle BFC which is equal to the Angle of Inclination ABC, which is suppos'd.

Suppose the Angle of Inclination ABG to be 10 degrees, in which case the Angle BFE will be 170 ; and that one wants to know the broken Angle EBF : The Analogy is this :

As the Side BE	300
To the Sinus of the opposite Angle BFE	17365
So is the Side EF	200
To the Sinus of the broken Angle EBF	11577.

which will be found to be about 6. 39'. and which being substracted from the Angle BFC, or the Angle of Inclination ABG, which we supposed to be 10 degrees, the Remainder is 3. 21'. for the Angle of Refraction HBE, or for the Angle E, which will serve for finding the Semidiameter BF, by this Analogy :

As the Sinus of the Angle BFE	17365
To the opposite Side BE	300
So is the Sinus of the Angle E	5843
To its opposite Side BF	101

But if the Semidiameter BF is already known, as containing 100 parts, the Content of the Line EF in the same parts may be found by making the following Analogy in the same Triangle BEF :

As

As the Semidiameter BF	101
To the Line EF	200
So is the Semidiameter BF	100
To the ſame Line EF	198

To which if you add the Semidiameter FC or 100, you have 298 for the Line CE.

By this Method did we calculate the following Table, in which you'll find oppoſite to the Angle of Inclination ABG the Quantity of the broken Angle EBF, and of the Angle of Refraction HBE, with that of the Line CE, the Diameter CD of the Sphere of Glaſs being ſuppoſed 200 parts.

ABG	EBF	HBE	CE	ABG	EBF	HBE	CE
1	0.40	0.20	300	11	7.18	3.42	297
2	1.20	0.40	300	12	7.58	4. 2	297
3	2. 0	1. 0	300	13	8.38	4.22	297
4	2.40	1.20	300	14	9.16	4.44	296
5	3.20	1.40	300	15	9.56	5. 4	295
6	4. 0	2. 0	299	16	10.35	5.25	295
7	4.40	2.20	299	17	11.14	5.46	294
8	5.19	2.41	298	18	11.53	5. 7	293
9	5.59	3. 1	298	19	12.32	6.28	292
10	6.39	3.21	298	20	13.11	6.49	292

We have not prolong'd the Table beyond the 20th degree of Inclination, this being ſufficient to let you ſee to what Proportion the Line CE decreaſes; by which you'll obſerve that it decreaſes very ſlowly, as being always equal to about 3 Semidiameters, ſince the greateſt difference is but about the 25th part of a Diameter, whence it comes that the Line DE is almoſt equal to the Semidiameter of the ſame Sphere, *i. e.* to the Line DF.

This Line DE, which is found to be 98 parts for an Inclination of 20 degrees, as appears by ſubſtracting CD from CE, will ſerve for finding the Focus O, as I am about to ſhew you, after taking notice that the broken Angle EBF is about double of the Angle of Refraction HBE, and that by conſequence this Angle of

Refraction HBE is almost equal to the third part of the Angle of Inclination ABG, as appears at first view in the foregoing Table.

Now to find the Focus O, we must consider that the Lines DE, DF, are almost equal, the Angles IEF, IFE, are almost equal, and consequently that the Angle EIL which is equal to them, is about the double of each, and by consequence of the Angle E. This Supposition laid down, if we consider the Line OI as a Ray of Incidence, so that the Angle OIL will be an Angle of Inclination, in which case the Line IB will be a Ray of Refraction, the Angle OIE an Angle of Refraction, and the Angle EIL a broken Angle; we will find that this broken Angle EIL is likewise the double of the Angle of Refraction EIO, as we observed before. Whence it follows that the two Angles E, EIO, are equal one to another, and consequently that the Lines OE, OI, are likewise equal; and forasmuch as the Line OI is almost equal to the Line OD, the Line OE will be likewise almost equal to the Line OD; and so the *Focus* O is about the middle of the Line DE, and consequently the Line DO is about equal to half the Line DE, or half the Semidiameter DF. If then you take upon the prolong'd Diameter the Line DO equal to half the Semidiameter DF or to the quarter of the Diameter CD, you have in O the *Focus* you demand.

Remark. The Angle EBF, which is the broken Angle with respect to the Ray of Incidence AB that advancing from the Air to enter the Glass refracts to the Line BE the Ray of Refraction: This Angle, I say, EBF becomes an Angle of Inclination with respect to the Ray of Incidence IB, which advancing out of the Glass to enter the Air, refracts reciprocally in the Line AB a Ray of Refraction: and in regard this Angle EBF is double the Angle of Refraction HBE, 'tis plain that when the Ray of Incidence flies out of the Glass to enter the Air, the Angle of Inclination is double the Angle of Refraction; which we desire the Reader to take notice of, upon the consideration that 'twill be of use in the insuing Problem.

PRO-

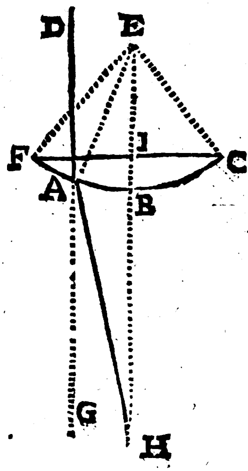
PROBLEM XVII.

Of the Lens's of Glass proper to produce Fire with the Rays of the Sun.

THE Sparks of Glass which are capable of producing Fire, when expos'd directly to the Rays of the Sun, may be flat on one side and convex on the other, as the Segment of a Sphere; or else convex on both sides, as your Old Mens Spectacles and the Microscopes that magnify Objects very much, and are of use for discovering the smallest and minutest parts of Nature; or else convex on one side and concave on the other, which are not so useful as the former, because they can't produce Fire but when their Convexity points directly to the Sun, for when their concave part is turn'd to the Sun, the Rays of Refraction instead of *converging*, *diverge*, that is they separate one from another, and so for want of Union can't produce Fire, as shall be shewn in the Sequel.

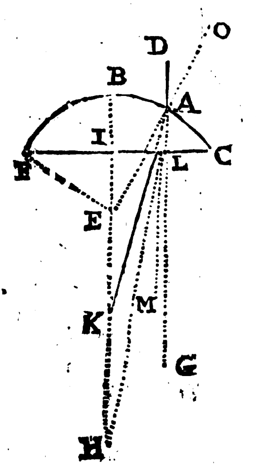
Of the Lens's of Glass made in the form of a Sphere.

To begin with the first sort, *viz.* those made in form of a Sphere; Let's expose to the Sun the plain surface FC of the *Lens* of Glass FBC, the Convexity of which FBC has its Center E in the Axis of its Incidence EBH, which divides the Arch FBC into two equal parts at the Point B, and its Chord FC likewise into two equal parts at the Point I. In this Axis of Incidence is the Focus H of all the Rays of Incidence which are parallel to the Axis of Incidence EH, and consequently perpendicular to the refracting Surface FC. This *Focus* H or its Distance BH from the Convexity of the Glass, is adjusted after the following manner.



Let DA be a Ray of Incidence, which being parallel to the Axis of Incidence EH will cut the refracting substance FC at right Angles, and consequently will go thro without refracting, till it arrives at the Point A of the convex Surface, where 'twill refract upon its egress from the Glass, and instead of going straight to G, 'twill turn off by the Ray of Refraction AH, which will cut the Axis of Incidence EH at the Point H; where all the other Rays of Incidence that are parallel to the Ray DA, will unite in Refraction, at least if the Arch BC or BF do's not exceed 20 degrees; for, as we shew'd in the foregoing Problem, the Rays of Refraction wou'd not unite at the same Point H, but nearer to the Point B, if these Arches exceeded 20 degrees. So the Point H will be the Focus, that being the Place where the Rays of the Sun uniting by Refraction are able to produce Fire.

This granted, we must consider that the Angle of Inclination DAE, or its equal AEH being double the Angle of Refraction GAH or AHE its equal, as we proved in the foregoing Problem, the Sine of the Angle AEH will be almost double the Sine of the Angle AHE, by reason of the smalness of these Angles: And forasmuch as in a rectilineal Triangle the Sides are proportional to the Sines of their opposite Angles, the Side AH will be almost double the Side AE, and since the Side AH is very near equal to the Side BH, it follows that the distance BH of the Focus H from the convex Surface FBC is almost double the Semidiameter AE or BE, and consequently the whole Distance EH is about the triple of this Semidiameter.



But if you turn the convex Part FBC towards the Sun, the Ray DA and all the other Rays parallel to the Axis of Incidence EB, will refract twice before they unite in the Point K, which will be the *Focus* when once they enter the Glass in the Line AH, which approaches

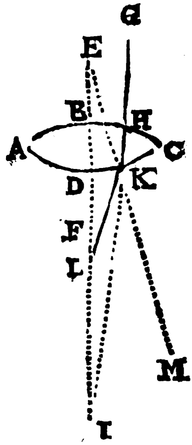
approaches to the Perpendicular EAO, and a ſecond time when they go out of the Glaſs in the Line LK, which recedes from the Perpendicular LM.

From what has been ſaid in the foregoing Problem, it appears, That in the firſt Refraction the Angle of Inclination DAO or AEB is the triple of the Angle of Refraction GAH or AHE, and by conſequence the Line AH is the triple of the Semidiameter EA: And foraſmuch as the Line AH is almoſt equal to BH, this Line BH will alſo be almoſt the triple of the ſame Semidiameter AE or BE, as before; which gives us to know, that the *Focus* would be in H if there were but one Refraction: But ſince there are two,

'Tis evident from the Remark made in the foregoing Problem, that in the ſecond Refraction HLM or KHL is double the Angle of Refraction KLH, and conſequently the Line KL is double the Line KH; and ſince the Line KL is almoſt equal to KB, when the thickneſs BI of the Spark is but ſmall, as we here ſuppoſe it to be, that Line KB is alſo almoſt double the Line KH, and by conſequence the whole Line BH is about the triple of the Line KH: And ſince we have prov'd the Line BH to be likewiſe the triple of the Semidiameter BE, it follows that this Semidiameter BE is equal to the Line KH, and conſequently the Line KB is double the Semidiameter BE, or equal to the whole Diameter. If then you meaſure the length of the Semidiameter EB from the Center E to K, this Point K will be the Focus you look for.

Of the Lens's of Glass that are Convex on both Sides.

We come next to the Glasses that are convex on both Sides. To find the *Focus* of the Lens of Glass ABCD, of which the Axis EI contains the Center E of the Covexity ADC, and the Center F of the Convexity ABC; draw any Ray of Incidence GH parallel to the Axis EI, and having taken upon that Axis the Line BI triple to the Semidiameter BF, draw the straight Line HI, which will give the Point K of the second Refraction, thro which K draw from the Center E the strait Line EKM, which will be perpendicular to the refracting Surface ADC; and so IK being consider'd as a Ray of Incidence, the Angle IKM will be an Angle of Inclination, and that being double the Angle of Refraction, as we remark'd in the foregoing Problem, if at K you make the Angle IKL equal to half the Angle IKM, you will have in L the Focus you Look for, with respect to the Convexity ABC expos'd to the Sun.



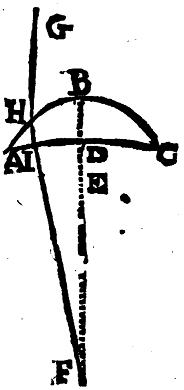
When the Semidiameters ED, BF, are equal to one another, *that is*, when the Convexities ABC, ADC, are equal Portions of the Surface of the same Sphere; the *Focus* will be found about the Center F of the Convexity AC pointing to the Sun. But let the Semidiameter ED, BF, be equal or unequal, the distance of the Focus L will always be the same, turn which Side you will to the Sun.

Of the Glasses that are Convex on one Side and Concave on the other.

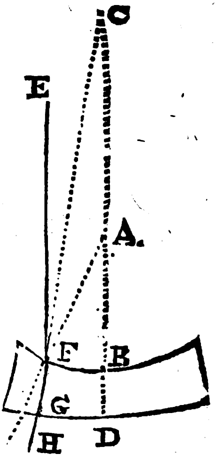
As for the Glasses which are convex on one Side and concave on the other, the Focus of such a Glass will be found after the same manner with that of the last sort, when the convex Side is turn'd to the Sun; but there's a more compendious way of finding it, when the Diameter of the Concavity is triple the Diameter of the Convexity, for then the Focus is a Diameter and a half or three Semidiameters distant from the Convexity which we suppose turn'd to the Sun, *i. e.* 'tis at the Center of the Concavity, the thickness of the Lens being consider'd as very small

Let's

Let't suppose a Lens of Glass ABCD, in which the Semidiameter EB of the Convexity ABC, which faces the Sun, is the third part of the Semidiameter FD of the concave part ADC. Upon this Supposition, I say, all the Rays of Incidence parallel to the Axis BF, as GH, will unite by Refraction at the Center F of the Concavity, because the Ray GH in passing thro the Glass will refract to the straight Line HI, which being continued will pass thro the Point F, at the distance of three Semidiameters from the Convexity ABC, as above; now the Ray of Refraction HF being perpendicular to the concave Surface ADC, will not refract at I upon its egress from the Glass, but will continue in a direct Line to F, and consequently F is the *Focus* we seek.



But if you turn the concave Side to the Sun, the Focus will be found as above; and may likewise be found after a more compendious way, when the Semidiameter AB of the Concavity is the third part of the Semidiameter CD of the Convexity; for in that case the *Focus* will be found at the Center C of the Convexity, if the thickness of the Glass BD be inconsiderable; which is always a necessary Supposition. But there's no use to be made of such a Glass expos'd to the Sun, for its Rays of Refraction separate instead of uniting. So that the Point C is but improperly term'd a *Focus*, for the Rays of Refraction can't assemble in that Point which looks to the Sun, but they separate in Lines that tend only to that Point.



This

Remark.

This *Focus* C which can't produce Fire, is call'd the *Virtual Focus* to distinguish it from the *True Focus*, in which the Rays of the Sun by Refraction are capable to produce Fire. The true Focus may be found by the following Analogy, which supposes the thickness of the Glass, the Convexity of which points to the Sun, to be very small and as it were insensible.

As the difference of the Semidiameters of the Concavity and of the Convexity,
To the Semidiameter of the Convexity;
So is the Diameter of the Concavity
To the Distance of the Focus.

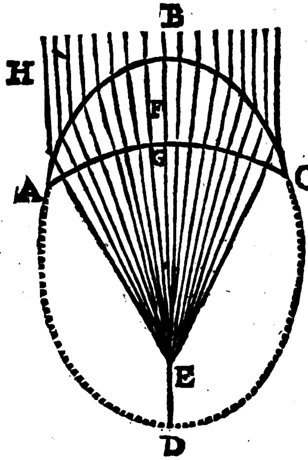
In a Glass that's convex on both Sides, the *Focus* is always *true*, and may be found by the following Analogy, which supposes, as well as the former, the thickness of the Glass to be very small.

As the Sum of the Semidameters of the two Convexities,
To the Semidiameter of the Convexity that faces the Sun;
So is the Diameter of the other Convexity,
To the Distance of the Focus.

Of Glasses Concave on both Sides.

This Analogy will serve likewise for finding the *Focus* of a *Lens* that's concave on both Sides; but in regard such a *Focus* is only *virtual*, as well as in those which are flat on one Side and concave on the other, we shall now wave all further consideration of 'em.

If you make a *Lens* of Glaſs ABCG, concave on one Side and convex on the other, ſo as that the Convexity ABC is the Surface of a part of a Spheroid produced



by the circumvolution of the the *Ellypſis* ABCD, round its great Axis BD, which is to the Diſtance EF of the two Focus's E, F, of the *Ellypſis*, as 3 to 2; and the Center of the Concavity AGC is the Focus E. If you make ſuch a Glaſs, I ſay, and expoſe its Convexity directly to the Rays of the Sun, all the Rays of Incidence that are parallel to the great Axis BD will unite by Refraction in the Focus E, which by conſequence will be the true *Focus* of this Spherico-Ellyptick Lens. Its Convexity may likewiſe be made hyperbolick; but that's too ſpeculative for Mathematical Recreations. See *Dechales's Dioptricks.*

PRO-

Remark. This *Focus* C which can't produce Fire, is call'd the *Virtual Focus* to distinguish it from the *True Focus*, in which the Rays of the Sun by Refraction are capable to produce Fire. The true Focus may be found by the following Analogy, which supposes the thickness of the Glass, the Convexity of which points to the Sun, to be very small and as it were insensible.

As the difference of the Semidiameters of the Concavity and of the Convexity,
To the Semidiameter of the Convexity;
So is the Diameter of the Concavity
To the Distance of the Focus.

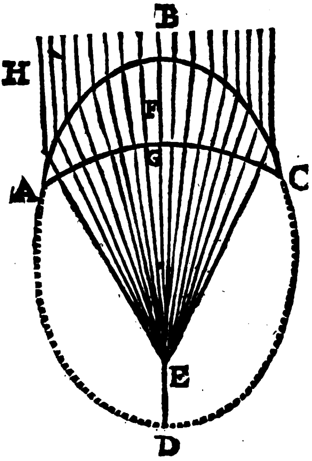
In a Glass that's convex on both Sides, the *Focus* is always *true*, and may be found by the following Analogy, which supposes, as well as the former, the thickness of the Glass to be very small.

As the Sum of the Semidameters of the two Convexities,
To the Semidiameter of the Convexity that faces the Sun;
So is the Diameter of the other Convexity,
To the Distance of the Focus.

Of Glasses Concave on both Sides. This Analogy will serve likewise for finding the *Focus* of a *Lens* that's concave on both Sides; but in regard such a *Focus* is only *virtual*, as well as in those which are flat on one Side and concave on the other, we shall now wave all further consideration of 'em.

If

If you make a *Lens* of Glaſs ABCG, concave on one Side and convex on the other, ſo as that the Convexity ABC is the Surface of a part of a Spheroid produced



by the circumvolution of the the *Ellypſis* ABCD, round its great Axis BD, which is to the Diſtance EF of the two Focus's E, F, of the *Ellypſis*, as 3 to 2; and the Center of the Concavity AGC is the Focus E. If you make ſuch a Glaſs, I ſay, and expoſe its Convexity directly to the Rays of the Sun, all the Rays of Incidence that are parallel to the great Axis BD will unite by Refraction in the Focus E, which by conſequence will be the true *Focus* of this Spherico-Ellyptick Lens. Its Convexity may likewiſe be made hyperbolick; but that's too ſpeculative for Mathematical Recreations. See *Dechales's Dioptricks*.

PRO-

PROBLEM XVIII.

To represent in a dark Room the Objects without, with their natural Colours, by the means of a Lens of Glass that's convex on both Sides.

HAving shut the Door and Windows of the Room, so as to stop all the Avenues of Light, except a small Hole made in one of the Windows that looks to some frequented place or some fine Garden; apply to that Hole a *Lens* of Glass that's Convex on both sides, but not very thick, that its *focus* may be the more distant, as in your old Men's Spectacles: And the Images of the Objects without that pass by the Glass, being receiv'd upon a piece of Linnen stretch'd Perpendicular, or very white Pastboard placed about the focus of the Glass, will appear thereon with their natural Colours, and those even more lively than the Natural, especially when the Sun shines upon 'em, but so as not to shine upon the Glass; for if too much Light flash'd against the Glass, 'twould hinder the pleasant distinction of the Images of the External Objects; which will otherwise appear so distinctly with all their Motions, that not only Men may be distinguish'd from other Animals that pass, but even Men from Women, the Fowls flying in the Air will be observ'd, and the least Air of Wind will discover it self by the trembling of the Plants or Leaves of Trees Perceptible upon the Linnen or Pastboard.

Remark. Even without a Glass one may distinguish upon the Wall or Cieling of a Room, the Images of external Objects, and especially those in Motion; but then these Images do not appear near so fine nor so distinct, because their Colours are dull and dead. But see them which way you will, they will still appear inverted; which may be help'd several ways, though that is to no purpose; for it do's not inlarge the pleasure of seeing them with a Glass in their natural Colours, nor impair the use to be made of it, namely the representing in Miniature upon Pastboard, Landskips, and every thing that has the opportunity of conveying its form to the Pastboard; *viz.* by running a Pencil over

all

all the Traits of that Repreſentation, which will appear as in Perſpective, and of which the parts will be ſo much the better proportion'd that the *Lens* is thin in the middle, and the Hole through which the Species paſs to enter the Glaſs is ſmall. That Hole ought not to be very thick, and therefore it ought to be made in a very thin round plate of Metal applyed to the hole of the Window, which ought to be ſomewhat large, for giving the freeer paſſage to the Species or Images of the External Objects, that lie ſideways to it.

If you ſhut the Windows of a Room, and leave the Door open, you may there ſee what paſſes without by ſeveral plain Looking-glaſſes which communicate the Species by Reflexion, one to another, *&c.*

I forgot to tell you, that by this way of repreſenting upon a Surface the Images of Objects with a *Lens* of Glaſs, the Phyſicians explain the ſenſe of ſeeing; they take the hollow of the Eye for the cloſe Room, the bottom of the Eye or the Retina for the Surface that receives the Species, the Cryſtalline humour for the *Lens* of Glaſs, and the perforation of the Apple for the hole in the Window, through which the Species or Forms of the Objects paſs.

PROBLEM XIX.

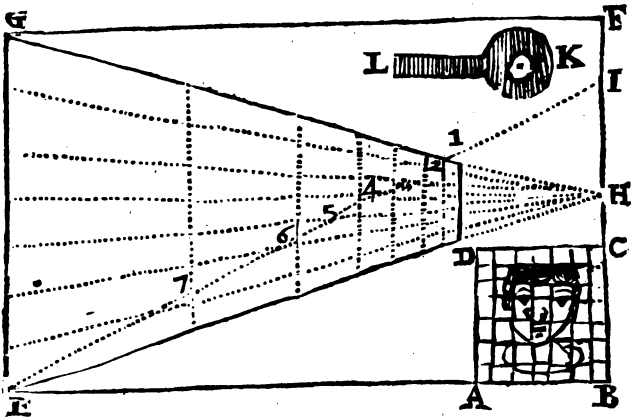
To repreſent on a Plain a diſguiſed or deform'd Figure, ſo as to appear in its natural Poſition, when view'd from a determin'd Point.

YOU may diſguiſe or miſ-ſhape a Figure, for example a Head, in ſuch a manner, that upon the Plain where 'tis drawn, there ſhall be no proportion obſerv'd in the Forehead, and yet when ſeen from a certain Point, it ſhall appear in its juſt Proportions. The way of doing it is this.

Having made upon Paper a juſt draught of the Figure you deſign to diſguiſe, deſcribe a Square round it, as ABCD, and reduce it to ſeveral little Squares by dividing the ſides into ſo many equal Parts, ſeven for Inſtance, and drawing ſtraight Lines along and a-croſs to the oppoſite Points of Diviſion, as the Painters do when

when they go to copy a Picture, and contract it or bring it into a smaller Compass.

This done, describe at pleasure upon the propos'd Plan the Oblong EBFG, and divide one of the two



lesser sides, EG, BF, into as many equal parts as there are already divided in the sides of the Square ABCD, *viz.* seven. EG being here thus divided, divide the other side FB into two equal parts at the Point H, from which draw to the Points of Division in the Opposite, as many straight Lines, the two last of which will be EH, GH.

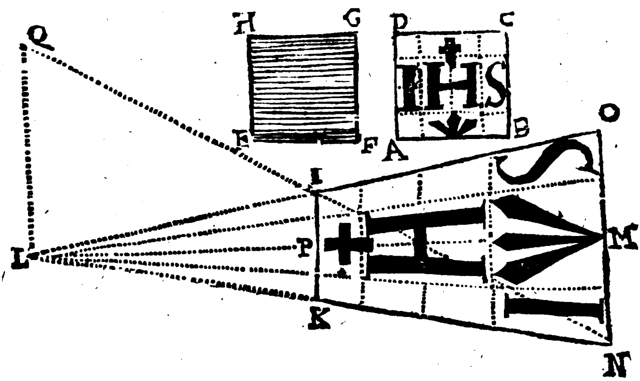
In the next place having taken at pleasure upon the side BF, the Point I above the Point H, for the height of the Eye above the Plain of the Picture, draw from that Point I to the Point E the straight Line EI, which here cuts those that go from H, at the Points, 1, 2, 3, 4, 5, 6, 7; through which do you draw as many straight Lines parallel to one another, and to the base EG of the Triangle EGH, which by this contrivance is divided into as many *Trapeziums*, as there are Squares in the Division of the Square ABCD. So if you transfer into the Triangle EGH, the Figure in the Square ABCD by bringing each Trait into the same Respective *Trapezium*'s or Perspective Squares, which are represented by the natural Square of the great Square ABCD, the deform'd Figure is describ'd; and you'll find it conform to its Prototype, *i. e.* to the appearance in the Square ABCD, when you look upon it through

through a hole that's narrow towards the Eye, but widens much on the ſide towards the Picture, ſuch as K, which I ſuppoſe to be raiſed perpendicularly upon the Point H, ſo that its height LK is equal to the height HI, which ought not to be very great, that the Figure may appear ſo much the more deform'd. *See Prob.* XXI.

PROBLEM XX.

To deſcribe upon a Plain a deform'd Figure that appears in its natural Perfection, when ſeen by Reflexion in a plain Looking-glaſs.

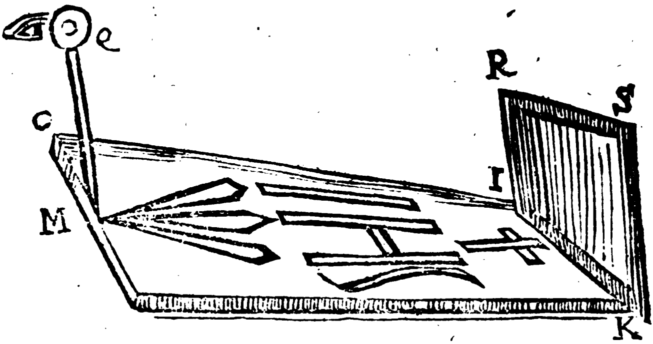
HAving drawn, as above, your propos'd Figure in a Square, ſuch as ABCD, divided into ſeveral other Squares, which in this example are ſixteen in number; and ſuppoſing the Glaſs to be an exact Square, naked and without a Frame, as EFGH, draw upon the plain of the Picture the Line IK equal to the ſide EF of the Looking-glaſs, to the end that



the Figure may entirely fill or take up the Glaſs EFGH; and having divided this Line IK into two equal parts at the Point P, draw the indefinite LM Perpendicular to it, and paſſing thro' its middle Point P, ſo that the two parts PL and PM are equal and as long as you will.

Then

Then raise from the Point L, the Line LQ Perpendicular to the Line LM, and equal to the double of the Line IK, or of the side of the Glass EF; and from the Point M, raise the Line NO Perpendicular to the same Line LM, and likewise double the Line IK; then joyn or draw the Right Lines LN, LO, which will pass thro' the Points, I, K, and make the Triangle LNO. Now, divide this Triangle LNO, as in the foregoing Problem, into as many Perspective Squares as there are natural ones in the Square ABCD, and after the same manner as above transfer into them the Figure in the Square ABCD, which will appear deform'd upon the plain of the Picture, but natural and like its Prototype when seen from the Point Q, rais'd Perpendicularly upon the Point L, as we shew'd in the foregoing Problem. But if you will you may see it with its natural features by Reflexion in the Glass IRSK placed upon the Line IK, when you look to the Glass through a small Hole raised Perpendicularly upon the Point M to the height of MQ, equal to LQ in the preceding Cut.



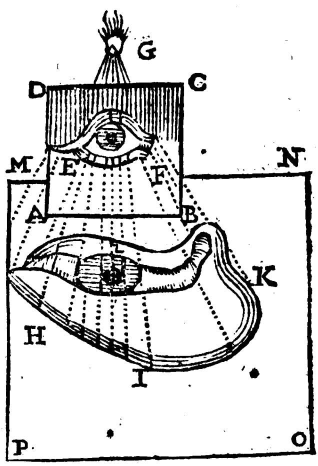
PROBLEM XXI.

To describe upon a Horizontal Plain a deformed Figure which appears Natural upon a vertical Transparent Plain, placed between the Eye and the deformed Figure.

TIS evident, That if you put in Perspective any Figure whatsoever, upon Paper considered as an Horizontal Plain; and raise at Right Angles upon the Ground

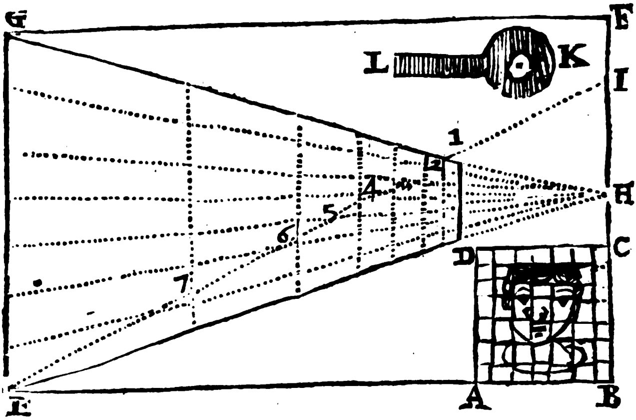
Ground Line a Transparent Plain, for example of Glass; the Eye being placed opposite to the Point of sight, upon a height equal to the distance between the Ground Line and the Horizontal Line, and distant from the Transparent Plain representing the Picture, by a distance equal to that suppos'd in the Perspective, will see the disguised Figure appear in the Glass in its just Proportions. Those who understand Perspective will readily understand what I say; and those who are unacquainted with it, may resolve the Problem Mechanically after the following manner.

Having drawn upon a piece of Past-board your propos'd Figure in its just Proportions, for Example the Eye EF, prick the Pastboard, and set it up at Right Angles upon the Plain MNOP, where you have a mind to draw the Figure disguised: Put behind the prick'd Pastboard a light, of what height and at what distance you please, as at G; and then the Light passing through the holes of the Pastboard ABCD will convey the Figure to the Plain MNOP, and there represent it all over disfigured, as HIKL, which you're to mark down with your Pencil or otherwise. Now this disfiguring Representation will appear in the natural proportions upon a Glass set up in the room of the Pastboard ABCD, and look'd into by the Eye placed at G. Nay 'twill appear conform to its Prototype EF, to the naked Eye thro' a little hole at the Point G.



when they go to copy a Picture, and contract it or bring it into a smaller Compass.

This done, describe at pleasure upon the propos'd Plan the Oblong EBFG, and divide one of the two



lesser sides, EG, BF, into as many equal parts as there are already divided in the sides of the Square ABCD, *viz.* seven. EG being here thus divided, divide the other side FB into two equal parts at the Point H, from which draw to the Points of Division in the Opposite, as many straight Lines, the two last of which will be EH, GH.

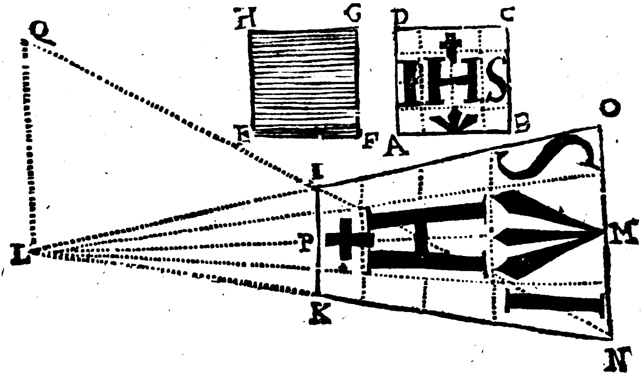
In the next place having taken at pleasure upon the side BF, the Point I above the Point H, for the height of the Eye above the Plain of the Picture, draw from that Point I to the Point E the straight Line EI, which here cuts those that go from H, at the Points, 1, 2, 3, 4, 5, 6, 7; through which do you draw as many straight Lines parallel to one another, and to the base EG of the Triangle EGH, which by this contrivance is divided into as many *Trapeziums*, as there are Squares in the Division of the Square ABCD. So if you transfer into the Triangle EGH, the Figure in the Square ABCD by bringing each Trait into the same Respective *Trapezium*'s or Perspective Squares, which are represented by the natural Square of the great Square ABCD, the deform'd Figure is describ'd; and you'll find it conform to its Prototype, *i. e.* to the appearance in the Square ABCD, when you look upon it through

through a hole that's narrow towards the Eye, but widens much on the side towards the Picture, such as K, which I suppose to be raised perpendicularly upon the Point H, so that its height LK is equal to the height HI, which ought not to be very great, that the Figure may appear so much the more deform'd. *See Prob.* XXI.

PROBLEM XX.

To describe upon a Plain a deform'd Figure that appears in its natural Perfection, when seen by Reflexion in a plain Looking-glass.

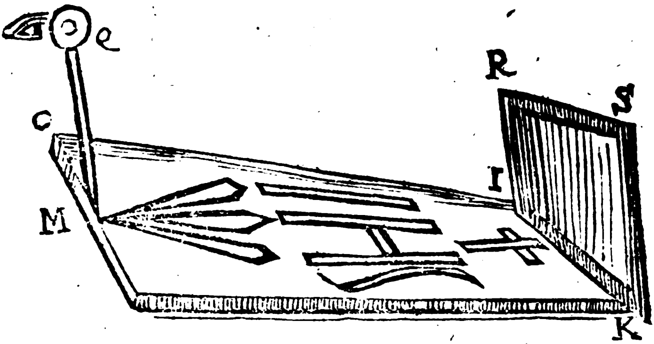
HAving drawn, as above, your propos'd Figure in a Square, such as ABCD, divided into several other Squares, which in this example are sixteen in number; and supposing the Glass to be an exact Square, naked and without a Frame, as EFGH, draw upon the plain of the Picture the Line IK equal to the side EF of the Looking-glass, to the end that



the Figure may entirely fill or take up the Glass EFGH; and having divided this Line IK into two equal parts at the Point P, draw the indefinite LM Perpendicular to it, and passing thro' its middle Point P, so that the two parts PL and PM are equal and as long as you will.

Then

Then raise from the Point L, the Line LQ Perpendicular to the Line LM, and equal to the double of the Line IK, or of the side of the Glass EF; and from the Point M, raise the Line NO Perpendicular to the same Line LM, and likewise double the Line IK; then joyn or draw the Right Lines LN, LO, which will pass thro' the Points, I, K, and make the Triangle LNO. Now, divide this Triangle LNO, as in the foregoing Problem, into as many Perspective Squares as there are natural ones in the Square ABCD, and after the same manner as above transfer into them the Figure in the Square ABCD, which will appear deform'd upon the plain of the Picture, but natural and like its Prototype when seen from the Point Q, rais'd Perpendicularly upon the Point L, as we shew'd in the foregoing Problem. But if you will you may see it with its natural features by Reflexion in the Glass IRSK placed upon the Line IK, when you look to the Glass through a small Hole raised Perpendicularly upon the Point M to the height of MQ, equal to LQ in the preceding Cut.



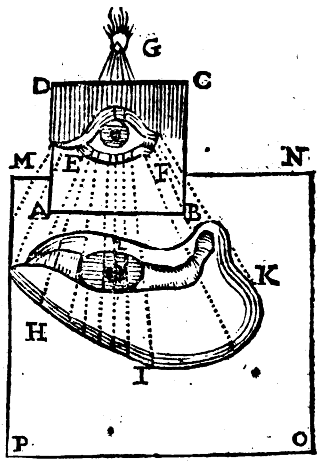
PROBLEM XXI.

To describe upon a Horizontal Plain a deformed Figure which appears Natural upon a vertical Transparent Plain, placed between the Eye and the deformed Figure.

'TIS evident, That if you put in Perspective any Figure whatsoever, upon Paper considered as an Horizontal Plain; and raise at Right Angles upon the Ground

Ground Line a Transparent Plain, for example of Glass; the Eye being placed opposite to the Point of sight, upon a height equal to the distance between the Ground Line and the Horizontal Line, and distant from the Transparent Plain representing the Picture, by a distance equal to that suppos'd in the Perspective, will see the disguised Figure appear in the Glass in its just Proportions. Those who understand Perspective will readily understand what I say; and those who are unacquainted with it, may resolve the Problem Mechanically after the following manner.

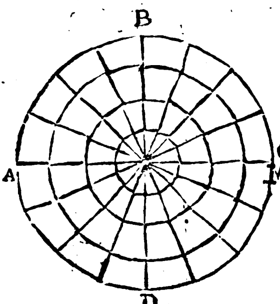
Having drawn upon a piece of Past-board your propos'd Figure in its just Proportions, for Example the Eye EF, prick the Pastboard, and set it up at Right Angles upon the Plain MNOP, where you have a mind to draw the Figure disguised: Put behind the prick'd Pastboard a light, of what height and at what distance you please, as at G; and then the Light passing through the holes of the Pastboard ABCD will convey the Figure to the Plain MNOP, and there represent it all over disfigured, as HIKL, which you're to mark down with your Pencil or otherwise. Now this disfiguring Representation will appear in the natural proportions upon a Glass set up in the room of the Pastboard ABCD, and look'd into by the Eye placed at G. Nay 'twill appear conform to its Prototype EF, to the naked Eye thro' a little hole at the Point G.

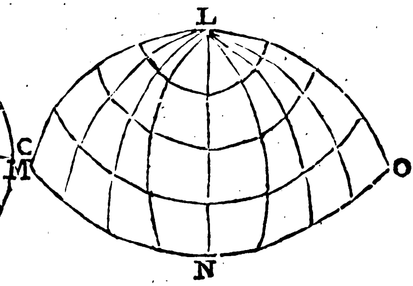


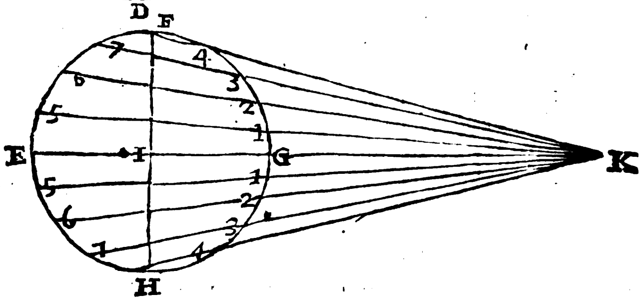
PROBLEM XXII.

To describe upon a Convex Surface of a Sphere a disguis'd Figure that shall appear natural when look'd upon from a determin'd Point.

Having drawn upon Paper the just Proportions of the Figure you have a mind to disguise, surround it with a Circle ABCD, the Diameter of which AC, or BD is equal to the Diameter of the Sphere propos'd; and divide its Circumference into what number of equal parts you will, sixteen for instance, and draw as many straight Lines from the Center of







the Circle to the Points of Division. Divide likewise the Diameter AC or BD into a certain number of equal Parts, eight for Instance, and describe from the same Center through the points of Division the Circumferences of Circles, which with the Right Lines drawn from the Center, will divide the Circle ABDC into 64 little Spaces.

Describe

Describe again another Circle EFGH, equal to the former ABCD, and draw from its Center I the Right Line IK equal to the distance of the Eye from the Center of the Sphere propos'd, so that the part GK may be equal to the height of the Eye above the surface of the same Sphere; and having drawn through the same Center I the Diameter FH Perpendicular to the Line IK, divide this Diameter FH into as many equal parts as you did the Diameter of the first Circle ABCD, *viz.* eight equal parts; then draw from the Point K through the Points of Division, as many straight Lines, which will give you upon the Semicircle FGH the Points, 1, 2, 3, 4, and upon the other Semicircle FEH, the Points, 5, 6, 7.

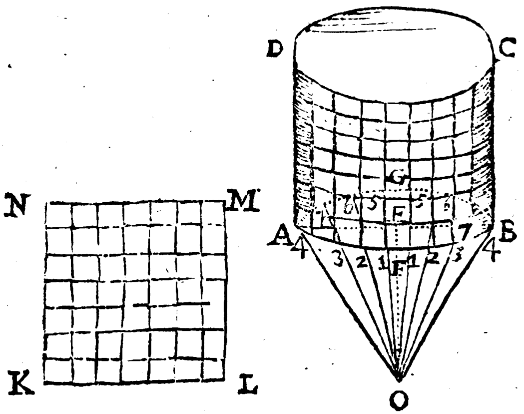
This Preparation being made; describe from the Point L as the Pole, upon the Convex Surface of the propos'd Globe, Parallel Circles, with the aperture or distances G1, G2, G3, G4 and GF, the greatest of which will be MNO, of which the half is only visible in the Scheme. Divide this half into as many equal Parts, as there are in the Division of the Semicircle of ABCD, *viz.* eight parts, in order to describe through the Points of Division and through the Pole L as many great Circles, which with the former will divide the Hemisphere LMNO in as many small spaces as you did the Circle ABCD; into which you are to transfer the Representation of the Circle ABCD, and there you will find its form disfigured, though 'twill reassume its primitive Aspect when beheld from a Point raised Perpendicularly upon the Point L, and remov'd from the Point L equally with the Line GK.

What we have done upon the Convex Surface of a Sphere, may be done after the same manner upon the Concave Surface of the same Sphere; with this only difference that the Parallel Circles describ'd above from the Pole L with the Apertures, G1, G2, G3, &c. must here be describ'd with the Intervals, E5, E6, E7, and EF. *that is to say*, instead of making use of the Semicircle FGH, which the Eye placed at the Point K sees as Convex, you must make use of the other Semicircle FEH, which the Eye placed at the same Point K sees as Concave. Remark.

PROBLEM XXIII.

To describe upon the Convex Surface of a Cylinder a deform'd Figure, that appears handsom and well proportion'd when seen from a determin'd Point.

Having inclosed after the usual manner, the Figure you have a mind to disguise, in a Square KLMN divided into several other little Squares; and having determin'd the Point of the Eye at O, at a reasonable distance from the propos'd Cylinder ABCD, the Base of which is the Circle AFBG; draw from the Center E of that Base through the determin'd Point



O, the Right Line EO; then draw Perpendicular to it, and through the Center E the Diameter AB, which divide into as many equal parts as those of the side KL in the Square KLMN; then draw from the Point O through the Points of Division as many straight Lines, which will give upon the Circumference of the Semicircle seen by the Eye, AFB, the Points, 1, 2, 3, 4; and upon the Circumference of the other Semicircle not seen by the Eye, *viz.* AGB, the Points, 5, 6, 7.

Then

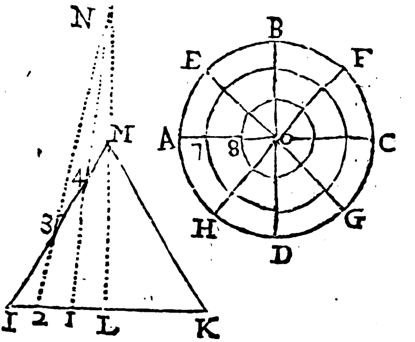
Then draw upon the Surface of the Cylinder, thro' the Points, 1, 2, 3, 4, Lines Parallel to one another, and to the *Axis* of the same Cylinder, or to the side AD or BC: And having divided one of these Parallels into as many equal parts as the Diameter AB, describe upon the Surface of the same Cylinder thro' the Points of Division, the Circumferences of Circles parallel to the Circumference AFBG; which with the foregoing parallel straight Lines will form little Squares; and into these do you transport the Figure of the Square KLMN, which will appear disfigured upon the Surface of the Cylinder ABCD, but conform to its Prototype when viewed through a little hole at O, where the Eye was supposed to be in the Construction.

What we have now been doing upon the Convex Surface of the Cylinder ABCD, may be done after the same manner in the Concave Surface; by making use of the Semicircle AGB, as we have done of the Semicircle AFB, *i. e.* by raising Perpendiculars from the Points, 5, 6, 7, into the Concave Surface, as we have done from the Points, 1, 2, 3, 4, into the Convex Surface, &c. Remark.

PROBLEM XXIV.

To describe upon the Convex Surface of a Cone a disguis'd Figure, which appears natural when look'd upon from a determin'd Point.

Describe round the Figure you intend to disguise, a Circle at pleasure, as ABCD, and divide its Circumference into as many equal parts as you please; as into eight, in order to draw from these Points of Division, A, E, B, F, &c. to the Center O, as many Semidiameters; one of which, as AO, being divided, for example, into three equal parts, by the Points 7, 8, do you describe from the Center O, through these Points of Division, 7, 8, as many Circumferences of Circles, which with the foregoing Semidiamiters will divide the Space terminated by the first and the greatest Circumference ABCD, into 24 small Spaces, which will be of use in copying the Picture therein



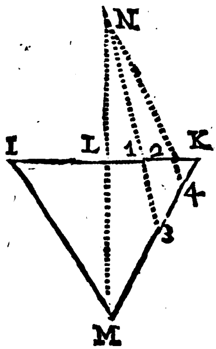
therein contain'd, and disfiguring it upon the Convex Surface of a Cone, when that Surface is divided into as many little Spaces, and that after the following manner.

Having drawn by it self the Line IK equal to the Diameter of the Base of the Cone propos'd, and divided it into two equal parts at the Point L, draw perpendicular to it, through the Point L, the Line LM equal to the height of the Cone, and joyn or draw the Right Lines, MI, MK, which will represent the Sides of the Cone, which I suppose to be a Right Cone, as if the Cone had been cut by a Plain drawn through its *Axis*, so that the *Isosceles* Triangle IKM will represent the Triangle of the *Axis*.

This done, prolong the Perpendicular LM to N, (above the Point M, which represents the Point of the Cone,) as far as you would have the Eye to be rais'd above that Point, so that the Line MN will be equal to the distance of the Eye from the top of the Cone. Having divided the half IL of the Base IK into as many equal parts as the Semidiameter AO of the Prototype, draw from the Point N through the Points of Division, 1, 2, the Right Lines N1, N2, which will give upon the side IM the Points 4, 3. In fine describe from the tip of the Cone with the Apertures M3, M4, the Circumferences of Circles upon the Convexity of the Cone, which will represent the Circumferences of the Prototype ABCD; and having divided the Circumference of the Base of the

the Cone into as many equal parts as the Circumference ABCD, draw from the top of the Cone thro' the Points of Division as many straight Lines which with the foregoing Circumferences will divide the convex Surface of the Cone into 24 small disfiguring Spaces representing those of the Prototype ABCD, into which you're to transfer the Figure of the Prototype, which will appear disfigur'd upon the convex Surface of the Cone, but will appear natural to the Eye placed at the distance MN directly above the Vertex of the Cone.

Remark. What we have now been doing upon the Convex Surface of a Cone, seated on its Base, may be practis'd after the same manner on the concave Surface of a hollow Cone, standing on its *Vertex*; With this only difference, that you must prolong the Perpendicular LM beyond the Point L to N, so that the Line MN may be equal to the distance of the Eye from the Point of the Cone, which in this Case must serve for its Base, that the Eye placed at N may see into it, &c.

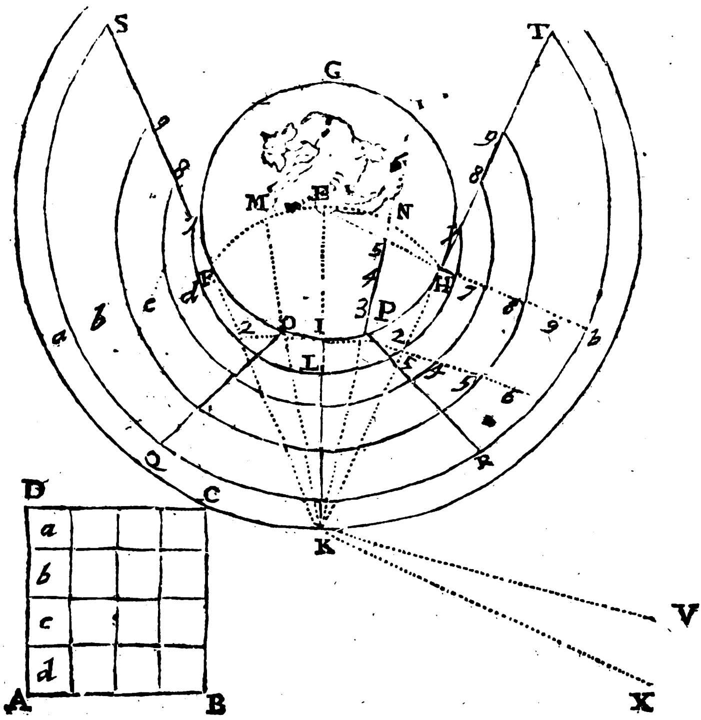


PROBLEM XXV.

To describe upon an Horizontal Plain a disguis'd Figure, which will appear in its natural proportions upon the Convex Surface of a Right Cylindrick Looking-Glass, the Eye seeing it by Reflexion from a Point given.

First of all inclose the Figure you have a mind to disguise, in a Square, such as ABCD; and divide the Square into sixteen other small Squares, in order to transfer from them the Figure of the Prototype into such other disfiguring Squares to be describ'd upon the Convex Surface of a Cylindrical Glass, the Base of which is the Circle FGHI, having E for its Center

If K is the Seat of the Eye, that is, the Point that answers upon the Horizontal Plain Perpendicularly to the Eye, which may be distant from the Cylinder a Foot or two, and be placed a little higher than the Cylinder, in order to see by Reflexion the more parts of the Horizontal Plain: Draw from the Point K to the Center E the Right Line KE, and from its



middle Point L describe through the same Center E the Arch of a Circle FEH, which will mark upon the Circumference FGHI the two Points F, H; and thro' these you are to draw the Right Lines KFS, KHT, which will touch the Circumference at the same Points F, H.

Then divide each of the two equal Arches EF, EH, into two equal parts, at the Points M, N, and draw

draw from the Point K through the Points M, N, the Right Lines KM, KN; which will mark upon the Circumference FIH the two Points, O, P; and from these two you are next to draw the Right Lines OQ, PR, so, that the Angle of Reflexion FOQ may be equal to the Angle of Incidence POK, the Line KO being taken for a Ray of Incidence, and in like manner the Angle of Reflexion HPR may be equal to the Angle of Incidence OPK, the Line KP being taken for a Ray of Incidence; and then the five Lines IK, OQ, PR, FS, HT, will represent the Lines of the Prototype, which are Parallel to the two sides AD, BC, represented by the two Tangents FS, HT. It remains only to divide these Lines into four equal parts in Representation, which I shall do the shortest way, without the possibility of any considerable Error.

Having drawn through the Point I, where the Line KE cuts the Circumference FIH, the Line 1, 2, Perpendicular to the same Line KE, which will be terminated at the Points 1, 2, by the two Tangents KF, KH, draw from the Center E through the Point H the Right Line Ho, equal to the Line, 1, 2, and divide it into four equal parts at the Points, 7, 8, 9. Then draw through the Point K the Right Line KX equal to the height of the Eye and Parallel to the Line Ho, or Perpendicular to the Tangent KH; and having applied a straight Ruler to the Point X, and to the Points of Division, 7, 8, 9, o, mark the Points upon the Line HT, where 'tis cut successively by the Ruler; and you'll find the Line HT divided into the Points, 7, 8, 9, T, parts equal in appearance to those of the Line, 1, 2, which is divided by the Lines drawn from the Point K, into four parts almost equal one to another. At last, carry the divisions of the Tangent HT upon the other Tangent FS.

To divide the Line PR into four equal parts in Representation of those of the Line, 1, 2, draw thro' the Point P the Line P6 perpendicular to the Line KP, and equal to the Line, 1, 2; and divide this Perpendicular P6 into four equal parts at the Points 3, 4, 5. In like manner draw from the Point K the Line KV equal to the height of the Eye, and Parallel to the Line P6, or Perpendicular to KP; and having applied,

applied, as before, a Ruler to the Point V, and to the Points of Division, 3, 4, 5, 6, mark upon the Line KP prolong'd the Points, 3, 4, 5, 6, where 'tis cut by the Ruler. In fine, transfer the Divisions of the Line PN, upon each of the two Lines, PR, OQ, and draw four Circumferences of Circles through the Points equidistant from the Circumference FGHI, mark'd upon the four Lines FS, OQ, PR, HT. These four Circumferences with the Right Lines FS, OQ, IK, PR, HT, will form 16 Squares, into which if you transfer the Figure of the Prototype ABCD, 'twill appear deform'd upon any Horizontal Plain, but in its just proportions upon the Convex Surface of the Cylindrical Glass, placed Right upon its Base FGHI, when seen by Reflexion, by the Eye rais'd perpendicularly upon the Point K to a height equal to the Line KV or KX.

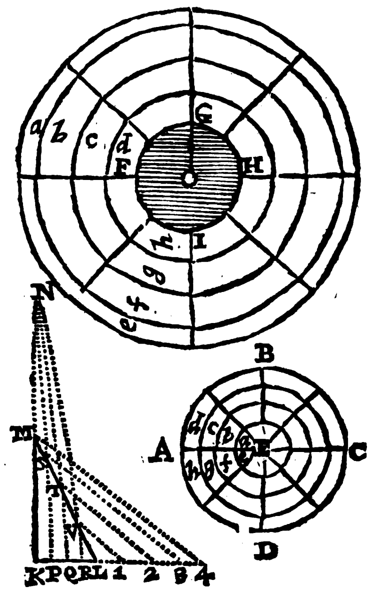
PROBLEM XXVI.

To describe upon an Horizontal Plain a disguis'd Figure that appears in its just proportions upon the Convex Surface of a Conical Glass, set up at Right Angles upon that Plain, being seen by Reflexion from a Point given in the prolong'd Axis *of this Specular Cone.*

IN the first place, describe round the Figure you mean to disguise, the Circle ABCD, of what bigness you will; and divide its Circumference into what number of equal Parts you will; in order to draw from the Center E to the Points of the Division as many Semidiameters, one of which, as AE, or DE ought to be divided into a certain number of equal parts, in order to describe from the Center E, thro' the Points of Division, as many Circumferences of Circles, which with the foregoing Semidiameters will divide the Space terminated by the first and greatest Circumference, ABCD, into severnl little Spaces, which will serve for copying the Picture therein contain'd, and for disfiguring it upon an Horizontal Plain round the Base FGHI of a Conick Glass, and that after the following manner.

Taking

Taking the Circle FGHI whose Center is O, for the Base of the Cone; describe apart the Right Angled Triangle KLM, in which the Base KL is equal to the Semidiameter OG of the Base of the Cone, and the height KM is equal to the height of the Cone. Prolong the Altitude KM to N, so, that the part MN may be equal to the distance of the Eye from the top of the Cone, or the whole Line KN may be equal to the height of the Eye above the Base of the Cone: And having divided the Base KL into as many equal parts as the Semidiameter AE, or DE of the Prototype, draw from the Point N to the Points of Division P, Q, R, as many straight Lines, which will mark the Points S, T, V, upon the Hypothenuse LM, which represents the side of the Cone. At the Point V make the Angle LV 1 equal to the Angle LVR; at the Point T make the Angle LT 2 equal to the Angle LTQ; at the Point S make the Angle LS 3 equal to the Angle LSP; and at the Point M which represents the Vertex of the Cone, make the Angle LM 4 equal to the Angle LMK; and so you have upon the prolong'd Base KL the Points, 1, 2, 3, 4.



In fine describe from the Center O of the Base FGHI of the Conical Glass, with the Distances K1, K2, K3, K4, Circumferences of Circles, which will represent those of the Prototype ABCD; and of which the greatest ought to be divided after the same manner into as many equal parts as the Circumference ABCD; in in order to draw from the Center O to the Points of Division, Semidiameters, which will give upon the Horizontal Plain as many little difform Spaces as in the Prototype ABCD; into which by Consequence you may transfer the Figure of the Prototype, and so 'twill be extreamly disguis'd upon the Horizontal Plain; and yet appear by Reflexion in its just Proportions upon the Surface of a Conical Glass placed upon the Circle FGHI, when the Eye is placed Perpendicularly above the Center O, and distant from the Center O the length of the Line KN.

To avoid Mistakes in transferring what you have in the Prototype ABCD to the Horizontal Plain, observe that what is remotest from the Center ought to be nearest the Base FGHI of the Conical Glass, as you see by the same Letters, *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, of the Horizontal Plain and of the Prototype. The same thing is to be observ'd with respect to a Cylindrical Glass, as you see by the same Letters *a*, *b*, *c*, *d*, of the Horizontal Plain and of the Prototype, in the Cutt annex'd to the foregoing Problem.

PROBLEM XXVIII.

To describe an Artificial Lantern, by which one may read at Night at a great distance.

MAke a Lantern in the Form of a Cylinder or of a small Cask laid on one side; put in one of its two ends a Concave Parabolick Glass, in order to apply to its *Focus* the flame of a Wax-Candle, the Light of which will reflect to a great distance in passing through the other End that ought to be open, and will appear with such a Splendour, that by it one may read at Night very small Letters at a great distance, with Telescopes; and those who see the light of the Candle at a great distance, will take it to be a great Fire, which will be still the lighter if the Lantern is tinn'd within, and made in the form of an Ellypsis.

Remark. The Magical Lantern. We likewise make use of such a Glass for a Magical Lantern, so call'd, because by means of it we can make any thing appear on the white Wall of a dark

dark Room; ſuch as Monſters and fearful Apparitions, which the Ignorant impute to Magick. The Light reflected by vertue of this Glaſs paſſes through a Hole in the Lantern, in which there's a Lens of Glaſs; and between them there's a thin piece of Wood containing ſeveral little Glaſſes painted with monſtrous and formidable Figures, which they move up and down through a ſlit in the Body of the Lantern, and which caſt their Repreſentation to any oppoſite Wall with the ſame Colours and Proportions, but much inlarged.

PROBLEM XXVIII.

By the means of two plain Looking-Glaſſes to make a Face appear under different Forms.

HAving placed one of the two Glaſſes horizontally, raiſe the other to about Right Angles over the firſt; and while the two Glaſſes continue in this Poſture, if you come up to the Perpendicular Glaſs, you'll ſee your Face quite deform'd and imperfect; for 'twill appear without Forehead, Eyes, Noſe or Ears, and nothing will be ſeen but a Mouth and a Chin rais'd bold. Do but incline the Glaſs never ſo little from the Perpendicular, and your Face will appear with all its parts excepting the Eyes and the Forehead. Stoop it a little more, and you'll ſee two Noſes and four Eyes; and then a little further, and you'll ſee three Noſes and ſix Eyes. Continue to incline it ſtill a little more, and you'll ſee nothing but two Noſes, two Mouths and two Chins; and then a little further again, and you'll ſee one Noſe, and one Mouth. At laſt incline a little further, that is, till the Angle of Inclination comes to be 44 Degrees, and your Face will quite diſappear.

If you incline the two Glaſſes the one towards the other, you'll ſee your Face perfect and intire; and by the different Inclinations, you'll ſee the Repreſentation of your Face, upright and inverted alternately, &c.

PRO-

PROBLEM XXIX.

By the means of Water to make a Counter appear, that while the Vessel was empty of Water was hid from the Eye.

TAke an empty Vessel and put a Counter in it at such a distance from the Eye, that the height of the sides of the Vessel keeps it hid; you may make the Eye to see this Counter without altering the place of either the Eye, the Vessel or the Counter, *viz.* by pouring Water into it; for as Sight which is perform'd in a straight Line, do's upon encountring a thicker *Medium* refract towards a Perpendicular, so in this case the Water pour'd into the Vessel being a thicker *Medium* than the Air, will make the Rays darted from the Eyes to refract towards the Line that's Perpendicular to its Surface; and so the Eye will see the Counter at the bottom of the Vessel, which without that Refraction could not be seen.

PROBLEM XXX.

To give a perfect Representation of an Iris *or Rainbow upon the Cieling of a dark Room.*

FOR solving this Problem, you must take a Triangular Prism, which the Artists call barely a Triangle, and which, as all the World knows, gives the appearance of divers Colours when applied to the Nose, and makes the Objects appear invested with Colours like unto those of the *Iris* or Rainbow. Now, if you place such a Prism in your Chamber Window, when the Sun shines upon it, the Rays of the Sun passing thro' the Triangular Glass, will form upon the Cieling of the Room a Rainbow; which will be a pretty Sight, especially if the Cieling of the Room is done Archwise; for that will make the Figure round, and like unto the natural Rainbow in the Clouds.

PRO-

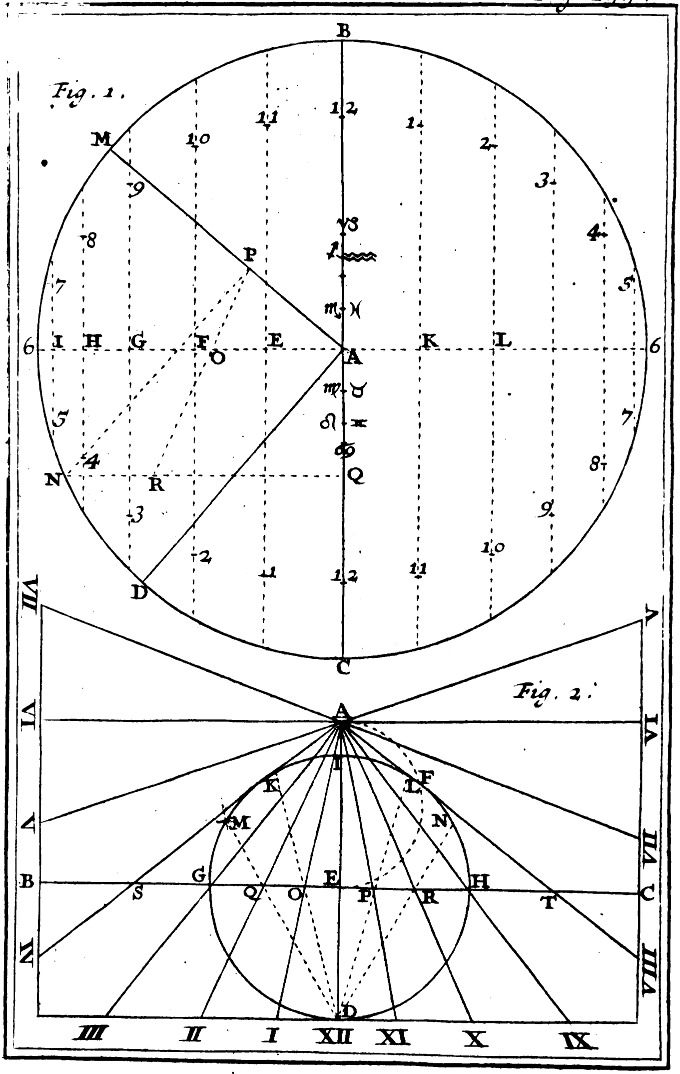


Fig. 1.
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I H G F E K L
Fig. 2.
VII VI V IIII III II I XII XI X IX VIII VII VI V

PROBLEMS OF DIALLING.

DIALLING is the pleasantest Part of the Mathematicks, but is grounded upon a profound Theory, which is not fit for Mathematical Recreations; so that our present Province calls only for the easiest and most diverting Problems.

PROBLEM I.

To describe an Horizontal Dial with Herbs upon a Parterre.

YOU may make an Horizontal Dial of Plants upon a Parterre, after the usual manner, by marking the Hour-lines with Box or otherwise; and putting in the room of a Cock or Gnomon some Tree planted straight upon the Meridian Line, which by its Shadow will point to the Hours as in the ordinary Sundials. But instead of a Tree, one may take his own Heighth for the Style, planting himself upright at the Place mark'd upon the Meridian Line,

You may likewise lay down such a Dial by a Table of the Altitudes of the Sun, or a Table of the Verticals of the Sun, or else after the following manner.

Thro the Point A taken at discretion upon the Horizontal Plain, draw the Meridian Line BC; and from the same Point A describe at pleasure the Circle 6B6C; divide the Circumference of that Circle into 24 equal Parts, from 15 to 15 degrees, for the 24 Hours of the natural Plate 1. Fig. 1.

natural Day, beginning from the Meridian BC; then joyn the two opposite Points that are equally remote from the Meridian by straight Lines parallel to one another and to the Meridian BC, or perpendicular to the Diameter 6, 6, which determines upon the Circle the Points of 6 a-clock at Night and 6 in the Morning.

Upon each of these parallel Lines mark the Points of the Hours which will fall upon the Circumference of an Ellypsis after the following manner. At the Center A with the Line A6 make the Angle 6AD of the Elevation of the Pole (here supposed to be 49 degrees for *Paris*;) and take the perpendicular Distance between the Point 6 and the Line AD, upon the Meridian BC on each side the Center A to 12 and 12: Take likewise the perpendicular Distance between the Point I and the same Line AD, upon each of the two Parallels nearest to the Line BC, from E and K, on each side, to 1 and 12; and in like manner the perpendicular Distance between the Point H and the same Line AD, upon each of the two Parallels next to the last mention'd, from F and L on each side to the Points 2 and 10, and so throughout the rest.

This done, mark the beginning of each sign of the Zodiack which answers to about the 20th Day (*N. S.*) of each Month; mark it, I say, on each side the Center A (which represents the beginning of ♈ and ♎) upon the Meridian Line BC, after the following manner.

At the Center A make with the Meridian AB the Angle BAM of the Elevation of the Pole, the Line AM being perpendicular to the Line AD. Take the Arch DN equal to the Declination of the Sign you are about to mark, as 23 degrees and a half for ♋ and ♑; 20 degrees and a quarter for ♊, ♌, and for ♒, ♐, and 11 degrees and a half for ♉, ♍, and for ♓, ♏. Draw from the Point N the Line NP parallel to the Line AD, and the Line NQ parallel to the Line A6, and lay out the Part A12 from P to the Line NQ at R, so that the Line PR may be equal to the Part A12, or to the perpendicular Distance of the Point 6 from the Line AD, and the Part OP terminated by the two Lines A6, AM, will be the Distance of the Sign propos'd from the Center A, which represents the two Equinoctial Points.

The

The Dial being thus drawn with its Ornaments, you may know the Hours upon it by the Rays of the Sun, provided you place your ſelf about the degree of the current Sign of the Sun; with this difference, that, whereas in the Horizontal Dial the Cock is determin'd to a certain ſize, here it may be of what ſize you will; and indeed it ought to be a little long, becauſe if it be ſhort the Shadow may in Summer prove ſo ſhort as not to reach to the Hour-Points mark'd upon the Parallels. If you deſign to make uſe of your own Heighth for a Gnomon, you muſt not deſcribe too large a Circle round the Center A, for fear the Hour-Points ſhould be too remote.

PROBLEM II.

To deſcribe an Horizontal Dial, the Center of which and the Equinoctial Line are given.

LET the given Center be A and the Equinoctial Line BC. Draw thro the Center A the Line AD perpendicular to BC, for the Meridian Line. Deſcribe upon the Line AE the Semicircle AEF; upon which take the Arch EF equal to the double of the Elevation of the Pole (for example 98 degrees for *Paris*, where the Pole is elevated about 49 degrees.) From the Point E deſcribe thro the Point F the Circumference of a Circle, which will give upon the Equinoctial BC the Points G, H, of 3 and 9 Hours, and upon the Meridian AD the two Points I, D, each of which may be taken for the Center Diviſor of the Equinoctial BC, upon which you are to mark the Points of the other Hours after the following manner. Plate I. Fig. 2.

Set the Compaſſes with the Aperture or Extent of EF, upon the Circumference of the Circle deſcrib'd from the Center E; ſet 'em, I ſay, from the Points G and H to K and L, and from I on each ſide to M and N; and draw from the Point D, thro the Points K, L, M, N, the ſtraight Lines which upon the Equinoctial BC will mark the Points O, P, Q, R, for 1, 11, 2 and 10 Hours. If you ſet the Compaſſes with the ſame extent EF, from M and N, to the Points S and T upon the Equinoctial BC; you have in S the Point of 4, and in T the Point of 8. At last set your Compasses with the same Aperture EF from the Points S, T, twice to the Right and Left upon the same Equinoctial Line BC, and you have the Points of 5 and 7 which are out of the Plain of the Dial, &c.

PROBLEM III.

To describe an Horizontal Dial by the means of a Quadrant of a Circle.

Plate 2. Fig. 3. I Suppose the Quadrant of a Circle is divided into 90 degrees as ABC, within which you must draw the Line DE perpendicular to the Semidameter AB, or parallel to the other Semidiameter AC; which may be distant from A the Center of the Quadrant, more or less, according as you wou'd have your Dial larger or smaller. That Line DE will be unequally divided by the straight Lines drawn from the Center A to the Points at every 15 degrees which represent the Hour-Points of the Equinoctial Line of the Horizontal Sun-dial to be drawn as followeth:

Draw upon the Horizontal Plain the Meridian Line FG, and having taken there at pleasure the Point F for the Center of the Dial, take from that Center upon the Meridian FG, the Part FH equal to the Part AI terminated by the Line DE upon the Line of the Elevation of the Pole, which we here suppose to be 30 degrees, computing from C; then draw thro' the Point H the Line KL perpendicular to the Meridian FG, and that Line KL shall be taken for the Equinoctial Line; upon which you are to transfer or lay down from H on each side the divisions of the Line DE beginning from D, in order to have the Hour-Points, thro which you are to draw from the Center F the Hour-Lines, &c.

If you desire to find the Root and Length of the Gnomon, draw in the Quadrant from the Point D which represents the end of the Gnomon, the Line DO perpendicular to the Line AI of the Elevation of the Pole, which represents the Meridian Line of the Horizontal Dial; and make HM equal to AO, or FM equal to IO; and so you have in M the Foot of the Gnomon, the Length of which is equal to the Perpendicular DO, for the Point I represents the Center of the Dial.

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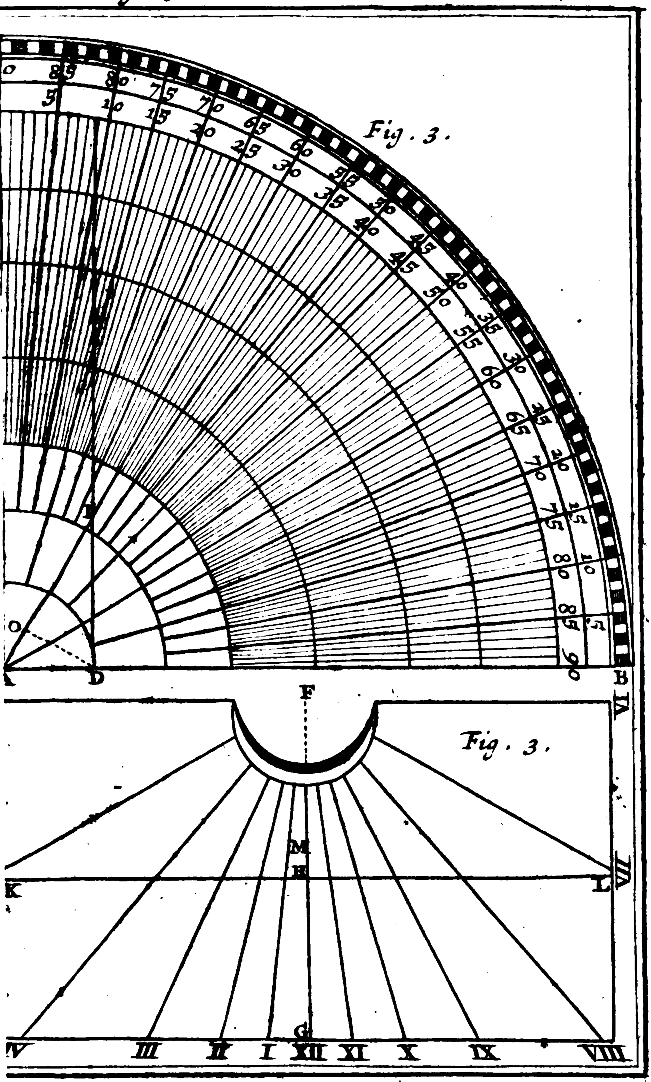
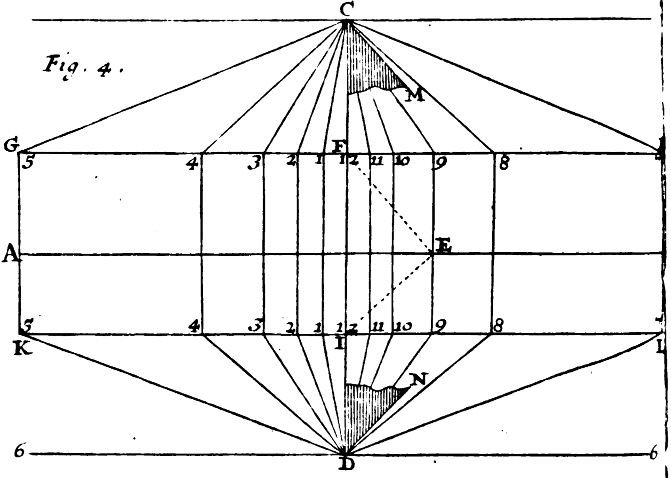


Fig. 3.
Fig. 3.
A
D
B
O
F
M
H
K
L
G
IV
III
II
I
XII
XI
X
IX
VIII

Fig. 4.


C
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G
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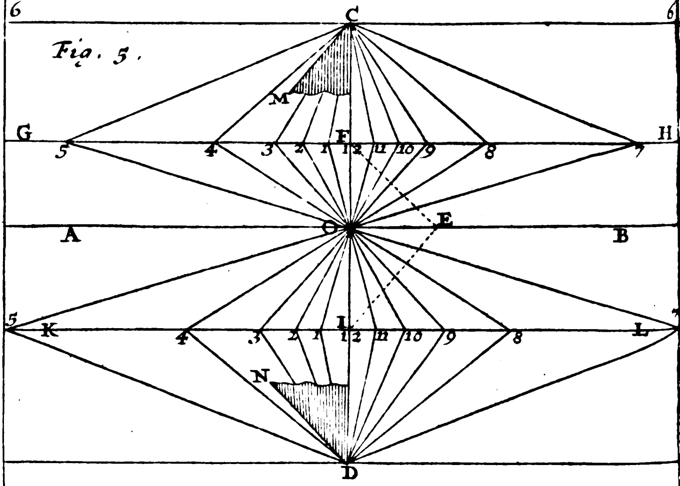
6
D
6

Fig. 5.


6
C
6
M
G
F
H
5 4 3 2 1 12 11 10 9 8 7
A
O
E
B
5
K
I
L
4 3 2 1 12 11 10 9 8
N
D

PROBLEM IV.

To describe an Horizontal Dial, and a Vertical South Dial, by the means of a Polar Dial

IF the Polar Dial is supposed in a Plain parallel to a Circle of six Hours, so that the Equinoctial Line AB is perpendicular to the Meridian Line CD, and to all the other Hour-Lines which are parallel one to another and to the Meridan: At the Point E of 9 Hours upon the Equinoctial, make with the same Equinoctial AE, the Angle AEF of the Complement of the Elevation of the Pole; and thro the Point F where the Line EF cuts the Meridian CD, draw GH perpendicular to the same Meridian CD, which Perpendicular will be cut by the Hour-Lines of the Polar Dial at certain Points, thro which you are to draw to the Center C the Hour-Lines of the Horizontal Dial; and this Center C is found upon the Meridian CD by taking the Line FC equal to the Line EF. Plate 3. Fig. 4.

If from the same Point E you draw the Line EI perpendicular to the Line EF, or, which is the same thing, if at the Point E you make the Angle AEI of the Elevation of the Pole upon the Horizon, and thro the Point I, where the Line EI cuts the Meridian CD, draw the Line KL perpendicular to the Meridian or parallel to the Equinoctial; that Line KL which represents the first Vertical, will be cut by the Hour-Lines of the Polar Dial at Points, thro which you are to draw to the Center D the Hour-Lines of the South Vertical Dial, that Center D being found in like manner (as above) upon the Meridian CD, by making the Line ID equal to the Line IE.

Take notice that the Axis CM of the Horizontal Dial is parallel to the Line EF, and in like manner the Axis DN of the Vertical Dial is parallel to the Line EI.

PROBLEM V.

To describe an Horizontal Dial and a vertical South Dial, by the means of an Equinoctial Dial.

Plate 3. Fig. 5. IF the Equinoctial Dial is supposed to be describ'd upon a Plain parallel to the Æquator, so that the Line of 6 Hours AB is perpendicular to the Meridian Line CD; make at the Point E taken at discretion upon the Line of 6 Hours AB, the Angle AEF of the Elevation of the Pole; and thro the Point F where the Line EF cuts the Meridian CD, draw GH perpendicular to the Meridian CD; which Perpendicular will be cut by the Hour-Lines of the Equinoctial Dial in Points, thro which you're to draw the Hour-Lines of the Horizontal Dial from the Center C. This Center C is found by taking FC equal to the Line EF.

For the Vertical Dial, draw from the same Point E, the Line EI, perpendicular to the Line EF; or, which is the same thing, make at the Point E the Angle AEI of the Complement of the elevation of the Pole; and thro the Point I, where the Line EI cuts the Meridian CD, draw KL parallel to the Line of six Hours AB; which Parallel will be cut by the Hour-lines of the Equinoctial Dial that come from the Center O, in Points thro which you are to draw the Hour-lines of the Vertical Dial, from its Center D; this Center being found by taking ID upon the Meridian CD, equal to EI.

You'll observe, that the Axis CM of the Horizontal Dial is parallel to the Line EI, and that the Axis DN of the Vertical Dial is parallel to the Line EF.

PROBLEM VI.

To describe a Vertical Dial upon a Pane of Glass so as to denote the Hours without a Gnomon.

I Once made such a Dial for a Friend after the following manner.

I took off a Pane of Glass that was soldered on the out-side to the Frame of a Window, and calculating the Thickness of the Frame for the Gnomon, had the Pane

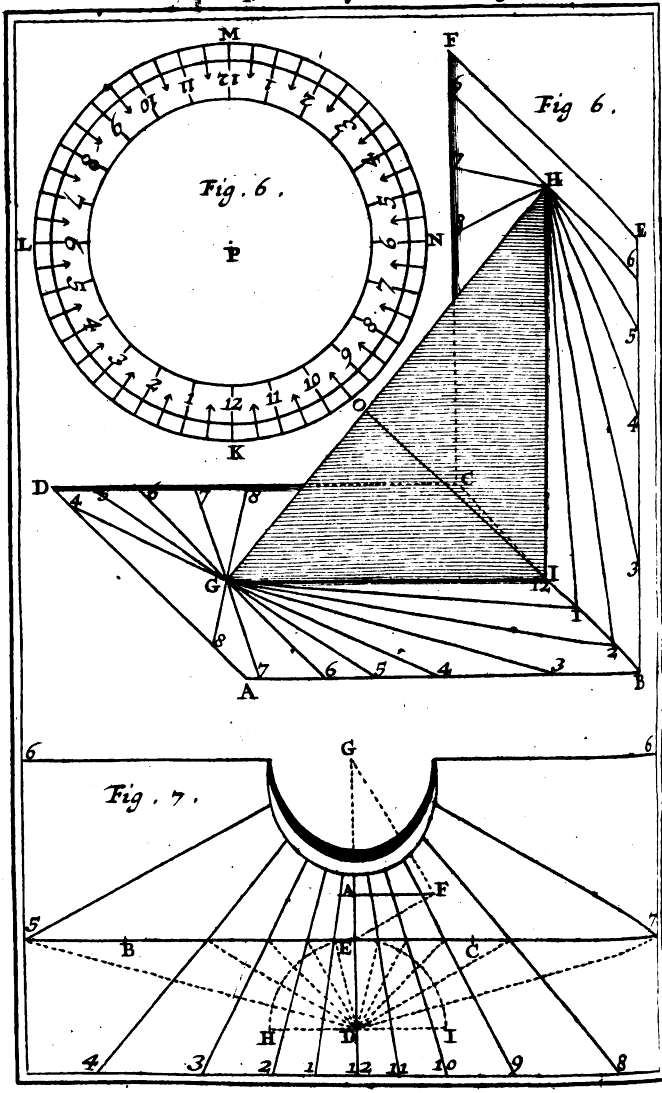


Fig. 6.
Fig 6.
Fig. 7.

Pane glew'd on again to the in-side of the Frame, alloting to the Meridian Line a Situation perpendicular to the Horizon, as it should be in Vertical Dials, and on the out-side I caus'd to be glew'd to the Frame opposite to the Dial, a strong piece of Paper un-oil'd, that so the Rays of the Sun might penetrate it the less, and keep the Surface of the Dial darker. Then to distinguish the Hours without a Style, I made a little Hole in the Paper with a Pin, over-against the Foot of the Style mark'd upon the Dial: And thus the Hole representing the tip or end of the Style, and the Rays of the Sun passing thro it, cast upon the Glass a small Light that pointed out the Hours very prettily in the obscurity of the Dial.

PROBLEM VII.

To describe three Dials upon three different Plains, denoting the Hours of the Sun, by only one Gnomon.

PRepare two Rectangular Plans ABCD, BEFC, of an equal breadth BC; join them by that Line BC which shall represent their common Section, so that they make a right Angle; and for that reason, the one ABCD being taken for an Horizontal Plain, the other BEFC may be taken for a Vertical Plain. Plate 4. Fig. 6.

This done, or rather before you join the two Plans, divide their common breadth BC into two equal Parts at the Point I; and to that Point I draw in the Plain ABCD the Line GI perpendicular to the Line BC, and in the Plain BEFC draw the Line HI perpendicular to the same Line BC: And then each of the two Lines HI, GI, shall be taken for the Meridian of its Plain.

Now, taking the Plain ABCD for an Horizontal Plain, describe an Horizontal Dial upon it, the Center of which G may be taken at pleasure upon the Meridian GI; and upon the other Plain BEFC describe a Vertical South Dial, of which the Center H will be found upon the Meridian HI by means of a right-angled Triangle GIH, the Angle IGH being equal to the elevation of the Pole. This Triangle GHI the right Angle of which is in I, ought to be made of some ſtrong Subſtance, that it may be applied to the Plains, ſo as to keep them in the right Angle, as you ſee in the Figure; and then the Hypothenuſe GH may ſerve for an Axis to the Horizontal Dial of the Plain ABCD, and to the vertical Dial of the Plain BEFC.

Theſe two Plains ABCD, BEFC being thus join'd and detain'd in that poſition by the third Triangular Plain GIH; draw from I the right Angle of that third Plain, the Line IO perpendicular to the Axis GH; and with that IO as a Radius, make a round fourth Plain KLMN, with its Circumference divided into 24 equal parts, in order for an Equinoctial Dial, both ſuperior and inferior, ſo that the Hour-lines of the one may anſwer to the Hour-lines of the other.

This Plain KLMN ought to be cut on the inſide as the Circle of a Sphere, and ſlit along the Meridian that by that Slit it may fit the Triangular Plain GIH upon the Line IO, the South Point K touching the Point I; in which caſe the Axis GH will paſs thro the Center P of the Equinoctial Dial, and be perpendicular to its Plain, and conſequently will likewiſe be the Axis of that Dial; the Plain of which being turn'd direct South, ſo that the Center G points exactly South, which will be parallel to the Æquator, and then the Shadow of the common Axis GH, will ſhew the Hours by the Rays of the Sun upon each of the three Dials, excepting the time of the Equinoxes, at which time 'twill only ſhew 'em in the Horizontal and Vertical Dials.

To turn the Center G of the Horizontal Dial directly South, ſo, that the Meridian Line of each of theſe Dials may be in the Plain of the Meridian, and that the Axis GH may anſwer to the Axis of the World; you may make uſe of a Compaſs with the declination of the Magnet mark'd in it. Or elſe, you may mark the Points of the beginning of each Sign of the Zodiack, on the Axis GH on each ſide of O, which repreſents the Equinoctial Points, or the beginnings of ♈ and ♎ according to the declination of the Signs, making at the Point, with the Line IO, Angles equal to that Declination: For thus, by giving the Plain ABCD an Horizontal Situation, and turning it till the Shadow of the Circumference KLMN falls upon the degree of the Sign current of the Sun, the Center G will point directly South, and each Meridian Line

will

will lie in the Plain of the Meridian Circle. I do not say, that the North Signs are to be mark'd from O to G; for those who understand the Sphere, know that in our Zone the Point G represents the North Pole.

PROBLEM VIII

To draw a Dial upon an Horizontal Plain, by means of two Points of a Shadow mark'd upon that Plain at the times of the Equinoxes.

IF the two Points of the Shadow are B, C; join them by the straight Line BC, which will represent the Equinoctial Line; and that the Error may be less sensible, the two Shadow-points must not be far distant one from another, because the declination of the Sun changes sensibly round the Equinoxes; and at the same time they must not be too near, neither, because 'tis difficult to draw an exact straight Line between two Points that lie too close together. Plate 4. Fig. 7.

Having thus drawn the Equinoctial Line BC, draw by the foot of the Style A the Line GD perpendicular to it, and that will be the Meridian Line, upon which you must mark the Center D of the Æquator, and the Center G of the Dial, after the following manner. Having drawn by the foot of the Gnomon A, the Line AF perpendicular to the Meridian Line or parallel to the Equinoctial Line, and equal to the Gnomon, joyn the Radius of the Æquator EF, and take upon the Meridian the Line ED equal to EF; then D will be the Center of the Æquator; and if you draw from the Point F the Line FG perpendicular to the same Radius of the Æquator EF, you have upon the Meridian Line the Center of the Dial at the Point G.

It remains only to mark the Hour-points upon the Equinoctial BC, which may be done by *Probl.* 2. or else thus: Having describ'd from D the Center of the Æquator, with what extent of the Compasses you will, the Semicircle HEI, and divided its Circumference into 12 equal parts, from 15 to 15 degrees; draw from the same Center D to the Points of Division as many straight Lines, which being prolong'd will give upon the Equinoctial Line BC the Points of the Hours.

Or, an easier way may be this; Take upon the Equinoctial Line from the Point E, on each side of it, a Line equal to the Radius of the Æquator EF, extending from E to the Points of 3 and 9 a Clock; then take the distance of these two Points, and lay it from D on each side, to the Points of 4 and 8; and again from these Points, on each side, to the Points of 5, 11, 1 and 7: For thus you'll have all the hour Points upon the Equinoctial, excepting those of 2 and 10, which you'll find by dividing the distance of 4 and 8 into three equal parts, or thus.

Remark. You'll observe that the distance between the South Point E, and the Point of 4 or 8 hours upon the Equinoctial Line, is the half of the Distance between the Points of 1 and 5, or the Points of 11 and 7; and that the Distance between the the Points of 2 and 9, or 10 and 3, is the half of the Distance between the Points of 2 and 5, or 10 and 7; and Consequently that the Distance between the Points of 2 and 9, or 10 and 3, is equal to the third part of the Distance between the Points of 5 and 9, or 3 and 7. Whence it follows that the Points of 2 and 10 may be found, otherwise than as above, by dividing the Distance of Points of 5 and 9, or 3 and 7, into three equal Parts.

If besides the hour Points of the Equinoctial Line BC, you would have the half-hour Points, divide the Semicircle HEI into twice as many equal Parts, *i. e.* into 24 equal Parts, and for the quarter Points into 48, and so on or again; to find the half-hour Points, set one Point of the Compasses upon the hour Points of the Equinoctial Line BC that fall in odd Numbers, Namely, those of 1, 11, 3, 9, 5, and 7, and extend the other Points to the Center of the Æquator D; and so you have the Intervals or Extents, being taken from the same hour Points, on each side, upon the Equinoctial Line, will give the half-hour Points; and these in like manner the quarters, and so on.

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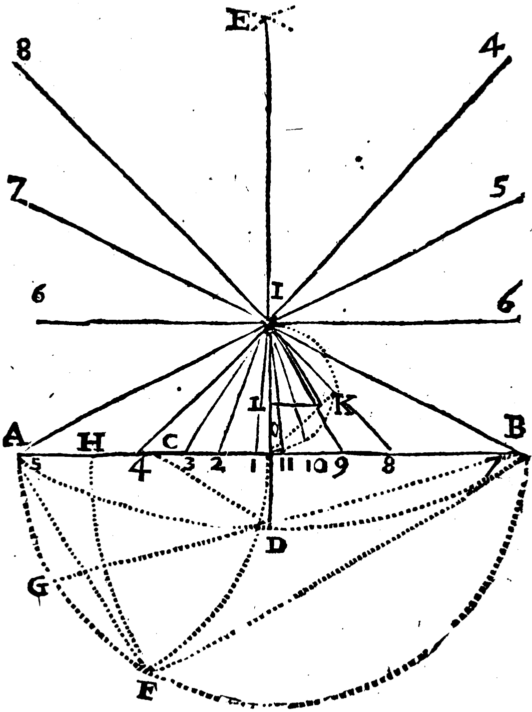
PROBLEM IX.

To draw a Dial upon an Horizontal Plain, in which the Points of 5 and 7 a Clock are given upon the Equinoctial Line.

IT happens oftentimes that by taking too long a Gnomon with respect to the Breadth of the Plain, the Points of 5 and 7 upon the Equinoctial Line fall out of the Plain, and so the Dial can't be Compleat. 'Twill therefore be proper to determine these two



Points, as A, B, upon the Equinoctial, the middle Point of which O will be the South Point.

Having

Having drawn through the South Point O the Meridian Line DE Perpendicular to the Equinoctial BC, you must first find the Center D of the Æquator upon the Meridian DE; and by that the Center of the Dial I, in order to draw the Hour-lines through the Points that you're to mark upon the Equinoctial Line AB, as in the foregoing Problem, by means of the Center of the Æquator D, which we shall here shew you how to find three different ways.

The first Method for finding the Center of the Æquator. Having describ'd from the South Point O, through the Points, A, B, of the hours 5 and 7, the Semicircle AFB, and having drawn from the Point A through the same Point O, the Arch of a Circle OF; divide the Arch AF into two equal Parts at the Point G, and draw the straight Line BG, which will give you the Meridian Line DE, the Center of the Æquator D.

The second Method. Having drawn as above, the Semicircle AFB, and the Arch of a Circle OF, describe from the Point B through the Point F the Arch of a Circle FH, and the Line OD equal to the part AH, and so you make have D for the Center of the Æquator.

The third Method. Describe from the Points A and B, of the Hours of 5 and 7, with the Aperture of the Compasses equal to the distance AB, two Arches of Circles, which here cut one another upon the Meridian at the Point E; and from that Point E describe, with the same extent of the Compasses, the Arch ADB, which gives upon the Meridian DE the Center of the Æquator D.

To find the Center of the Dial, make at the Center of the Æquator, the Angle ODC of the Complement of the elevation of the Pole, and upon the Meridian DE take OI equal to the Line CD; and that Point I will be the Center of the Dial, where all the Hour-lines are to meet.

If you want to find the foot and length of the Gnomon; having drawn upon the Line OI the Semicircle OKI, take the length of OD upon its Circumference, from O to K; and draw from the Point K the Line KL perpendicular to the Diameter OI, in order to have in L the foot or root of the Style, the length of which will be the Perpendicular LK.

'Tis evident, that the Line OK is the Radius of the Æquator, and the Line IK represents the Axis of the Dial, so that the Angle LIK is equal to the Elevation of the Pole.

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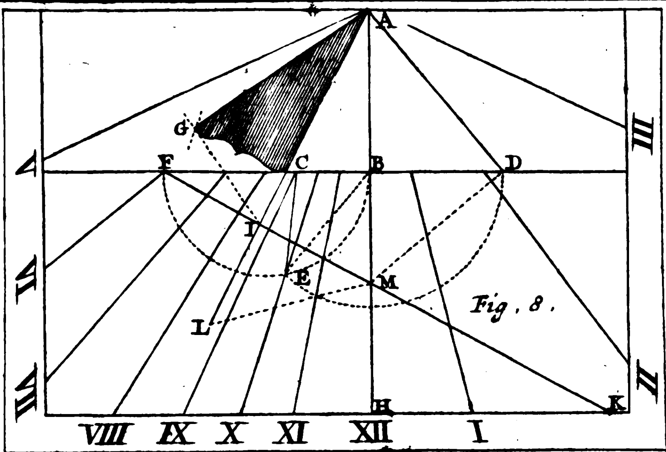
A
G
F
C
B
D
I
E
M
L
H
K
Fig. 8.
VIII
IX
X
XI
XII
I

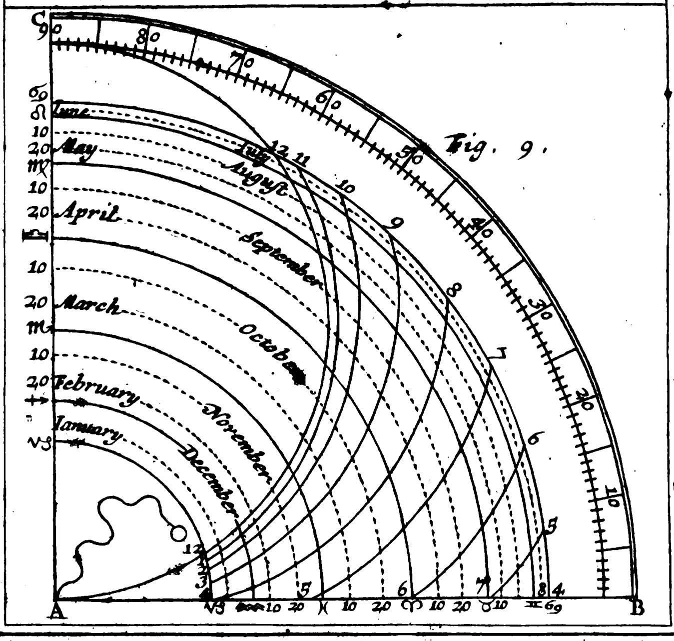
Fig. 9.
June
May
April
March
February
January
July
August
September
October
November
December
A
B

PROBLEM X.

A Dial being given, whether Horizontal or Vertical, to find what Latitude 'tis made for, after knowing the length and root of the Gnomon.

IN the first place, if the Dial is Horizontal, draw by the root of the Gnomon A, the Line AF equal to the Gnomon and Perpendicular to the Meridian; and from G the Center of the Dial to the Point F, the Line FG which will represent the *Axis* of the Dial, and make with the Meridian the Angle FGA equal to the Latitude sought for. Plate 4. Fig. 7.

The same is the method for finding the Latitude of a South or North Vertical Dial, that do's not decline, which is known when the Meridian Line passes thro' the root of the Gnomon, and then the Angle made by the *Axis* of the Dial with the Meridian, will be the Complement of the elevation of the Pole, for which the Dial was made.

If the Vertical Dial looks directly East or West, so as to be Meridian, which is known when the hour Lines are Parallel one to another; measure the Angle made by one of these Hour-lines with the Horizontal Line or any other Line Parallel to the Horizontal, and that Angle will be the elevation of the Pole in question.

If the Vertical Dial declines, which is known when the Meridian Line do's not pass by the root of the Gnomon, as AH, which do's not pass by the Root of the Style C; draw through the Point C the Horizontal Line FD Perpendicular to the Meridian AH, which runs straight down or Perpendicular in all Vertical Dials; and the Line CE Parallel to the Meridian AH or perpendicular to the Horizontal Line FD and equal to the Gnomon. Then take the length of the Hypothenuse EB, (which may be call'd the *Line of Declination*, since the Angle CEB is the declination of the Plain) upon the Horizontal Line from B to D, from which to the Center of the Dial A draw the straight Line DA, which with the Horizontal Line FD will make at the Point D, the Angle BDA, the quantity Plate 5. Fig. 8.

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tity of which will denote the Latitude or the Elevation of the Pole for which the Dial was calculated.

Remark. If you would know the Elevation of the Pole upon the plain of the Dial, that is, how many degrees the Pole is elevated above the Horizon, to which the Plain of the Square is parallel; draw the Substylar Line AC, and describe from C the root of the Gnomon, with the Aperture CE the Arch of a Circle; aud another Arch upon the Center of the Dial A with the Interval AD; and so you have G the Point of the common Section of the two Arches; from which draw to the Center A the *Axis* of the Dial AG, which with the Substylar AC will make the Angle CAG of the Elevation of the Pole.

If you would likewise know the difference of the Meridians of the Horizon of the Place, and the Horizon of the Plain, that is, the difference of Longitude between that of the Horizon for which the Dial was made, and that of the Horizon Parallel to the Plain of the Dial; having prolong'd the Substylar AC to L, draw from the Point F the Section of the Line of six hours and the Horizontal Line, the Line FK perpendicular to the Substylar, which Perpendicular FK will be the Equinoctial Line; then take the length of IG the Radius of the Æquator, upon the Substylar, from I to L, where the Center of the Æquator will fall. From this Center L to the Point *M* the Section of the Meridian and Equinoctial Lines, draw the Right Line LM, which with the Substylar AC, will make the Angle CLM, and that gives the difference of Longitudes.

The Center of the Dial A being here above the Equinoctial Line, we know that the Plain of the Dial declines from the South to the East, because the Root of the Gnomon C is between the Meridian Line and the Morning hours, or those before Noon. We know likewise, that at the time of the Equinoxes, the Dial will be illuminated by the Sun at three in the Afternoon, because the Line of the hour of three being prolong'd, do's not cut the Equinoctial Line on the Afternoon side. In fine, we know that at all times the Plain of the Dial is not shone upon by the Sun at those hours, the Lines of which in the Dial do not cut the Horizontal Line on the side of the same hours.

PRO-

PROBLEM XI.

To find the Root and length of a Gnomon in a Vertical declining Dial.

IF a Vertical declining Dial is drawn upon a Wall without a Gnomon, or any mark for its place or for the Point calculated for its Root, you may find the Root and length of the Gnomon, thus.

If you prolong the Meridian Line BH and any other hour Line, you have upon that Meridian the Center of the Dial, as A, where you'll have the Angle BAD of the Complement of the Elevation of the Pole, by vertue of the Horizontal Line FD, drawn through the Point B taken at discretion upon the Meridian AH, and perpendicular to the same Meridian; which Horizontal Line FD cuts the Line AD at D.

This done, draw from the Point D the Line DM perpendicular to AD, which Perpendicular will give upon the Meridian AH the Point M; through which and the Point F of six hours upon the Horizontal Line, you're to draw the Equinoctial Line FK, and from the Center A the Line AL Perpendicular to FK; and this AL will represent the Substylar Line, and so give upon the Horizontal FD the Root of the Gnomon at C.

To find the length of the Gnomon, draw from its Root found C, the Indefinite Line CE Perpendicular to the Horizontal FD, and describe from the Point B through the Point D, an Arch of a Circle, which will determine upon the Perpendicular CE the length of the Gnomon sought for; and by that you may know the declination of the Plain, represented by the Angle CEB, the Elevation of the Pole upon the Plain represented by the Angle CAG, and the difference of Longitudes represented by the Angle ILM, as we shew'd in the foregoing Problem.

Sometimes you have not the point F of six hours upon the Horizontal Line, *viz.* when the Declination of the Plan is very small; and so you can't draw the Equinoctial Line FK. In this case you may draw that Line by the Point M, by making with the Meridian BH Remark.

BH the Angle BFM to be found by the Declination of the Plain, and the Elevation of the Pole, by the following Analogy.

As the Sine Total,
To the Sine of the Declination of the Plain ;
So is the Tangent of the Complement of the Elevation of the Pole.
To the Tangent of the Complement of the Angle demanded.

Those who understand Trigonometry, knowing the Declination of the Plain and the Elevation of the Pole, will readily find by the three following Analogies, the Angle of the Line of six Hours with the Meridian, the difference of Longitudes, and the Elevation of the Pole upon the Plain.

As the whole Sine,
To the Sine of the Declination of the Plain ;
So is the Tangent of the Elevation of the Pole upon the Horizon,
To the Tangent of the Complement of the Angle of the Line of six hours with the Meridian.

As the Sine Total
To the Sine of the Elevation of the Pole upon the Horizon ;
So is the Tangent of the Complement of the Declination of the Plain
To the Tangent of the Complement of the difference of Longitudes.

As the whole Sine,
To the Sine of the Complement of the Declination of the Plain ;
So is the Sine of the Complement of the Elevation of the Pole upon the Horizon,
To the Sine of the Elevation of the Pole upon the Plain.

If you can't have the Center of the Dial, which may happen when the Elevation of the Pole is very great, or when the Plain declines much, which will hinder you to know the Declination of the Plain, and deter-

determine the Root and Length of the Gnomon by the foregoing Method; in this case measure the Angle of the Line of six Hours with the Horizontal Line; and by means of that Angle, and the Elevation of the Pole, you may know the Declination of the Plain, by this Analogy,

As the whole Sine,
To the Tangent of the Complement of the Elevation of the Pole;
So is the Tangent of the Angle of the Line of six Hours with the Horizontal,
To the Sine of the Declination of the Plain.

The Declination of the Plain being thus known, describe round the part FB terminated by the Line of six Hours and the Meridian, the Semicircle FEB; then take from F the Arch EF equal to the double of the Complement of the Declination of the Plain; and draw from the Point E the Line EC perpendicular to the Horizontal FD, which Perpendicular EC gives the length of the Gnomon, and determines its Root at C.

If you want to draw by the Root of the Gnomon found C, the Substylar Line, draw first the Equinoctial Line FK from the Point of six hours F, making with the Horizontal Line FD the Angle found by this Analogy,

As the whole Sine,
To the Sine of the Declination of the Plain;
So is the Tangent of the Complement of the Elevation of the Pole,
To the Tangent of the Angle demanded.

If from the Root of the Gnomon C, you draw the Line CL Perpendicular to the Equinoctial Line FK, the Perpendicular CL will represent the Substylar Line; which may likewise be drawn by making with the Horizontal FD, at the Point C, the Angle found by this Analogy,

As

As the whole Sine,
To the Sine of the Declination of the Plain;
So is the Tangent of the Complement of the Elevation of the Pole,
To the Tangent of the Complement of the Angle propos'd.

or else take upon the Horizontal Line FD, BD equal to BE; and at the Point D make the Angle BDM of the Complement of the Elevation of the Pole upon the Horizon, in order to have upon the Meridian the Point M, through which and the Point F of six hours you're to draw the Equinoctial Line FM, and from the Point C the Line CL perpendicular to the Equinoctial; and that Perpendicular is the Substylar inquir'd for.

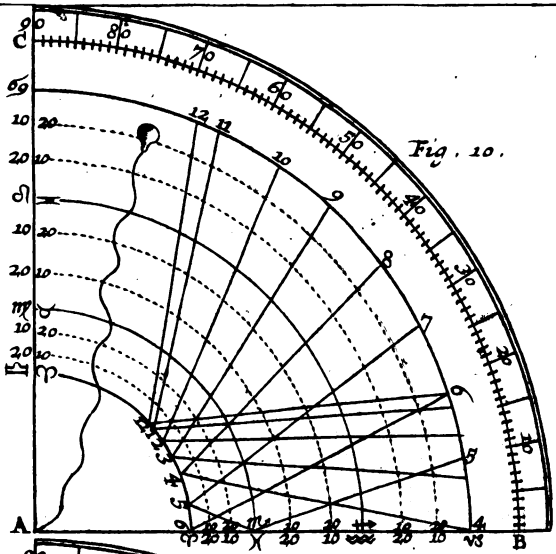
PROBLEM XII.

To describe a Portable Dial in a Quadrant.

Plate 5. Fig. 9. TO describe a Portable Dial in the Quadrant of a Circle ABC, the Center of which is A, and the Circumference BC is divided into 90 Degrees: Draw round the Diameter AC the Semicircumference of a Circle which shall be taken for the Meridian Line; by the means of which and of this Table, (which shews the height of the Sun for every day of the Year, from 10 to 10 Degrees of the Signs of the Zodiack, in the Latitude of 49 Degrees being that of *Paris*) you may describe first the Parallels of the Signs, and from thence the other hour-lines by Circles; and that, after the following manner.

To describe, for Example, the Tropick of ♋, knowing by this Table, that the Sun being in ♋ is elevated upon the Horizon at Noon 64 Degrees, and a half, apply a Ruler from the Center A to the 64th Degree of the Quadrant BC, reckoning from B to C; and through that Point at which the Ruler cuts the Meridian Line, describe upon the Center A a Quadrant or Quarter of a Circle which will represent the Tropick of Cancer. And so of the rest.

To


90
80
70
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20
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C
A
B
12
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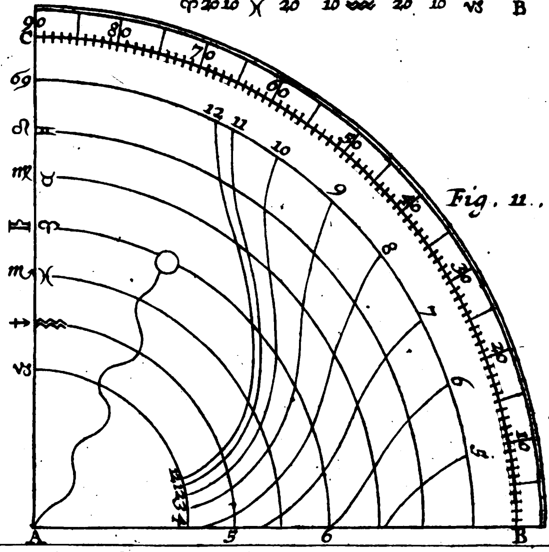
9
8
7
6
5
4

Fig. 10.


90
80
70
60
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C
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5

Fig. 11.

Hou.	XII	XI	X	IX	VIII	VII	VI	V	*Morn.*
Sign.	D M	D M	D M	D M	D M	D M	D M	D M	*Sign.*
♋	64.32	61.56	55.19	46.36	37. 1	27.12	17.32	8.22	♋
10	64. 9	61.33	55. 1	46.18	36.44	26.36	7.12	8. 4	20
20	63. 2	60. 3	54. 4	45 28	35.39	26. 8	16.22	7.12	10
♌	61.13	58.49	52.54	44. 7	34 40	24 51	15. 7	5.50	♊
10	58 48	56.30	50.29	42.14	32.54	23. 7	13.21	3.57	20
20	55 52	53.42	47.57	39.55	30.42	20.58	11.12	1.40	10
♍	52.21	50.30	45. 1	37.14	28.10	18.29	8.40		♉
10	48.51	46.58	41.44	34.13	25.19	15.43	5.54		20
20	44 58	43.12	38 15	31. 0	22.18	12.48	2.59		10
♎	41. 0	39.20	34.37	27.28	19. 9	9.47			♈
10	38. 2	35.26	30.58	24.12	15.58	6.42			20
20	33. 9	31.40	27.24	20.55	12.51	3.44			10
♏	29.29	28. 4	23.58	17.42	9 10	0.54			♓
10	26. 8	24.46	20.51	14.45	7. 5				20
20	23.12	21 52	18 5	12.12	4.42				10
♐	20.47	19.30	15 48	10. 3	2.42				♒
10	18.58	17.42	14. 6	8 27	1.12				20
20	17.51	16.30	13 3	7.27	0.18				10
♑	17 29	16.19	12.44	7. 8	0. 2				♑
Hou.	XII	I	II	III	IV	V	VI	VII	*Even*

This Method of representing the hour Lines by the Circumferences of Circles, will not stand a Geometrical Rigour; but still may be very usefully imployed, in regard the Error is but small. But in stead of Circles you may have straight Lines, in which the Error will not be so considerable; by describing first from the Center A, with what extent of the Compass you will, the two Quadrants ♋ ♑, ♈ ♎, the first of which shall be taken for one of the Tropicks, and the other for the Æquator; and then finding upon each of the two Quadrants one Point of each Hour, in order to joyn two Points of the same Hour by a straight Line, after this manner. Remark. Plate 6. Fig. 18.

To find, for Instance, the Noon-point upon the Æquator ♈ ♎, in which the Sun is elevated upon the Horizon 41 Degrees; apply to the Center A, and to the 41 Degree of the Quadrant BC, a straight Ruler, which will mark the Noon-point 12 upon the Æquator. In like manner, the Sun in ♋ being elevated upon the Horizon at Noon 64 Degrees and a half, apply to the Center A, and to the 64th Degree of the Quadrant BC, the same Ruler, and 'twill mark upon the Quadrant ♋ ♑, (which is consider'd as the Tropick of Cancer) a second South or Noon-point, which being joyn'd to the first gives the Meridian Line, that will serve for the six North Signs, from the Vernal to the Autumnal Equinox.

If the same Quadrant ♋ ♑, be taken for the Tropick of Capricorn, you'll find the Noon-point after the same manner; and by drawing a straight Line thro' this Point and the Noon-point found above upon the Æquator ♈ ♎, you have a second Meridian Line, which will serve for the six South Signs, from the Autumnal to the Vernal Equinox.

The same is the method of marking the other hour-lines, both for the six North and six South Signs; as you may understand by the bare sight of the Figure. The Parallels of the other Signs are describ'd by the Meridian Line, as above; and the hours are known upon the Dial, as upon that last describ'd.

Plate 6. Fig. 11. In short, the exactest way of making this Dial, is as followeth. Describe at pleasure from the Center A seven Quadrants, equidistant from one another if you will; and look upon these as the beginnings of the twelve Signs of the Zodiack, the first and the last representing the two Tropicks, and that in the middle the Æquator. Upon each of these Parallels of the Signs, mark the points of the hours, according to the due height of the Sun at such hours in the beginning of each Sign, taken from the Table inserted above: Then joyn with curve Lines all the Points of the same hour, and so your Dial is compleated, upon which you may distinguish the hour of the Day as above; only, instead of a little Stylus rais'd at right Angles upon the Center A, you may make use of two little Pins, the holes of which answer perpendicularly, and with an equal height upon the Line AC, upon another that is parallel to it; for by this means, instead of having the Line AC cover'd by the shadow of the *Stylus*, you'll make the Rays of the Sun

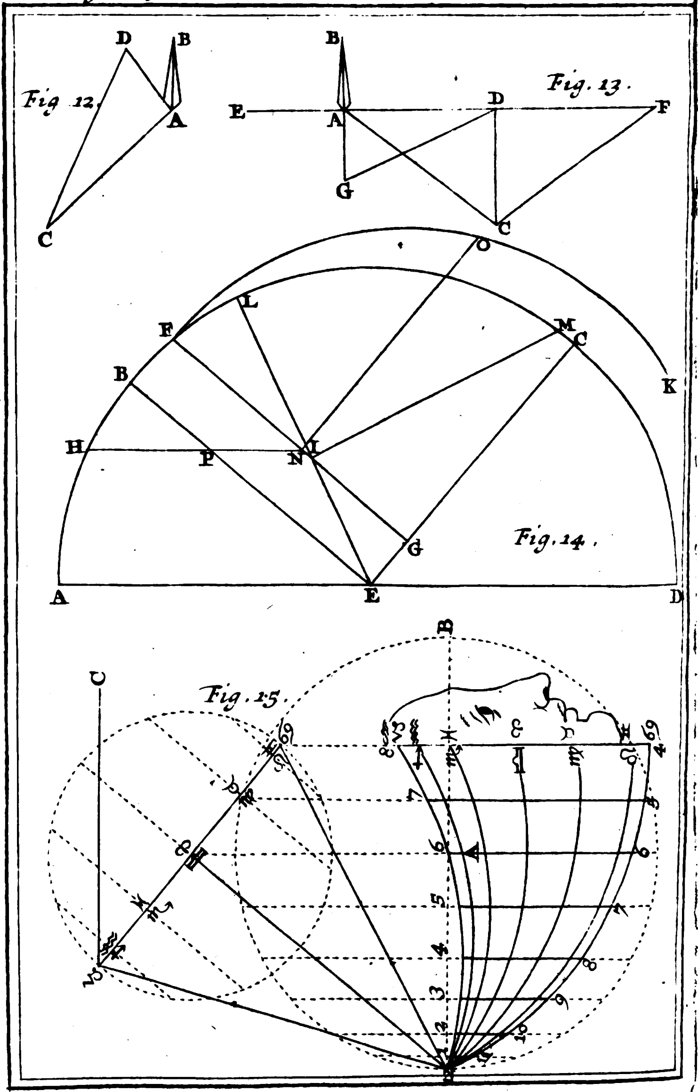
Fig 12.
D
B
A
C
Fig. 13.
B
E
A
D
F
G
C
O
L
F
M
C
B
K
H
P
N
I
G
Fig. 14.
A
E
D
B
C
Fig. 15.
1
2
3
4
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6
7
8
9
10
11

Sun pass through the holes of each Pin; and for the readier perception of the hour, you may put to the Thread that hangs from the Center A, a small Bead, which you're to advance upon the Sign and Degrees of the Sun mark'd upon the Line AC, when you want to know what a Clock it is; for when the Rays pass through the holes, and the thread swings at liberty from the Center A, the Bead will shew the hour, without the necessity of observing where the thread cuts the Degree of the Sign current of the Sun.

One may easily perceive, that with such a Dial, the hour may be known without the Sun, provided you know the place of the Sun in the Zodiack, and its height above the Horizon. For Example; in the beginning of ♈ or ♎ the Sun being elevated upon the Horizon 27 Degrees and a half, a straight Ruler applyed to the Center A, and the $27\frac{1}{2}$ Degree of the Quadrant BC, will cut the Parallel of ♈ and ♎ at the Point of 9 in the Morning, or three in the Afternoon; which shews that 'tis 9 a Clock in the Morning if the Altitude of the Sun was taken before Noon, or 3 in the Afternoon if the Altitude was taken after Noon. To know the hours upon a Dial without the Sun.

You may *know the hours of the Day without a Sundial, by means of the Altitude of the Sun and the Table inserted above*, after this manner. Look in the Table for the given Altitude of the Sun, or that which is next to it in the Column of the Sign current of the Sun, or that of the next tenth Degree; and then you will find opposite to it, the hour at top if the Observation is made in the Morning, and at the bottom, if in the Afternoon. To know what a Clock 'tis without a Dial.

One may likewise know the hours without a Sundial, by Geometry and Trigonometry, as we are about to shew you; after setting forth that the Altitude of the Sun may be taken by a single Quadrant, as you have seen, or else by the shadow of a Style or Gnomon elevated at right Angles upon an Horizontal or Vertical Plain, and that after this manner.

In the first place, if the shadow of the *Stylus* AB rais'd perpendicular upon an Horizontal Plain, is AC; draw from the root of the Cock A, the Line AD Plate 7. Fig. 12.

equal to the Cock AB, and perpendicular to the shadow AC; and from the Point D to the extremity C of the shadow AC draw the right Line CD; and the Angle ACD will be the Altitude of the Sun sought for.

Plate 7. Fig. 13. In the next place, if the plain be Vertical, draw to the extremity C of the shadow AC, the direct Line, CD; and from the root of the Cock A the Horizontal Line EF perpendicular to CD. Then draw from the root A the direct Line AG equal to the Cock AB, and having taken upon the Horizontal Line, the part DF equal to DG, draw the Line CF; and the Angle DFC will give the Altitude of the Sun upon the Horizon.

To know the hours by Geometry. Plate 7. Fig. 14. The Altitude of the Sun being known by this, or other means, *the hour of the Day may be found by Geometry*, thus. Describe at discretion the Semicircle ABCD, the Center of which is E, and the Diameter AD. Then take on one side of it the Arch DC of the Elevation of the Pole, and on the other side the Arch AB of the Complement of the Elevation of the Pole; after which draw EB, EC, which will be perpendicular to one another, and of which the first EB will represent the Æquator, and the second EC the Axis of the World, because the Point E represents the Center of the World, the Point C the Pole elevated upon the Horizon represented by AD, and the Circle ABCD the Meridian and the Colurus of the Solstices, the Colurus being suppos'd to agree with the Meridian.

In this Supposition, we'll take the Arch BL of the greatest Declination of the Sun, or 23 degrees and a half, from B to C if the Sun is in the Northern Signs, and from B towards A if in the Southern; then we'll draw from the Center E to the Point L the Line EL, which will represent the Ecliptick according to the Rules of the Orthographical Projection of the Sphere. This done, make the Arch LM equal to the distance between the Sun and the nearest Solstice; and from the Point M draw MI perpendicular to the Ecliptick EL, which is here cut by it at I; and through this Point I you're to draw FG parallel to the Æquator EB; this FG will represent the Parallel of the Sun, and cuts the Axis EC at the Point

Point G; from whence as a Center you're to draw thro' the Point F the Arch FOK.

In fine, having taken the Arch AH equal to the Altitude of the Sun; draw from the Point H the Line HN parallel to the Horizon AD; which HN will represent the *Almacantarat* of the Sun, and give upon the Parallel FG its place at N; from whence you're to draw the Line NO perpendicular to the Line FG; and then the Arch FO being converted into Time, computing 15 Degrees to an hour, will give the hour in question before or after Noon.

The Arch BF shews the Declination of the Sun; which may be taken yet more exactly by means of its greatest Declination, *viz.* 23 degrees and a half, and its distance from the nearest Equinox; and that by the following Analogy;

As the Sine Total,
To the Sine of the greatest Declination of the Sun;
So is the Sine of its distance from the nearest Equinox
To the Declination sought for.

'Tis evident, that when the Sun has no Declination, which happens at the time of the Equinoxes, instead of drawing the Perpendicular NO from the Point N, you must draw it from the Point P where the Æquator is cut by the *Almacantarat* HI, in order to have the hours of that Day. But in this case the hour may be found more exactly by the following Analogy.

As the Sine of the Complement of the Elevation of the Pole,
To the Sine of the Altitude of the Sun;
So is the whole Sine,
To the Sine of the distance of the Sun from six hours.

When the Sun has a Declination, substract it from 90 degrees if 'tis Northern, or add it to 90 if 'tis Southern, and then you have the distance of the Sun from the Pole; by means of which and of the Elevation of the Pole, with the altitude of the Sun, you may find the hour of the day by Trigonometry, as followeth.

To find the hour of the day by Trigonometry.

Add these three, the Complement of the Altitude of the Sun, the Complement of the Elevation of the Pole, and the distance of the Sun from the Pole; and substract separately from half their Sum, the Complement of the Elevation of the Pole, and the Distance of the Sun from the Pole; in order to have two differences which with the Complement of the Elevation of the Pole, and the distance of the Sun from the Pole, will serve for making these two Analogies,

As the Sine of the distance of the Sun from the Pole,
To the Sine of one of the two Differences;
So is the Sine of the other Difference,
To a fourth Sine.

As the Sine of the Complement of the Elevation of the Pole,
To the fourth Sine found;
So is the whole Sine
To a seventh Sine.

which being multiplied by the whole Sine, the square Root of the Product will be the Sine of half the distance between the Sun and the Meridian.

PROBLEM XIII.

To describe a portable Dial upon a Card.

THE Dial we are about to describe is call'd the *Capuchin*, with allusion to the resemblance it bears to a *Capuchin*'s Head with his cowl turn'd upside down. We do it upon a piece of Pastboard or Card, after this manner.

Having drawn at pleasure the Circumference of a Circle, the Center of which is A, and the Diameter B12, divide the Circumference into 24 equal Parts, from 15 to 15 Degrees, beginning from the Diameter B12; and joyn the two Division Points equidistant from the Diameter, by straight Lines parallel to one another, and perpendicular to the Diameter; which straight Lines will be the hour Lines, and of these that which passes through the Center A will be the Line of six hours.

Plate 7. Fig. 15.

This

This done, make, at the Point A with the Diameter B12, the Angle B12 ♈ of the Elevation of the Pole; and having drawn through the Point ♈ where the Line 12♈ cuts the Line of six hours, the indefinite Line ♋ ♑ perpendicular to the Line 12 ♈, terminate that Line ♋ ♑ by the Lines 12♋, 12 ♑, which ought to make with the Line 12 ♈, each of 'em, an Angle of 23 degrees and a half equal to the greatest Declination of the Sun.

You'll find upon this Perpendicular ♋ ♑ the Points of the other Signs, by describing from the Point ♈ as a Center through the Points ♋, ♑, a Circumference of a Circle, and dividing it into 12 equal Parts, from 30 to 30 Degrees, for the beginnings of the twelve Signs of the Zodiack, in order to joyn the two Division Points, that are opposite and equidistant from the Points ♋, ♑, by straight Lines parallel to one another, and perpendicular to the Diameter ♋ ♑, which will make upon that Diameter the beginnings of the Signs, from whence as Centers you're to draw through the Point 12 Arches of Circles that will represent the Parallels of the Signs, and by Consequence require the same Characters, as you see in the Figure.

These Arches of the Signs, will serve for distinguishing the hours by the Rays of the Sun, after the following manner. Having drawn at pleasure the Line C♑, parallel to the Diameter B12, raise at its extremity C in a true perpendicular a small Cock, and turn the plain of the Dial in such a manner, that the Point C pointing obliquely to the Sun, the shadow of the Cock may cover the Line C ♑, and then the thread swinging freely with its Plummet from the Point of the degree of the Sign current of the Sun mark'd upon the Line ♋ ♑, will shew the hour below upon the Arch of the same Sign.

That the Thread may be easily placed upon the degree of the Sign current of the Sun, the plain of the Dial must be slit along the Line ♋ ♑, for then you may easily advance the Thread to what Point you will of that Line and fix it there. And if you string a little Bead upon the Thread, you may know the hour of the day without the Arches of the Signs, by advancing the Bead to the Point 12, when the Thread — Remark.

is fix'd at the degree of the Sign current of the Sun, for then the Bead will shew the hour, if the Point C be turn'd directly to the Sun, so as to have the Line C♑ cover'd with the shadow of the Cock.

You might have mark'd the Signs more exactly upon the Line ♋♑, by making at the Point 12 on each side the Line 12 ♈, equal Angles to the Declination of these Signs: But in regard the Error is inconsiderable, when the Dial is small, as it commonly is, you had as good rest contented with the foregoing Method.

This Sundial derives its Origin from a certain Universal Rectilineal Dial formerly communicated to the publick by Father *Rigaud* the Jesuit, under the Title of *Analemma Novum*; the Construction and use of which are as followeth.

Father *Rigaud's* Universal Rectilineal Dial. Plate 8. Fig. 16.

Having describ'd, as above, the hour-lines, by vertue of a Circle divided into 24 equal Parts, the Center of which is A, and the Diameter ♈♎, to which all the hour-lines are Perpendicular; of which that passing through the extremity ♈ represents the South or Noon-line, and that passing through the extremity ♎ represents the Midnight-line: This done, I say, take the Diameter ♈♎ for the Æquator, and draw the Parallels of the other Signs in straight Lines, after the following manner.

The Diameter ♈♎ being the Æquator, make with that Line at the Center A, an Angle equal to the greatest Declination of the Sun, or of 23 Degrees and a half, by drawing ♋♑, which shall be taken for the Ecliptick, and will be cut by the hour-lines, from 15 to 15 Degrees, in Points, through which you're to draw straight Lines parallel to one another, and to the Æquator ♈♎, and these Right Lines will represent the beginnings of the Signs and their halves.

In fine, draw from the Center A to the degrees of the lower Semicircle straight Lines, from five to five, or from ten to ten Degrees; and prolong them till they meet, each of 'em, the two Meridian Lines ♋ 70, ♋ 20, to which you're to add Cyphers, so, that the Cyphers of one Meridian Line shall make with the corresponding Cyphers of the other, 90 Degrees, in order to have the Degrees of the Latitude mark'd upon each Meridian Line, which Degrees will direct us to the hours, thus. Draw

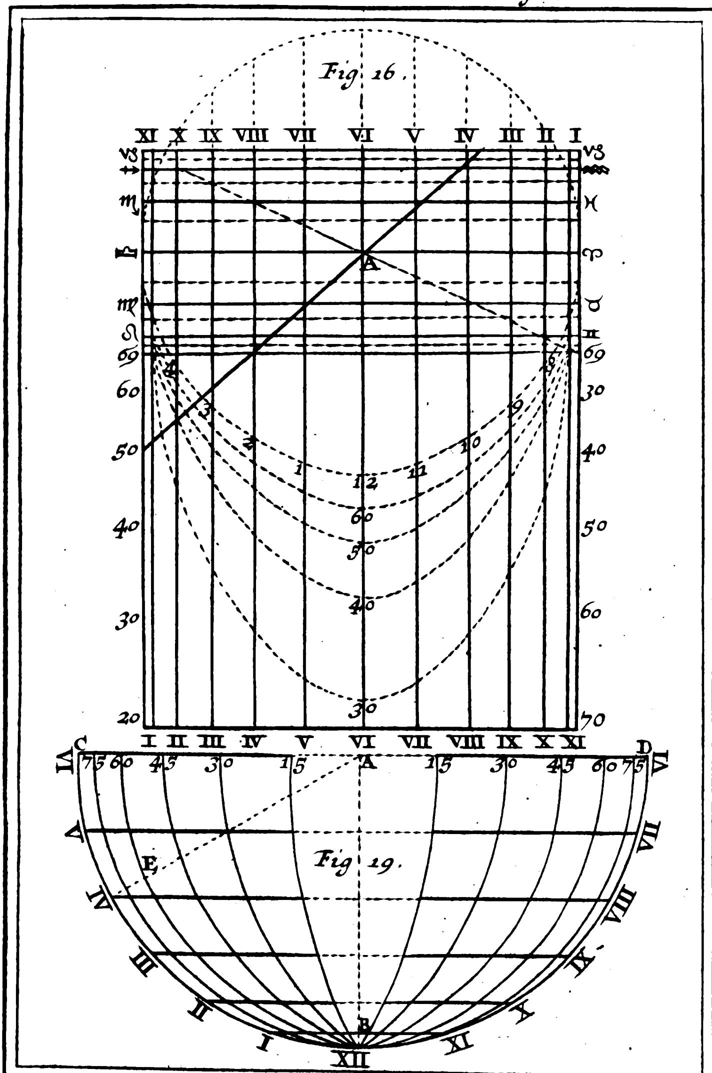


Fig. 16.
XI X IX VIII VII VI V IV III II I
A
60
50
40
30
20
30
40
50
60
70
I II III IV V VI VII VIII IX X XI
Fig. 19.
C
D
A
E
B
XII

Draw from the Center A to the degree of the Latitude of the place where you are, which is mark'd upon the Midnight-line ♋ 20, for instance the 5th degree; draw, I say, the Right Line A50, which representing that Horizon, will denote the hour of Sun-rise and Sun-set at the Point where it cuts the Parallel of the degree of the Sign current of the Sun: And at that Point fix a Thread with its Plummet and a Bead upon it, that so the Thread being extended from the same Point to the degree of the same Latitude mark'd upon the Noon-line ♋70, the Bead may advance upon that degree of Latitude; after which the Bead resting at that place of the Thread, let the Thread swing with its Plummet and its fix'd Bead, and so you'll know the hours, by the following means.

Raise a little Gnomon at Right Angles at the extremity ♎ of the Line ♈♎, or any other Line that's parallel to it; and turn the Point ♎ obliquely to the Sun, in such a manner, that the Thread may hang at liberty with its Plummet, and that the shadow of the Gnomon may cover the Line; for then the Bead will shew the hour.

This is what we are taught by Father *Rigaud*; to which I shall only add that we may make use of the universal Horizontal Dial, by taking the Line of six hours for the Meridian, and the Center A for the Center of the Dial, in which case the Line ♈♎ will be the Line of the hour; and by taking upon the hour-lines (from the Line of six hours ♈♎) the parts of the Horizon terminated by the hour-lines from the Center A. For thus you'll have Points upon the hour-lines, which being joyn'd by curve-lines, will yield Ellipses that will represent the Circles of Latitude; and upon these you'll distinguish the hours by the shadow of the Axis, which with the Meridian ought to make at the Center A an Angle equal to the elevation of the Pole.

But there's another and an easier way of drawing an Universal Elliptick Horizontal Dial, as we are about to shew you; after laying down in the next Problem two different ways of drawing an Universal Rectilineal Horizontal Dial.

PRO-

PROBLEM XIV.

To describe an Universal Rectilineal Horizontal Dial.

Plate 9. Fig. 17.

HAving drawn thro' the Center of the Dial, A, taken at pleasure upon an Horizontal Plain, the two perpendicular Lines AB, CD; and having accounted the first AB for the Meridian, and the second CD for the Line of six hours; describe at discretion upon the Center A the Quadrant EF; and after having drawn through the Point E the Line GH perpendicular to the Meridian, which shall represent the 90th degree of Latitude, and through the Point F the Line FK parallel to the same Meridian which shall represent the Line of 9 hours, and likewise the 30 Circle of Latitude with respect to the hour-lines that are perpendicular to it; divide the Quadrant EF into six equal Parts of 15 degrees each, that so by drawing Right Lines from the Center A through the Points of Division, you may have upon the Line GH the Points of the other hours, through which you are to draw the other hour-lines parallel to the Meridian, omitting on purpose the Lines of 5 and 7 hours, to avoid the excessive breadth of the Dial; nay to make it yet narrower, you may omit the Lines of 4 and 8, which represent the 60th degree of Latitude, with respect to the hour-lines that are perpendicular to them, and will supply the defect of the omitted hour-lines, I mean those parallel to the Meridian AB.

These same Lines that proceed from the Center A, being prolong'd, will mark upon the Line FK of 9 hours, Points through which you are to describe upon the Center A Arches of Circles, which will give upon the Meridian AB the Points 15, 30, 45, 60, 75; and through these you must draw as many straight Lines parallel to one another, and to the Line GH, or perpendicular to the Meridian AB, which straight Lines will represent the Circles of Latitude from 15 to 15 Degrees, with respect to the hour-lines parallel to the Meridian AB.

To

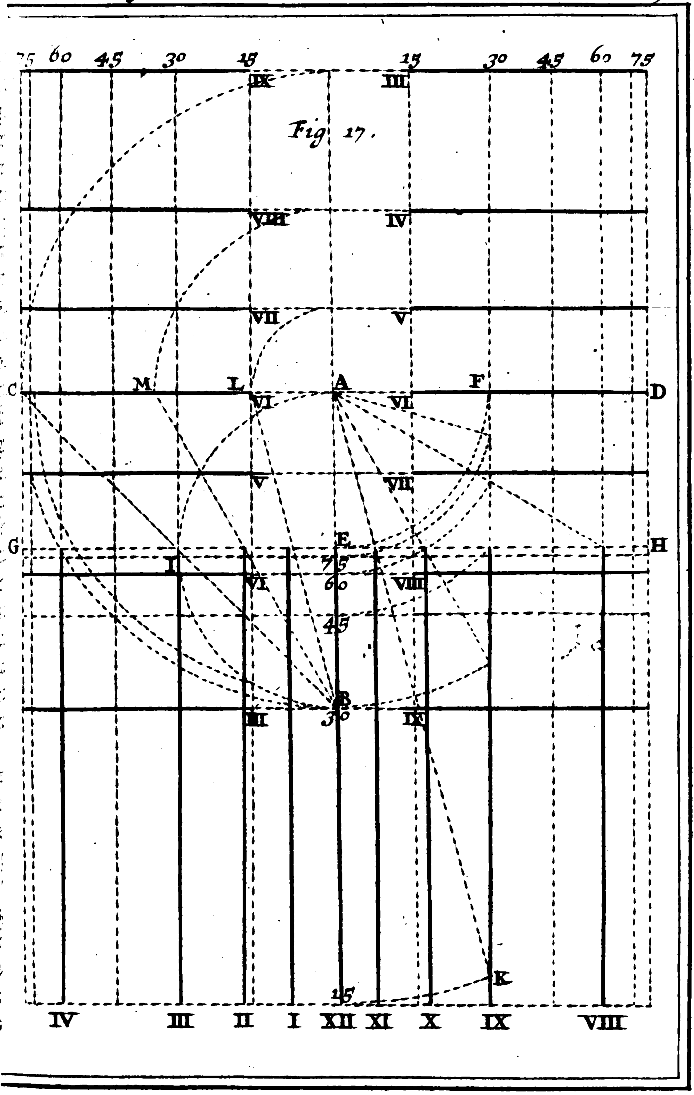


Fig. 17.
75 60 45 30 15 15 30 45 60 75
IX III
VIII IV
VII V
C M L A F D
VI VI
V VII
G E H
I 75
VI 60 VIII
45
B
III 30 IX
K
15
IV III II I XII XI X IX VIII

To find the other Circles of Latitude, and the other hour-lines to supply the defects of those that were omitted, describe from the Point B thro' the Center A the Semicircle AIB, and divide its Circumference into six equal Parts, from 30 to 30 degrees, in order to describe from the Center A through the Division Points, Arches of a Circle that will mark Points upon the Line of six hours, thro' which Points you must draw Lines parallel to the Meridian AB, which will represent Circles of Latitude of 15 degrees each.

To describe the hour-lines that correspond to the Circles of Latitude, and ought to be parallel to the Line of six hours, such as is the Line of 3 and 9 hours, which passes thro' the Point B, and represents the 30 Circle of Latitude with respect to the first hour-lines, draw from the Point B thro' the Points of Division of the Semicircle AIB, straight Lines which being prolong'd will give upon the Line of six hours the Points L, M, C; the distances of which AL, AM, AC, being taken upon the Meridian Line AB on each side the Center A, you will then have the Points thro' which you're to draw Lines parallel to the Line of six hours.

You may know the hours of the Day in this Universal Dial, after the same manner as in that last describ'd, *viz.* by turning the Center A directly South, and putting at the same Center A an Axis rais'd upon the Meridian to the extent of the Angle of the Latitude of the place; for then the shadow of that Axis will point to the hour upon the Line of the same Latitude.

There is yet another and an easier way of describing an Universal Rectilineal Dial upon an Horizontal Plain, *viz.* Having drawn, as above, through the Center of the Dial A, the two perpendicular Lines AB, GD; and having drawn thro the Point 90 taken at discretion upon the Meridian AB, the Line EF perpendicular to the same Meridian; describe from the Center A thro' the Point 90 the Semicircle C90 D, which here cuts the Line of six hours CD at the two Points C, D; thro' which and thro' the Point 90 you're to draw the straight Lines, C90, D90. Divide the Circumference of this Semicircle into twelve equal Parts, of 15 degrees each; and draw from the Center Plate 10. Fig. 18.

ter A thro' the Points of Division, straight Lines which will mark Points upon each of the two Lines C90, D90; and thro' these Points you're to draw the hour-lines parallel to the Meridian. These same Lines that go from the Center A being prolong'd will meet the Line EF in Points, thro' which you must draw from the Center A, Arches of Circles, which will mark upon the Meridian Line, the Points 30, 45, 60, 75; and from these Points to the two Points C and D you must draw as many Right Lines, which will represent the Circles of Latitude from 15 to 15 Degrees. The Dial being thus finish'd, you'll find the hour of the Day by it, as in the foregoing.

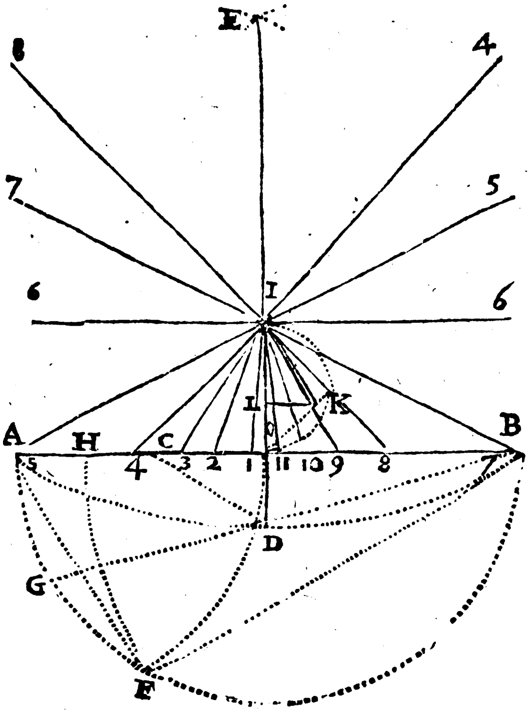
Remark. To make a particular Dial serve as Universal.

An Horizontal Dial calculated for any particular Latitude whatsoever, may be rendred Universal, two ways, namely by means of the Hour-lines, and by means of the Equinoctial Line divided into hours.

The first is perform'd by raising the Plain of the Horizontal Dial above the Horizon of the place where 'tis, towards the North if the Latitude of the place is greater than that for which the Dial was made, or towards the South if 'tis less; by raising it, I say, to the extent of the Degrees of the difference of the two Latitudes; and then the Axis of the shadow IK will shew the hours by the Rays of the Sun, when the Center I is turn'd due South.

In

In the second Method we place at the Point O, the Section of the Meridian DI, and the Equinoctial AB, we place there (I say) a small perpendicular Plain like the Rightangled Triangle OKL, which must be movable round the Point O, in such manner that the side OK may make with the Meridian OL



(which must be slit in that part) an Angle equal to the Complement of the Elevation of the Pole upon the Horizon of the place where it is; for then the shadow of the Axis KI will shew the hour upon the Equinoctial AB, the Center I being turn'd due South.

PRO-

PROBLEM XV.

To describe an Universal Elliptick Horizontal Dial.

Plate 8. Fig. 19.

HAving drawn, as in the foregoing Problem, from the Center of the Dial A taken at discretion upon the Horizontal Plain, the two perpendicular Lines, AB, CD; and having drawn upon the same Center the Semicircle CBD of what size you will; divide its Circumference into twelve equal Parts, of 15 degrees each, and joyn the two opposite Points of Division that are equidistant from the Line of six hours CD, by Right Lines perpendicular to the Meridian AB, or parallel to the Line of six hours CD, which will represent the other hour-lines, and upon these hour-lines you're to mark the Points of Latitude, thus;

To mark upon each hour-line, the Point, for example, of the 60 degree of Latitude, make at the Center A with the Meridian AB and the Line AE, an Angle of 60 degrees; and take the length of the perpendicular distances of the Points in which the Meridian is cut by the hour-lines from the Line AE; take this length, I say, upon the opposite hour-lines, from the Meridian AB on each side of it, in Points, which must be joyn'd by a Curve-line which will be the Circumference of a Semi-Ellipsis, and will represent the 60 Circle of Latitude. Thus 'tis, that we have represented the other Circles of Latitude, from 15 to 15 degrees, by which with the Rays of the Sun the hour of the Day may be known as above.

PROBLEM XVI.

To describe an Universal Hyperbolick Horizontal Dial.

Plate 10. Fig. 20.

HAving drawn, as above, from the Center of the Dial A, the two perpendicular Lines AB, CD, and having likewise drawn, as above, upon the same Center A, the Semicircle EFG divided into twelve equal Parts, of 15 degrees each; draw from the Center A through the Points of Division indefinite Lines, within

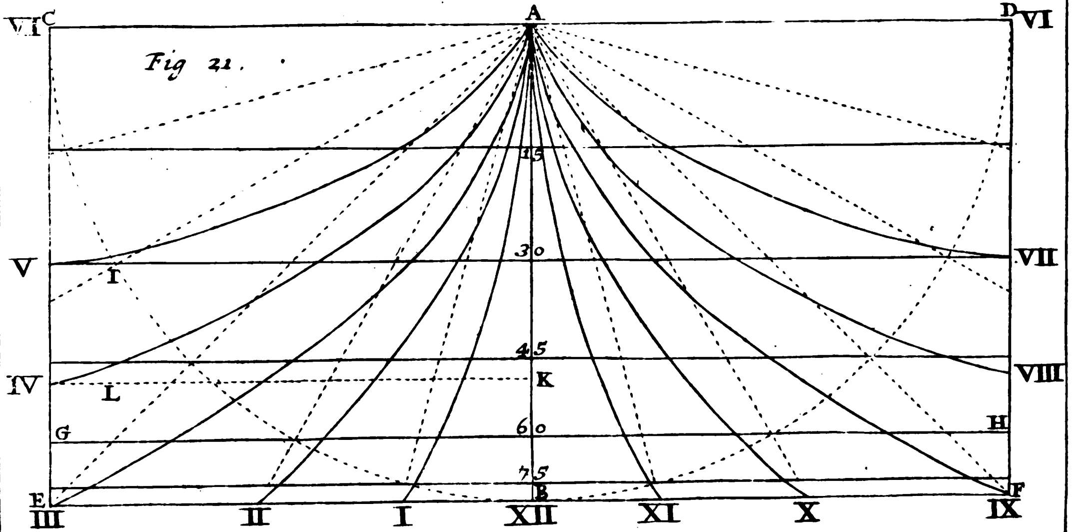


Fig 21.

within which, as between Asymptotes, you must describe thro' the Point F taken at discretion upon the Meridian AB, Hyperbola's which will represent the hour-lines.

This done, draw thro' the same Point F, the Line HI perpendicular to the Meridian AB; which perpendicular will represent the 90 Circle of Latitude, and will be cut by the Asymptotes drawn from the Center A, in Points, thro' which you are to describe from the same Center A, Arches of Circles, which will give upon the Meridian Line, the Points 75, 60, 45, 30, 15; and thro' these Points you must draw as many Lines perpendicular to the same Meridian, which will represent the Circles of Latitude from 15 to 15 degrees, by which the hour will be known as in the foregoing Dial.

Those who understand the Conick Sections, know, that in order to describe an Hyperbola through the Point F between the Asymptotes, AK, AL, (for instance) they need only to draw at Discretion thro' the Point F the Line MN, terminated in M and N by the two Asymptotes AK, AL; and take MO equal to FN, and so have O for the Point of the Hyberbola that is to be describ'd, *&c.* Remark.

Those who are unacquainted with the Conical Sections, may mark the Points of the hour-lines upon each Circle of Latitude, (as we shall shew in the insuing Problem) in order to joyn the Points belonging to the same hour, by Curve-lines, which will necessarily be Hyperbola's.

PROBLEM XVII.

To describe an Universal Parabolick Horizontal Dial.

HAving drawn, as above, thro' the Center of the Dial A, the two perpendicular Lines AB, CD; draw thro' the Point B taken at Discretion upon the Meridian AB, the Line EF perpendicular to the same Meridian, which will represent the 90 degree of Latitude; and describe, as in the foregoing Problem, upon the Center A, thro' the Point B, the Semicircle CBD, which must be divided into twelve Parts, in order Plate 11. Fig. 21.

order to joyn the opposite Division Points, that are equidistant from the Line of six hours CD, by Right Lines which will represent the Circles of Latitude from 15 to 15 degrees.

Upon each of these Circles of Latitude, for instance, the Line GH, which represents the 60 degree of Latitude, we must mark the hour-points, thus. From the Point 60 the Section of the Meridian AB and the Line GH, draw an Arch of a Circle that touches the Line AI, which with the Meridian AB makes at the Center A an Angle of 60 degrees; and with the same extent of the Compasses take upon the Meridian AB, the part AK, in order to draw thro' the Point K the Line KL perpendicular to the Meridian AB. This perpendicular KL will be cut by straight Lines drawn from the Center A thro' the twelve Divisions of the Semicircle CBD; 'twill be cut, I say, in Points, the distances of which from K are to be taken upon the Line GH, on each side the Point 60; and so you have the hour-points upon the Line GH, which in this case is consider'd as an Equinoctial Line in respect of the Axis AI.

The same is the method of marking the hour-points upon the other Lines of Latitude, consider'd as so many Equinoctial Lines: And the hour-points belonging to the same hour are to be joyn'd by Curve-lines, which will represent the hour-lines, and be Parabola's, having the Center A for the common Vertex, and the Line of six hours CD for the common Axis. The hour is observ'd upon this Dial, as upon the foregoing.

PROBLEM XVIII.

To describe a Dial upon an Horizontal Plain, in which the hour of the Day may be known by the Sun without the shadow of any Gnomon.

THIS Dial is commonly made two ways, *viz.* by the Table of the Verticals of the Sun from the Meridian to every hour of the Day, in the beginning of each sign of the Zodiack, such as this here annex'd, which is calculated for the Latitude of 49 Degrees;

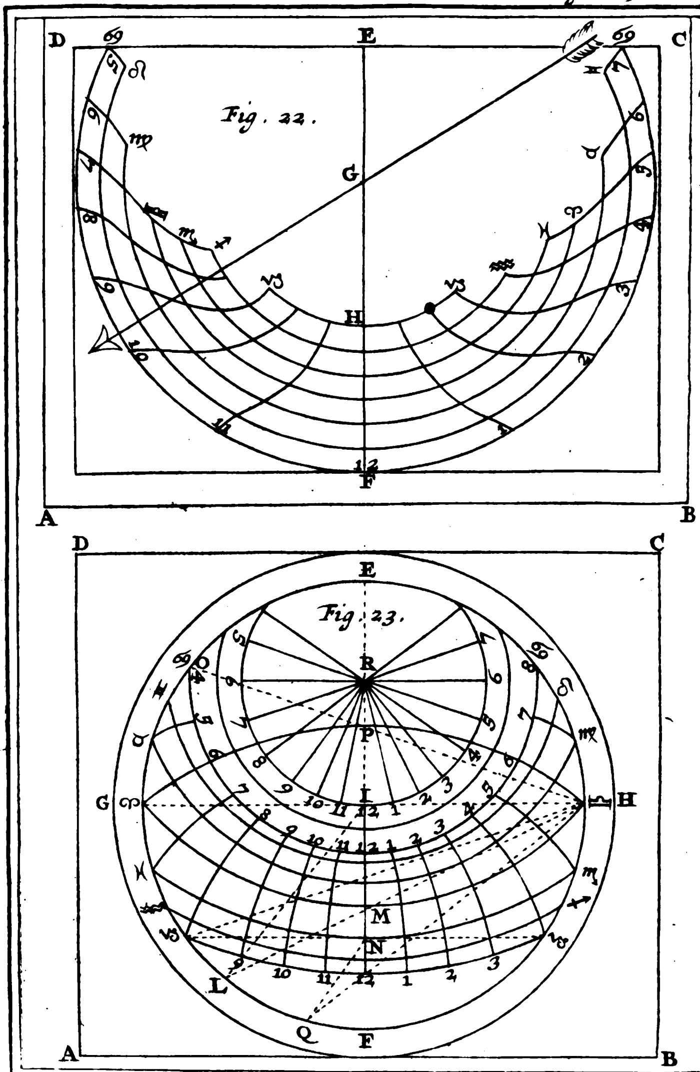


Fig. 22.
D
E
C
G
H
F
A
B
Fig. 23.
D
E
C
R
P
L
G
H
M
N
L
Q
F
A
B

degrees; or else without any Table, by the Stereographical Projection of the Sphere.

A Table of the Verticals of the Sun from the Meridian to every Hour of the Day.

H.	XI	X	IX	VIII	VII	VI	V	IV
S.	D.M	D.M.	D.M.	D.M.	D.M.	D.M.	D.M.	D.M.
♋	30.17	53.40	70.30	83.57	95.20	105.56	116.28	127.26
♌ ♊	27.58	50.33	67.34	81. 6	92.45	103.35	114.56	
♍ ♉	23.30	43.52	60.29	74.17	86.21	97.36		
♎ ♈	19.33	37.25	52.58	66.57	78.34			
♏ ♓	16.42	32.25	46.30	59.28	71.12			
♐ ♒	14.56	29.11	42.23	54.26				
♑	14.19	28. 2	40.48					
H.	I	II	III	IV	V	VI	VII	VIII

In the first place, to describe this Dial from the foregoing Table, whence 'tis call'd the *Azimuth Dial*; draw upon the Horizontal Plain, which I suppose to be moveable, the rectangle Parallelogram ABCD, and divide each of the two opposite sides, AB, CD, into two equal parts, at the Points E, F, which ought to be join'd by the right Line EF, that is to be taken for the Meridian; and upon that Meridian you are to take at discretion the Point G for the Root of the Gnomon, and the Points F, H, for the Solstice-Points of ♋ and ♑; thro which you must describe upon the Point G as Center, two Circumferences of a Circle for representing the Tropicks or the beginnings of ♋ and ♑. Plate 12. Fig. 22.

To represent the Parallels of the beginnings of the other Signs, divide the Space FH into six equal parts; and from the same Point G draw thro the Points of Division, other Arches of Circles to represent the beginnings of the Signs; and mark upon these Arches the Points of the Hours, by taking upon them the Degrees of the Vertical of the Sun (as they stand in the foregoing Table) every Hour of the Day from the beginning of the respective Sign: These degrees must be taken upon the Arches on each side the Meridian Line

EF, and the Points belonging to one Hour must be join'd by Curve-Lines, which will be the Hour-Lines. The Dial being thus finish'd, you may know the Hour of the Day without a Gnomon, after the following manner.

Apply to the Center G of the Arches of the Signs a magneted Needle rais'd upon a small Hinge, with freedom of Motion in turning round, as in the common Sea-Compasses; and turn the Point E directly to the Sun; so that each of the two sides, AD, BC, which are parallel to the Meridian Line EF, ceases to be shone upon by the Sun without giving any Shadow; for then the Needle will point to the Hour upon the degree of the Sign current of the Sun.

Dials made by the Stereographical Projection of the Sphere.

To describe this Dial by the Stereographical Projection of the Sphere, in which case it assumes the Name of an *Horizontal Astrolabe*; draw thro the Center I of the Square ABCD, the two perpendicular Lines EF, GH; one of which, as EF which is parallel to the side AD, being taken for the Meridian, the other GH parallel to the side AB will represent the first Vertical, because the Point I represents the Zenith; from which as from a Center, you're to draw at discretion the Circle E♈F♎, which will represent the Horizon.

Plate 12. Fig. 23.

Upon the Circumference of this Circle, take on one side the Arch EO of the Elevation of the Pole upon the Horizon, and on the other side, the Arch FL of the complement of the same Elevation of the Pole; and draw from the Point ♎ to the Points, O, L, the straight Line ♎O, (which will give upon the Meridian the Pole in P; thro which and thro the two Points ♈, ♎, you must run the Circumference of a Circle to represent the Circle of six Hours;) and the straight Line ♎L, which will give upon the Meridian the Point M, thro which and thro the two Points ♈, ♎, you must describe another Circumference ♈M♎, for the Æquator.

This Circle or Æquator ♈M♎ might be divided into Hours, from 15 to 15 degrees, by the Rules of the Stereographical Projection; by taking two Points diametrically opposite, and describing Circumferences through the Pole; but a shorter way, is, to take upon the Horizon E♈F♎, on each side, from the two Points, E, F, the Arches of the Horizon comprehended between the Meridian Circle, and the Hour-Circles

Circles, which are equal to the Angles made by the Hour-lines with the Meridian at the Center of an Horizontal Dial, and which in the Latitude of 49 degrees ought to be, 11. 26′. for 1 and 11 Hours; 23, 33′. for 2 and 10; 37. 3′. for 3 and 9; 32, 35′. for 4 and 8; 70. 27′. for 5 and 7. By this Direction we may describe Hour-lines or Circles, as above, which are only needful to be drawn between the two Tropicks; which together with the Parallels of the other Signs of the Zodiack, may be describ'd, thus:

To describe Parallels of the Signs, make use of their Declination, which is 23. 30′. for ♋, ♐; 20. 12′. for ♊, ♌, ♒, ♐; and 11. 30′. for ♉, ♍, ♓, ♏. By this their Declination you may find three Points of each Sign, one upon the Meridian EF, and two upon the Horiozn E♈F♎; and so describe thro these three Points a Circumference of a Circle for the Parallel of the respective Sign.

Now to find these three Points, for Example, for the Tropick of ♑; take from L which answers to the Equinoctial M, towards F (the Sign being Southern, for if 'twere Northern, you should take from L towards ♈) the Arch LQ of 23. 30. such being the Declination of ♑, and draw from the P oint ♎ to the Point Q, the straight Line ♎Q, which will give upon the Meridian EF, the Point 12 of ♑. If from the Point Q you draw the Line QN parallel to LF; and if thro the Point N where the Line QN cuts the Meridian, you draw the Line ♑N♑ perpendicular to the same Meridian, you'll have upon the Horizon E♈F♎, the two Points, ♑, ♑, thro which and thro the Point 12 you are to describe the Arch ♑12♑ which will represent the Tropick of Capricorn.

The same way do we represent the Parallels of the other Signs; and the Dial being thus finish'd we know the Hour of the Day as in the foregoing Dial, or else by raising at the Point I a very straight Style of what length you will, and turning the Point E directly to the Sun; for then the Shadow of the Gnomon points to the Hour upon the Sign current of the Sun. Or else thus:

Describe upon the same Meridian EF a common Horizontal Dial, the Center of which may be R, for example; and there put an Axis that rests upon the

Gnomon rais'd perpendicular at I; and turn the Plain of the Dial, so, that the Shadow of the Axis may show in its Dial the same Hour, that the Shadow of the Gnomon does in its own.

PROBLEM XIX.

To describe a Moon-Dial.

Plate 13. Fig. 24. TO describe a Moon-Dial upon any Plain whatsoever, for example an Horizontal Plain; draw upon that Plain an Horizontal Sun-Dial for the Latitude of the Place, according to *Probl.* 2. then draw at pleasure the two Lines 57, 39, parallel to one another and perpendicular to the Meridian A12; the first of which 57 being taken for the Day of full Moon, the second 39 will represent the Day of new Moon, when the Lunar Lines agree with the Solar; from whence it comes that the Hour-points mark'd upon these two Parallels by the Hour-lines, which go from the Center A, are common to the Sun and Moon.

This done, divide the Space terminated by the two parallel Lines 39, 57, into twelve equal parts, and draw thro the Division-Points as many Lines parallel to these two Lines, which Parallel Lines will represent the Days when the Sun by its proper Motion towards the East, removes successively by an Hour a Day, and on which by consequence it rises an Hour later every Day; so, that the first Parallel, 4, 10, will be the Day in which the Moon rises an Hour later than the Sun, and then the Point B, for example, of 11 Hours to the Moon, is the Point of Noon to the Sun: The next Parallel 5, 11, will represent the Day on which the Moon rises two Hours later than the Sun, and then the Point C, for example, of 10 Hours to the Moon will be the Point of Noon to the Sun.

'Tis evident, that if you join by a Curve-line the Points 12, B, C, and all the others retaining to Noon, which may be found by a Ratiocination like the last, that Curve will be the Lunar Meridian Line. The same Method is to be observed in drawing the other Hour-lines for the Moon, as the bare sight of the Figure will inform you.

In

Fig. 24.

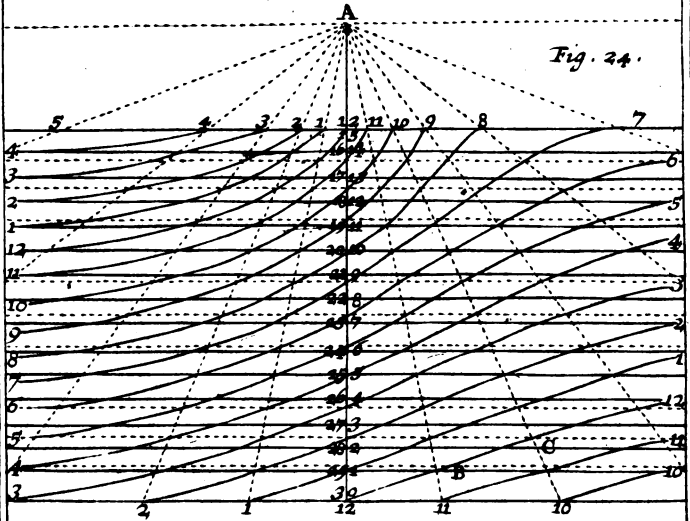
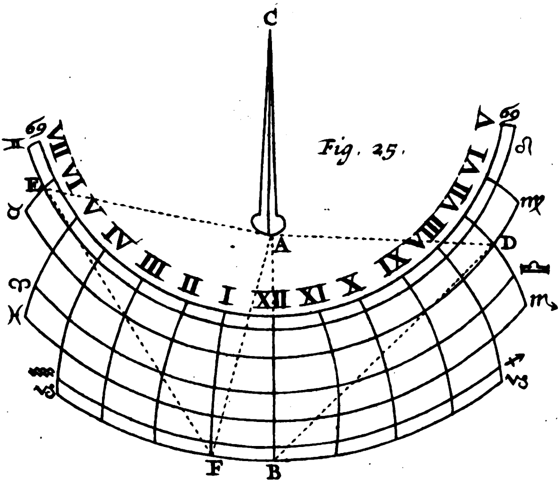


Fig. 25.



In regard the Moon ſpends about 15 Days between its Conjunction with the Sun and its Oppoſition, that is, between new Moon and full Moon when 'tis diametrically oppoſite to the Sun, ſo that it riſes when the Sun ſets; you muſt deface all the fore-going Parallels, excepting the two firſt, 57, 39, and inſtead of dividing their Interval into twelve parts, you muſt divide it into fifteen, and draw thro the Diviſion-Points, other Parallels repreſenting the Days of the Moon, to which by conſequence you muſt add ſuitable Figures, as we have done here along the Meridian; for by theſe Figures you may know the Hour of the Sun at night by the Rays of the Moon after the following manner:

Place at the Center of the Dial A, an Axis, that is, a Rod that at the Center A makes with the Subſtylar A 12 an Angle equal to the Elevation of the Pole upon the Plain of the Dial, which is the ſame, with the elevation of the Pole upon the Horizon in an Horizontal Dial; and then the Hour will be pointed to by the Shadow of the Axis upon the current Day of the Moon.

Since the Moon by its proper motion removes from the Sun three quarters of an Hour towards the Eaſt, ſo that it riſes every day three quarters of an hour later than the foregoing day; 'tis evident that knowing the Age of the Moon, you may with a common Sundial know the Hour of the Night by the Rays of the Moon, *viz.* by adding to the Hour mark'd upon the Dial by the Moon, as many times three quarters of an hour, as the Moon is days old. Now the Age of the Moon is found by the Rules laid down in our *Problems of Coſmography.* Remark.

PROBLEM XX.

To deſcribe a Dial by Reflection.

TO deſcribe a Dial upon a dark Wall or arch'd Roof that will ſhew the Hours by Reflection, draw a Dial upon an Horizontal Plain expos'd to the Rays of the Sun, in a Window, for inſtance, in ſuch manner that the center of the Dial looks directly North, and the Hour-lines have a contrary Situation to that of the

the common Sundials. A Dial being made after this manner, with a little straight Gnomon fitted, lay a Thread upon any point of each Hour-line, and extend it tight till it passes the end of the Gnomon and meets the Wall or Vault in a Point which will belong to the Hour that the Thread was apply'd to. Find by the same means as many other Points of each Hour-line, and join them by a right or curve Line, and the Dial is finish'd; upon which you'll know the Hours by Reflection, by placing at the end of the Gnomon of the Horizontal Dial, a small flat piece of Looking-Glass, laid exactly horizontally; or, which is the easier way, by putting instead of the Glass, Water, which naturally affects an Horizontal Situation, besides that when the Rays of the Sun are weak, 'twill by its motion give a more distinct Reflection upon the Wall or Plank where the Dial is.

PROBLEM XXI.

To describe a Dial by Refraction.

ONE may easily describe an Horizontal Dial by Refraction in the bottom of a Vessel full of Water, by the Table of the Verticals of the Sun inserted above, *page* 289, together with the Table of the Altitudes of the Sun given likewise above and the following Table, the first Column of which to the left contains the Angles of Inclination of the Rays of the Sun, that is, the degrees of the Complement of the Sun's height upon the Horizon, or of the distance of the Sun from the Zenith, to which there correspond in the second Column, the degrees and minutes of the Angles refracted in Water, that is, the diminution of the Angles of Inclination made in Water, when the Sun is remov'd so many degrees from the Zenith, which shortens the Shadow of the Gnomon that is to be cover'd with Water in order to know the Hours by the Rays of the Sun.

A Table

A Table of the Angles refracted in Water for all the Degrees of the Angles of Inclination.

A.	D. M	A.	DM.	A.	D.M.
1	0.46	31	33.38	61	42.52
2	1.33	32	24.41	62	43.23
3	2.20	33	25. 4	63	43.53
4	3. 7	34	25.47	64	44.21
5	3.54	35	26.30	65	44.50
6	4.40	36	27.13	66	45.17
7	5.27	37	27.55	67	45.44
8	6.13	38	28.37	68	46.10
9	7. 0	39	29.19	69	46.34
10	7.46	40	30. 0	70	46.58
11	8.20	41	30.41	71	47.21
12	9.18	42	31.22	72	47.43
13	10. 4	43	32. 2	73	48. 3
14	10.50	44	32.42	74	48.23
15	11.36	45	33.22	75	48.43
16	12.22	46	34. 2	76	49. 1
17	13. 9	47	34 41	77	49.17
18	13.55	48	35.19	78	49 33
19	14.40	49	35.57	79	49.47
20	15.25	50	36.35	80	50. 0
21	16.11	51	37.12	81	50.12
22	16.57	52	37.47	82	50.23
23	17.42	53	38.24	83	50.32
24	18.27	54	39. 0	84	50.41
25	19 12	55	39.35	85	50.48
26	19.56	56	40. 9	86	50.54
27	20.40	59	40.43	87	59.58
28	21.25	58	41.17	88	51. 1
29	22.10	59	41.46	89	51. 3
30	22.54	60	42.21	90	0. 0

Now the Dial to be thus used, is made after the following manner. Having drawn from the Root of the Gnomon A the Meridian Line AB, mark upon that Meridian the Points of the Signs; for example, the Plate 13. Fig. 25.

 Point

Point of the beginning of ♑, from the foregoing Table of the refracted Angles, and the Table of the Altitudes of the Sun upon the Horizon, by drawing from the Root of the Gnomon A the Line AD perpendicular to the Meridian AB, and equal to the Gnomon AC; and by making at the Point D the Angle ADB of the refracted Distance from the Zenith, which in the beginning of ♑ is at Noon about 48 degrees; making this Angle, I say, with the Line DB, which will mark upon the Meridian the Point B of ♑. And so of the rest.

To find the refracted distance of the Sun from the Zenith, look first upon the Table of the Altitudes of the Sun, where you find that in the beginning of ♑ the Sun at Noon is rais'd upon the Horizon 17. 29′ and consequently is distant from the Zenith 72. 31′. and taking this distance for an Angle of Inclination, you'll find by the Table of refracted Angles, That this Angle of Inclination is changed into an Angle of 48 degrees for the refracted Distance of the Sun from the Zenith.

The same is the method of finding by these two Tables, the refracted distance of the Sun from the Zenith in the beginning of any other Sign, and that not only at Noon, but at the other Hours of the Day; which will direct you to find the Points, and at the same time, the Points of the Signs from the Table of the Sun's Verticals, after the following manner.

To find, for example, the Point of the beginning of ♑ and of 1 a-clock, at which time the Sun is on a Vertical distant from the Meridian 14. 19′. make with the Meridian AB at the Root of the Gnomon A the Angle BAF of 14, 19′. by the Line AF which represents the Sun's Vertical. And having drawn from the same Root A, the Line AE perpendicular to AF and equal to the Gnomon AC, make at the Point E the Angle AEF equal to the refracted distance of the Sun from the Zenith, which will be found 48, 18′. And so you have in F upon the Vertical AF, the Point of 1 a-clock and of ♑.

By the same procedure you'll find the other Points of the Signs and other Hours; and if you joyn with a Curve Line those which retain to the same Hour, and in like manner those retaining to the same Sign, the

the Dial is finiſh'd; upon which you'll know the Hours by Refraction, when the whole Gnomon AC is cover'd with Water, and the Root of the Gnomon is turn'd directly South, ſo that the Point B ſets North; and at the ſame time the end of the Shadow of the Gnomon denotes the Sign in which the Sun is.

PRO-

PROBLEMS OF COSMOGRAPHY.

COSMOGRAPHY, according to its Etymology, is the Descciption of the World, *that is*, of Heaven and of Earth. 'Tis divided into the *General*, which considers the whole Universe in general, and advances the several Ways of describing and representing it, according to the divers Sentiments of Philosophers and Mathematicians: And the *Particular*, which is properly call'd *Geography*, because it represents in particular every part of the World, and especially the Earth, both in Globes and Planispheres and Maps of the World. I do not pretend upon this Occasion to write a particular Treatise of these two Parts; but only to lay before you some useful and agreeable Problems that depend upon 'em.

PROBLEM I.

To find in all parts and at all times, the four Cardinal Points of the World, without seeing the Sun, or the Stars, or making use of a Compass.

THE Four Cardinal Parts of the World, *viz.* the East, the West, the South and the North, are easily found by a Compass, the Needle of which being touch'd with a Loadstone, turns always one of its Points towards the South and the other towards the North, which is enough to direct us to East and West; for

for when one sets his Face to the North, the East is on his Right and the West on his Left hand.

The North is easily distinguish'd in the Night by the Stars, particularly by minding the Polar Star which is but two degrees distant from the Arctick Pole: And in the Day-time Astronomers mark the Meridian Line upon an Horizontal Plain, by means of the two Points of a Shadow mark'd before and after Noon upon the Circumference of a Circle describ'd from the Point of the Stylus, the Shadow of which is made use of to shew by its extremity upon that Circumference two Points equally remote from the Meridian.

But without all these Helps you may at all times and in all parts find out the Meridian Line, after the following manner.

Take a Platter or Basin full of Water, and when the Water is settled and still, put softly into it an Iron or Steel Needle, such as a common sewing Needle; and if the Needle is dry, and be laid all along upon the Surface of the Water, 'twill not sink; but after several turns will stop in the Plan of the Meridian Circle, so that it represents the Meridian Line; and by consequence one end of it will point to the South and the other to the North: But without seeing the Sun or the Stars, 'tis not easy to know which of the two ends points to the South, and which to the North.

Father *Kircher* lays down an easy way of knowing South and North. He orders you to cut horizontally a very straight Tree growing in the middle of a Plain at a distance from any Eminence or Wall that may shelter it from the Wind or the Rays of the Sun: In the section of that Trunk you'll find several curve Lines round the Sap which lie closer on one side than t'other; And, as he says, the North lies on that side where the Lines are most contracted, perhaps because the Cold arising from the North binds up, and the Heat from the South spreads and rarifies the Humours and Matter, of which these crooked Lines are form'd. These Lines, says that Author, are as the Circumferences of concentrical Circles in Ebony or Brasil Wood.

PRO-

PROBLEM II.

To find the Longitude of a propos'd Part of the Earth.

BY the Longitude of a place we underftand the diftance of its Meridian from the firft Meridian, which paffes thro the Ifland *de Fer*, the moft Weftern of the Canary Iflands. This diftance is computed from Weft to Eaft upon the Æquator, in imitation of the motion in Longitude of the Planets, which is likewife from Weft to Eaft, and is computed upon the *Deferens* of each Planet, which is call'd Excentrick, becaufe they fuppofe it to be excentrical to the Earth, for the explication of the *Apogæum* or the remoteft ftation of the Planet from the Earth, and the *Perigæum* or its neareft place of approach to the Earth.

In the Maps of the World or the general Maps, we have the degrees of Longitude mark'd upon the Æquator from 10 to 10 degrees, reckoning from the firft Meridian Eaftward to 360 degrees; fo that the firft Meridian is the 360th Meridian, the Geographers having thought it fit fo to compute their terreftrial Longitude, as the Aftronomers did their Celeftial upon the Ecliptick, from the Vernal Section, *i. e.* from the beginning of the Conftellation of *Aries*, where the Æquator and the Ecliptick cut one another, with refpect to the fix'd Stars.

'Tis evident that thofe who are under the fame Meridian, have the fame Longitude; and that thofe that are under the firft Meridian, have no Longitude at all; and in fine, that thofe who live more to the Eaftward are under different Meridians, and then the diftance of one Meridian from another is call'd *Difference of Longitude*; which gives us to know how much fooner 'tis Noon at one place than at another that lies more Weft; it being a ftanding Rule, that when the difference of Longitude comes to be 15 degrees, 'twill be Noon an Hour fooner in the Place that lies fo far more Eaft than the other, becaufe 15 degrees upon the Æquator make an Hour, the whole 360 making 24 Hours or a diurnal Circumvolution.

Thus we fee that in order to know the Longitude of any

any part of the Earth, we need only to know what Hour of their Computation corresponds to the Hour computed at the same time under the first Meridian; for if you convert that difference of Hours into Degrees, taking 15 Degrees for an Hour, 1 Degree for 4 Minutes of Time, and 1 Minute of Degrees for 4 Seconds of Time, you have the Longitude of the Place propos'd. To know this difference of Hours, you may make use of some visible Sign in the Heavens, observ'd at the same time by two Mathematicians, one under the first Meridian, the other under the Meridian of the Place propos'd. The Ancients for this end made use of the Eclipses of the Moon, and at present regard is had to the Eclipses of the first of the Satellites of *Jupiter*, which happen oftner, and whose Immersions or Emersions are with more facility observ'd with a Telescope.

When once you have discover'd the Longitude of a Place, you have no further Occasion to have recourse to the first Meridian for the Longitude of any other place, it being sufficient to know how far that Place is more to the East or West than the Place you know already. Neither is there any occasion for two Mathematicians, for making the Observation last mention'd, since one Man can observe in the place where he is, the Hour of the Emersion or Immersion of the Satellites, and compare that with the Hour of the Place whose Longitude he knows, set down in Mons. *Cassini's* Tables; for these Tables shew the Hour at *Paris* of the Immersion or Emersion.

From what has been said, we may learn the truth of that Paradox, *Quælibet Hora est omnis Hora*, which shou'd be understood of Places under different Meridians; for 'tis certain that when 'tis Noon at *Paris*, 'tis an hour after Noon at *Vienna* in *Austria*, and in all the other places that lie 15 degrees more East than *Paris*; at *Constantinople* 'tis two a-clock in the Afternoon, and so on. Remark.

Hence it follows that of two Travellers, one going West observing the Course of the Sun, and the other East contrary to the Course of the Sun, the first must have longer Days than the second, insomuch that after a certain time the second that goes Eastward will have reckon'd more Days than he that goes Westward. And

This

This gave rise to the story of two Persons that were Twins, one of whom travell'd to the East and the other to the West, and tho they both died at one time the one had liv'd more Days than the other.

As the Latitude is divided into Septentrional and Meridional, extending to 90 degrees towards the two Poles, on one side and t'other of the Æquator; so Longitude might have been divided into Oriental and Occidental, extending 180 degrees on one side and t'other the first Meridian: Which wou'd be very convenient to let us know, for example, that when 'tis Noon under the first Meridian, 'tis but 8 a-clock in the Morning in the Island of *Cuba*, the Western Longitude of which, is 60 degrees.

PROBLEM III.

To find the Latitude of any Part of the Earth.

BY *Latitude*, with respect to the parts of the Earth, we mean the distance of the Place propos'd from the Æquator, which is measured by an Arch of the Meridian of that Place between its Zenith and the Æquator. This Arch is always equal to the Elevation of the Pole, which is an Arch of the same Meridian between the Pole and the Horizon; and hence it comes that commonly Latitude is confounded with the Elevation of the Pole; so that those who have no Latitude, *i. e.* who live under the Æquator, have no Elevation of the Pole, the two Poles of the World being at their Horizon.

The Latititude of any Place of the Earth may be known at Noon time of Day by the Meridian Altitude of the Sun and its Declension; and in the Night time by the Meridian Altitude of some fix'd Star and its Declension, and even without its Declension, when the Star do's not set, and the Night is longer than 12 Hours, as I am about to shew you.

To find the Latitude of any Place by the Meridian Altitude of the Sun, add to that Meridian Altitude, the Declension of the Sun, if the Declension is meridional, which it is from the Autumnal to the Vernal Equinox; or if the Declension is Northern, which it is from

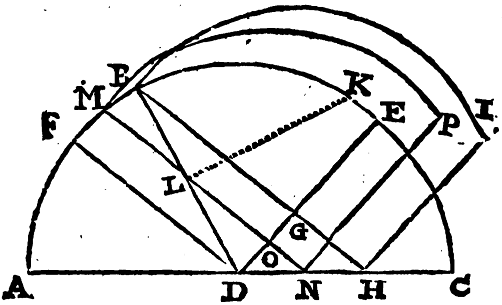
from the Vernal to the Autumnal Æquinox, substract it from the Meridian Altitude: And thus you have the Height of the Æquator, which substracted from 90 degrees, leaves remaining the Latitude sought for.

The same is the Operation in the Night time with respect to the Stars that are Southern or Northern without setting, as in the case of the Stars near the Pole that's elevated above the Horizon. As soon as Night is come take the Meridian Altitude of such a Star, and 12 Hours after in the Morning take the Meridian Altitude of the same Star; then add these two Altitudes together, and half the sum gives the Elevation of the Pole upon the Horizon.

PROBLEM IV.

To know the Length of the longest Summer Day at a certain Place of the Earth, the Latitude of which is known.

TO know, for example, at *Paris*, where the Elevation of the Pole is about 49 degrees, the longest Summer Day, which is of equal extent to the longest Win-



ter Night. Describe at pleasure from the Center D the Semicircle ABC, and upon one side of it take the Arch CE of the Elevation of the Pole upon the Horizon, which

which in this Example is 49 degrees; and on the other side the Arch AF of the Complement of the Elevation of the Pole, which in this supposition is 41 degrees; then draw from the Center D to the Points E, F, the Lines DE, DF, the first of which DE will represent the Circle of six Hours, and the second DF the Æquator, taking the Circle ABC for the Meridian of the Place propos'd, and the Diameter AC for the Horizon, according to the Rules of the Orthographick Projection of the Sphere.

This done take the Arch FB of the greatest Declination of the Sun, which is about 23 degrees and a half; and having drawn from the Point B parallel to the Line DF, the Line BH, which here cuts the Circle of six Hours at the Point G, and the Horizon at the Point H; describe from the Point G as a Center thro the Point B, the circular Arch BI, which is terminated in I by the Line HI parallel to the Line DE, or perpendicular to the Line BH. This Arch BI is here 120 degrees, or an Arch of 8 Hours, reckoning 1 Hour for 15 degrees, the double of which gives us to know that at *Paris* and at all other places where the Pole is elevated upon the Horizon 49 degrees, the longest Summer Day, or the longest Winter Night, is 16 Hours.

The Arch BI being 120 degrees or 8 Hours, shews that the Sun sets on the longest Summer Day, or rises on the shortest Winter Day, at 8 a-clock; and consequently that it rises on the longest Summer Day, and sets on the shortest Winter Day, at 4 a-clock; which happens when the Sun is in the Summer or Winter Tropick. And by the same method may we find the Hour of the rising and setting of the Sun, when 'tis in any other Sign of the Zodiack, for example, in the beginning of ♉ and of ♍, provided we know how to describe the Parallel of that Sign, which is done after the following manner.

Having drawn from the Center D which represents the Point of the Eastern and Western Æquinoctial, to the Point B, which represents the Solstice Point of ♋ or of ♑, the Line DB, which by consequence represents a Quadrant of the Eclyptick, and having taken upon the Meridian or the Colurus of the Solstices ABC, the Arch BK of 60 degrees, which is the distance of the propos'd Sign, from the beginning of ♋ represented by by the Point B, because we suppose the Colurus of the Solstices agrees with the Meridian; draw from the Point K the Line KL perpendicular to the Line DB, and thro the Point L the Line MN, which repesents the Parallel of ♉, and cuts the Horizon AC at N, and the Axis of the World DE at O; from which Point, as a Center, describe thro the Point M the Arch MS, which will be terminated in P by the Line NP parallel to the Line DE, or perpendicular to the Line MN; and this Arch NP being reduced to Hours, after knowing its degrees and minutes, will give the Hour inquir'd after.

The Arch FM is the Declination of the propos'd Sign, the distance of which to the nearest Equinox is suppos'd to be 30 degrees; the Arch DN is the oriental or occidental Amplitude of the same Sign, with respect to the Horizon AC, which we supposed to be 49 degrees oblique; and the Arch ON is the ascensional difference, which shews what space of Time, the Sun (being in the propos'd Sign) rises or sets before or after six a-clock upon the same Horizon. These Arches are Geometrically calculated in this Figure; but a more exact computation may be had by Trigonometry, after the following manner. Remark.

To know first of all the Arch FM, supposing the Arch FB or the Angle FDB, that is, the Obliquity of the Eclyptick, to be 23. 30′: Observe the following Analogy, in which we use Logarithms, these being very convenient in Spherical Trigonometry.

As the whole Sine	100000000
To the Sine of the distance between the Sign propos'd, and the nearest Equinox	96989700
So is the Sine of the Obliquity of the Eclyptick	96006997
To the Sine of the Declension sought for	92996697

Which will be found 11 degrees 30 minutes.

For the Amplitude DN, observe the Declension found but now, and make the following Analogy:

As the Sine of the Complement to the Height of the Pole	98169429
To the Sine of the Declension found	92996697
So is the whole Sine	100000000
To the Sine of the Amplitude sought for	94827268

Which will come to 17 degrees and about 41 minutes.

To find the Ascensional Difference NO, take in again the Declension found, and make the following Analogy:

As the whole Sine	100000000
To the Tangent of the Declension found	93084626
So is the Tangent of the Elevation of the Pole	100608369
To the Sine of the Ascensional Difference	93692995

Which comes to 13 degrees and 32 minutes; and these being reduced to time (by saying, if 15 degrees give 1 hour, or 60 minutes, how much will be given by 13. 32'. or 812'.) shew that the Sun, when in the beginning of ♉ or of ♍ sets at 6 a-clock and 54 minutes, and by consequence rises at 5 and 6 minutes.

PROBLEM V.

To find the Climate of a propos'd Part of the Earth, the Latitude of which is known.

BY a *Climate* we mean a space of the Earth, in the form of a Zone or Girdle, terminated by two Circles parallel to one another and to the Æquator; in which space, from the Parallel nearest the Æquator to that towards the Pole, the longest Summer's Day varies, *that is*, increases or decreases, half an Hour.

In regard the Climates are reckon'd from the Æquator, under which 'tis always twelve Hours Day and 12 Hours Night, towards one of the Poles; and those who are remote from the Æquator have above 12 Hours in their longest Day, and still the more the remoter they are

are; it follows that the first Climate terminates, where the longest Day is 12 Hours and a half; the second where the longest Day is 13 Hours, and so on, to the termination of the 24th Climate, where the longest Day is 24 Hours which happens under the Polar Arctick or Antarctick Circle, the Elevation of the Pole being there 66. 30′. Beyond that we reckon no Climates, because in advancing never so little further towards the Pole, the longest Day increases more than half an Hour; and upon that Consideration the Moderns have added to the 24 Climates above-mention'd, six of another Nature, from the Polar Circle to the Pole, in each of which the longest Day increases a whole Month.

Thus, to know in what Climate is any propos'd Place of the Earth, the Latitude of which is known; we need only to find by the fore-going Problem the longest Summer's Day, and from that substract twelve Hours; for the Remainder doubled gives the Climate. For example, at *Paris*, where the Elevation of the Pole is 49 degrees, the longest Summer's Day is 16 Hours, from which if you take 12, the Remainder is 4, the double of which 8 shews that *Paris* lies in the eighth Climate.

As the Longitudes distinguish the most Oriental or Occidental Countries, and Latitudes the bearings to South or North; so the Climates distinguish Countries by the length or shortness of their Days. For by the knowledge of the Climate, we may easily find the longest Summer's Day by an Operation contrary to the preceding, *viz.* by adding 12 to half the number of the Climate, the sum of which Addition is the quantity of the longest Day. Thus knowing that *Paris* is in the 8th Climate, I add 4 the half of 8, to 12, and so learn that 16 Hours is the measure of the longest Summer Day at *Paris*.

PROBLEM VI.

To find the Extent of a Degree of a great Circle of the Earth.

SUpposing the Earth to be round and its Center the same with that of the World, a degree of one of its Circles will answer to a degree of the like corresponding Circle in the Heavens; and so when a Person goes a degree of the Earth upon the same Meridian, directly South or North, his Zenith alters likewise to the extent of a degree in the Heavens under the corresponding Celestial Meridian; and by consequence the Elevation of the Pole is a degree alter'd. In like manner, if one travels a degree of the Earth on the Æquator directly East or West, his Zenith is a degree different from what it was under the Celestial Æquator, and consequently the Longitude is changed to the extent of a degree.

This Alteration being observ'd by the repeated Experience of several Astronomers in different parts of the Earth, we may from thence conclude that the Earth is round from South to North, and likewise from East to West; and that 'tis seated in the Center of the World, or at least in the middle of the Celestial circumvolutions. From the same Observation we learn the manner of finding in Leagues or any other Measure the quantity of a Degree of one of its great Circles, which are all equal, *viz.* by pitching upon two Places of the Earth situate under the same great Circle, for example under the same Meridian, the mutual distance of which and their respective Latitudes are exactly known; for if we substract the least of the two Latitudes from the greatest, we have the Arch of their common Meridian intercepted between the propos'd Places. By this means we learn that a certain number of Degrees and Minutes of a great Circle of the Earth, answers to a certain number of Leagues, which is sufficient to shew us the extent of a Degree of the same great Circle, and even the whole Circumference of the Earth; since we may argue by the Rule of three direct, If so many Degrees and Minutes answer to so many Leagues, how

how many Leagues will one Degree answer ; or 360 Degrees, if you want to know the whole Circuit of the Earth?

Let's suppose that two Places pitch'd upon, are *Paris* and *Dunkirk*, situate under the same Meridian, and distant from one another about 62 Parisian Leagues, of 2000 Toises each; The Latitude of *Paris* is 48. 51′. which substracted from that of *Dunkirk*, *viz*. 51. 1′. leaves remaining 2. 10′. or 130 Minutes for the Arch of the Meridian comprehended between *Paris* and *Dunkirk*. Now I know that an Arch of a great Circle of the Earth of 130 Minutes is 62 Leagues ; and in order to know from thence how many Leagues go to a Degree or 60 Minutes of the same Circle, I multiply these 60 Minutes by 62 the distance between *Paris* and *Dunkirk*, and divide the Product 3720 by 130 (the number of Minutes of the Arch of the Meridian common to both Places) and the Quotient gives about 28 *Paris* Leagues for the extent of a Degree of a great Circle of the Earth.

I said, *about* 28 *Leagues* ; upon the consideration that the Gentlemen of the *Royal Academy of Sciences* have found by Experiments that a Degree of the Earth is 57060 Toises of the *Chatelet* measure at *Paris* ; which 57060 Toises, amount to a little more than 28 Parisian Leagues of 2000 Toises each, as appears by dividing 57060 by 2000, for the Quotient is 28, with a Remainder of 1060 to be divided by 2000, which makes about half a League.

A Toise of the *Chatelet* of *Paris* is divided into 6 Foot, and if you divide that Foot into 1440 parts, the *Rheinland* or *Leyden* Foot will contain 1390 of 'em, the *London* Foot 1350, the *Boulogne* Foot 1686; and the *Florence* Fathom 2582.

PROBLEM VII.

To know the Circumference, the Diameter, the Surface, and the Soldity of the Earth.

THO' we can't actually measure the Circumference of the Earth, by reason of the high Mountains and vast Seas, which can't be brought into a straight Line ; yet

yet we may easily adjust it by the Rules of Astronomy; as well as its Diameter, Surface and Solidity, by the Principles of Geometry; as I am now about to shew you.

First of all, to know the Circumference of the Earth; having found by the foregoing Problem that a Degree of this Circumference is 28 Parisian Leagues, we multiply these 28 Leagues by 360, *that is*, the number of Degrees of the Circumference of the Earth, and the Product gives 10080 Parisian Leagues for the Circuit of the Earth.

In the second Place; To find the Diameter of the Earth, or the distance from us to our *Antipodes*: Considering that the Diameter of a Circle is to its Circumference, as 100 to 314, or as 50 to 157, and that the Circumference of the Earth is already found to be 10080 *Paris* Leagues, I multiply these 10080 Leagues by 50, and divide the Product 504000 by 157, and the Quotient gives 3210 Leagues for the Diameter of the Earth.

In the third place; To find in square Leagues the Surface of the Earth, we need only to multiply its Circumference, 10080 Leagues, by its Diameter, 3210 Leagues, and the Product gives 32356800 square Leagues upon the Surface of the Earth.

In the last place; To find in Cubical Leagues the Solidity of the Earth, we multiply its Surface, *viz.* 32356800 square Leagues, by 535 the sixth part of its diameter (3210) and the Product brings us 17310888000 cubical Leagues for the Solidity of the Earth.

In the foregoing Operations, we over-look'd the Fractions of the Diameter of the Earth, which gave us the Surface somewhat imperfect, and the Solidity yet more imperfect. But if you want to have the Surface and the Solidity more exactly, without having recourse to the Diameter of the Earth, mind only the Circumference of the Earth, which came precisely to 10080 Parisian Leagues, and proceed as follows,

First, to find the Surface of the Earth, the Circumference of which is 10080 Leagues, multiply 10080 into it self, and thus you have its Square 101606400, and that multiplied by 50 gives the Product 5080320000, which Product divided by 157 yields in the Quotient 32356814 square Leagues for the Surface of the Earth.

Again,

Again, for the Solidity of the Earth, multiply its Circumference 10080 into it self, and so you have its Square 101606400, which multiplied by the Circumference again gives its Cube 1024192512000, and this multiplied once more by 1250 yields the Product 1280240640000000 ; and this Product divided by 73947 gives in the Quotient 17312949004 Cubical Leagues for the Solidity of the Earth.

COROLLARY I.

Since the Circumference of the Earth is 10080 Leagues, we may readily infer from thence, that if the Earth moves round its Axis from West to East, so as to finish its Circumvolution in the space of 24 hours, a place upon the Earth situate under the Æquator which is a great Circle, must run by vertue of the Motion of the Earth 420 Leagues in an Hour ; for 10080 divided by 24, yields 420 in the Quotient : And 420 divided by 60 yields in the Quotient 7, which shews that the place proposed must run 7 Leagues in a Minute of Time.

COROLLARY II.

Since the Diameter of the Earth is 3210 Leagues, we conclude that its Semidiameter or the distance of its Center from its Surface is 1605 Leagues. *i. e.* the half of 3210. From whence this Consequence do's naturally arise, That, If 'twere possible to dig a deep Well to the Center of the Earth, the depth of that Well wou'd be 1605 Leagues or 3210000 Toises, as appears by multiplying 1605 by 2000, the number of Toises in a Parisian League.

COROLLARY III.

Since a Well as deep as the Center of the Earth wou'd be 3210000 Toises, in depth, we may from thence calculate the time that a Stone or any other Body thrown from the surface of the Earth down this Well, which is supposed to be empty, we may calculate, I say, the time that a Stone thus thrown wou'd spend in reaching to the bottom, provided we do but know by any solid Experiment in what measure of time a heavy Body falling freely in the Air, flies thro a determin'd space.

Suppose we that in a Minute of Time a heavy Body is found to descend 100 Toises; now to find the time requisite for descending 3210000 Toises in the same Medium, we multiply 3210000 by the square of the Time known, *that is*, 1 the square of 1 Minute; and divide the Product 3210000 by 100 the space run thro in a Minute; and the Quotient is 32100, the square Root of which is 179 Minutes, which make almost 3 Hours, for the Time that the same heavy Body will imploy in descending to the Center of the Earth.

Remark. Here we shall observe by the bye, that if this Well were continued to the Antipodes, so as to make a thorough Perforation in the Earth, the Body thrown down the Well from the surface of the Earth, wou'd not stop on a sudden at the Center of the Earth, tho indeed that be the lowest place; for the great velocity of the Motion with which 'tis carried to the Center, wou'd throw it beyond the Center, and make it reascend towards the Antipodes with a Motion that wou'd gradually slacken, and near the Surface of the Antipodes part of the Earth wou'd entirely cease, upon which the Body wou'd fall back again and over-reach the Center of the Earth advancing towards us; insomuch that for some time abstracting from the resistance of the Air, this heavy Body wou'd continue to move to and fro, by several Vibrations, almost of equal duration, tho' still lesser and lesser, till at last the *Mobile* finds a Resting-place in the Center of the Earth.

All we have said of the mensuration of the Earth, goes upon the supposition of its being perfectly round; tho' indeed strictly speaking 'tis not so, by reason of the height of the Mountains, which is only considerable with respect to us, for with respect to the Earth it self 'tis very inconsiderable; as appears from the following Table taken by Father *Kircher*, which lays down in Geometrical Paces the heighth of the most considerable Mountains in the World, as far as we are able to judge of it by the length of their Shadows.

Mount

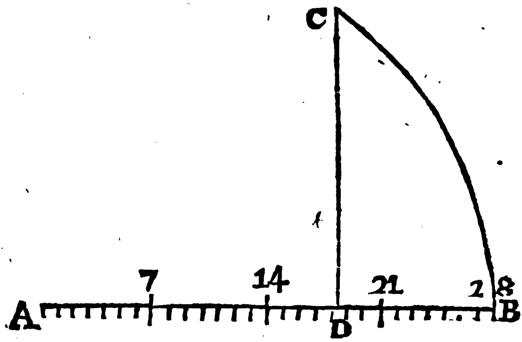
Mount *Pelion* in *Thessaly*	1250
Mount *Olympus* in *Thessaly*	1269
Catalyrium	1680
Cyllenon	1875
Mount *Ætna*, or Mount *Gibel* in *Sicily*	4000
The Mountains of *Norway*	6000
The *Peek* of the *Canaries*	10000
Mount *Hemus* in *Thrace*	10000
Mount *Atlas* in *Mauritania*	15000
Mount *Caucasus* in the *Indies*	15000
The Mountains of the Moon	15000
Mount *Athos* between *Macedonia* and *Thrace*	20000
Stolp, the highest of the *Ryphean* Mountains in *Scythia*.	25000
Cassius	28000

PROBLEM VIII.

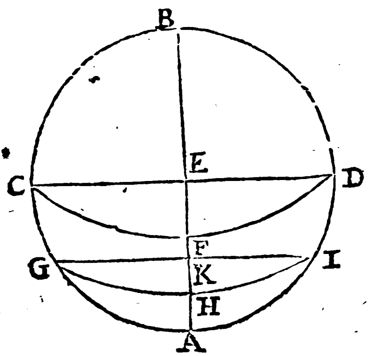
To know the extent of a Degree of a propos'd small Circle of the Earth.

AFter finding by the 6th Problem the extent of a Degree of a great Circle of the Earth, you may easily take the measure of a Degree of a small Circle, for example of a Circle parallel to the Æquator, which we commonly call barely a *Parallel*; this, I say, you may easily measure, provided the distance of the Parallel from the Æquator is known. This is of use to Geographers imployed in drawing Chorographical Maps and laying down the distances of two Places of the Earth situate under the same Parallel, *that is*, equidistant from the Æquator.

Suppose



Suppose one wants to know the measure of a Degree of the Parallel of *Paris*, which is about 49 Degrees distant from the Æquator, and the quantity of a degree of the Æquator to be 28 Leagues, we draw the Line AB of what Length we please, for one Degree of the Æquator, and divide it into 28 equal parts, each of which represents a League; then we describe from the Extremity A, distance B, an Arch of a Circle 49 Degrees; and draw from the Point C the Line CD perpendicular to the Line AB; and in regard this Line CD cuts off from the Line AB, the Part AD containing about 18 Leagues, we conclude, that one Degree of a Parallel distant from the Æquator 49 Degrees is 18 Parisian Leagues. This Measure may be taken with greater facility and exactness, by Trigonometry, after the following manner.



Let AB be the Axis of the World, of which A and B are the two Poles, and ACBD one of the two Colurus's: Let CFD be the Æquator, and GHI the Parallel, of which the Diameter GI is perpendicular to to the Axis AB, and DI or CG its distance from the Æquator is suppos'd to be 49 Degrees, in which case the Complement AG or AI will be 41 Degrees.

'Tis evident that with respect to the whole Sine CE the Semidiameter GK is the Sine of the Arch AG the Complement of the distance of the Parallel. 'Tis equally evident, that, CE the Semidiameter of the Æquator, or the whole Sine, is to its Circumference, as GK the Semidiameter of the Parallel or the Sine of the Complement of the distance of that Parallel is to its Circumference; and consequently that the whole Sine is to a Degree of the Æquator, as the Sine of the Complement of the distance of the Parallel is to a Degree of that Parallel; and forasmuch as a Degree of the Æquator is known to be 28 Parisian Leagues, the following Analogy will shew how many such Leagues are in a Degree of the Parallel.

As the whole Sine	100000
To a Degree of the Æquator	28
So is the Sine of the Complement of the distance of the Parallel from the Æquator	65606
To a Degree of that Parallel	18

Having thus discover'd the quantity of a Degree of the Parallel of *Paris*, you may easily know, if you will, the whole Circumference of that Parallel, by multiplying the found quantity 18 by 360, or, which is more exact, by the following Analogy:

As the whole Sine	100000
To the Circumference of the Earth	10080
So is the Sine of the Complement of the distance of the Parallel from the Æquator	65606
To the Circumference of the Parallel	6613

Here you see the Circumference of the Parallel of *Paris*, is about 6613 Parisian Leagues; from whence it follows, that if the Earth moves, the City of *Paris* or any other Point under the same Parallel travels from West to East 6613 Leagues in 24 Hours, and consequently 275 Leagues in one Hour, and about 4 Leagues and a half in a Minute of Time.

PRO-

PROBLEM IX.

To find the distance of two propos'd places of the Earth, the Longitudes and Latitudes of which are known.

THIS Problem may fall under three different Cases; for the two propos'd places may be under one Parallel, having the same Latitude, and different Longitudes: Or under one Meridian, having the same Longitude, but different Latitudes: Or else under different Parallels and different Meridians, having both Latitudes and Longitudes different. Of each of these Cases apart.

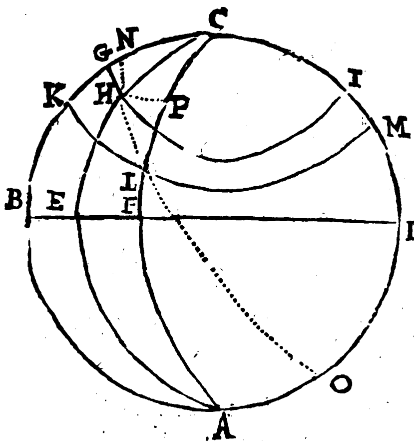
For the first Case; if the two propos'd places are under the same Parallel, as *Cologn* and *Mastricht*, the Parallel of which is distant from the Æquator North 50. 50′: *Cologn* lies more to the East than *Mastricht* by 6 Minutes of time, which are equivalent to 1. 30′ of the Æquator, or of the Parallel, under which these two Cities are situate; as appears by the Operation upon this Question; if 1 Hour or 60 Minutes are equivalent to 15 Degrees, what Degrees do 6 Minutes answer to? Now the Arch of the Parallel intercepted between *Cologn* and *Mastricht* being 1. 30′, which upon the Æquator is 42 *Parisian* Leagues (a Degree there being 28 Leagues as above) it remains to see by the following Analogy, how many such Leagues are in this Arch of the Parallel that is 50. 50′ distant from the Æquator, *i. e.* what is the distance of the two propos'd places.

As the whole Sine,	100000
To the Equivalent of 1. 30′ *upon the Æquator,*	42
So is the Sine of the Complement of the distance of the Parallel from the Æquator,	63158
To the distance in question.	$26\frac{1}{2}$

Thus you see the distance between *Cologn* and *Mastricht* is 26 *Parisian* Leagues and a half.

As to the second Case; if the two propos'd places are under the same Meridian, as *Paris*, the Latitude of of which is 48. 51′. and *Amiens*, the Latitude of which is 49. 54′. The Latitude of *Paris* being the least, substract it (*viz*. 48. 51′.) from 49. 54′. the Latitude of *Amiens*; and the Remainder 1. 3′. is the Arch of the Meridian taken in between *Paris* and *Amiens*; which convert into Leagues, by working this Question, by the Rule of Three. If one Degree or 60 Minutes of a great Circle of the Earth is equivalent to 28 *Parisian* Leagues, what is 1. 3′. or 63 Minutes; the answer of which is 29 Leagues.

In the last Case; if the two propos'd places differ both in Longitude and Latitude, as *Paris* and *Constantinople*, which last lies 29. 30′. more East, and 7. 45′. more South than *Paris*; imagine a great Circle to pass thro' these two Cities, and the Arch of the Circle comprehended between 'em will be found after the following manner.



Let ABCD be the first Meridian, and BD the Æquator equally distant from the two Poles A and C. Let AEC be the Meridian of *Paris*, and GHI its Parallel, the Point H representing *Paris*. Let AFC be the Meridian of *Constantinople*, and KLM its Parallel, the Point L representing *Constantinople*. Let HL be the Arch of the great Circle NHLO, that passes thro' the two proposed places H and L.

This

This Arch HL may be known by Trigonometry, in the oblique angled spherical Triangle HCL, of which we know the side HC (the Complement of EH the Latitude of *Paris*, or of 48. 51′.) to be 41. 9′. and the side CL (the Complement of FL, the Latitude of *Constantinople*, *i. e.* of 41. 6′.) to be 48. 54′. And the Angle comprehended HCL (or the difference of the Longitudes BCE, BCF, of the two propos'd places H, and L) to be 29. 30′.

Now to find the side or distance HL first in Degrees and Minutes, draw from the Angle H the Arch of a great Circle HP perpendicular to the opposite side CL, and make these two Analogies:

As the whole Sine	100000000
To the Sine of the Complement of the Angle HCL	99396968
So is the Tangent of the side HC	99414585
To the Tangent of the Segment CP.	99811553

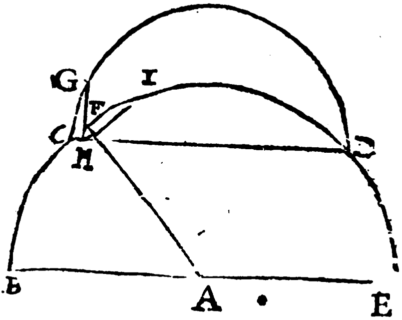
which you will find to be 37. 25′. and that being substracted from the Base CL or from 48. 54′. leaves a Remainder of 11. 29′. for the other Segment LP,

As the Sine of the Complement of the Segment CP	98999506
To the Sine of the Complement of the Segment LP	99912184
So is the Sine of the Complement of the side HC	9876789
To the Sine of the Complement of the side HL.	99680567

which you will find to be 21. 42′. and these you're to reduce to *Parisian* Leagues by the Rule of Three, saying, If one Degree or 60 Minutes of a great Circle of the Earth is equivalent to 28 *Parisian* Leagues; what is the equivalent of 21. 42′. or 1302 Minutes? So you learn that *Paris* is distant from *Constantinople* 607 Leagues.

Remark. When the two places propos'd lie at a considerable distance one from another, as in this Example, we may without any Calculation find that distance with almost equal exactness, in Degrees and Minutes of a great

great Circle of the Earth, by the Orthographical Projection of the Sphere, as I am now about to shew you.



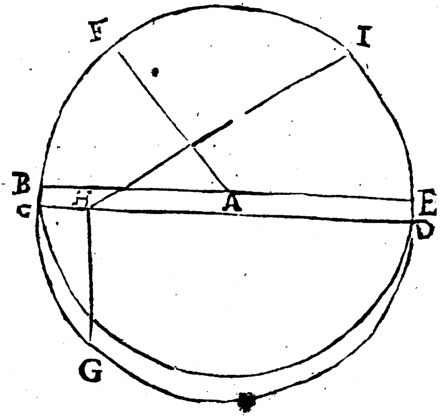
Describe from the Center A, with what extent of the Compasses you please, the Semicircle BCDE, which shall stand for the Meridian of *Paris*. Take upon that Semicircle the Arch BF of 48. 51'. such being the Latitude of *Paris*, so that F will represent the place where *Paris* stands, to which you draw from the Center A the Radius AF.

Take upon the same Semicircle the Arches BC, ED, each of 'em 41. 6'. such being the Latitude of *Constantinople*, and draw the Line CD which will represent the Parallel of *Constantinople*, and upon that Parallel you may determine the place where *Constantinople* lies, after the following manner.

Having describ'd round CD as Diameter, the Semicircle CGD, take upon its Circumference the Arch CG of 29. 30'. such being the difference of the Longitudes of *Paris* and *Constantinople*, and draw from the Point G the Line GH Perpendicular to the Diameter CD, and so you have H for the place where *Constantinople* stands. From this Point H draw the Line HI perpendicular to the Line AF, and by measuring the Arch FI, you'll find the distance sought for to be about 22 Degrees.

Here we took BC the Latitude of *Constantinople* in the same Hemisphere with BF the Latitude of *Paris*, with respect to the Line BE, which represents the Æquator; because these two Cities are in North Latitude.

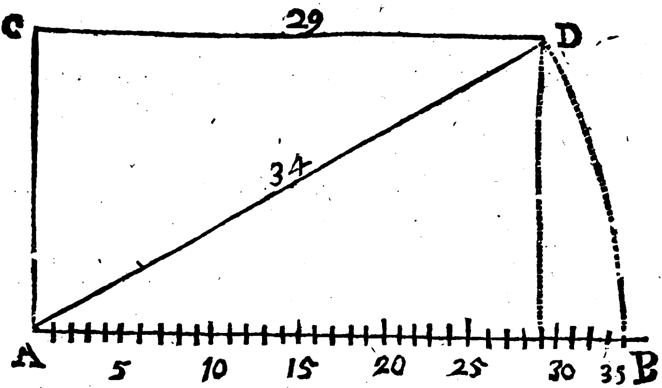
titude. But if one had been in South Latitude, as *Pernambouc* in *Brasil*, the South Latitude of which 7. 40′. it behoved us to have taken the Arch BC of 7. 40′. on the other side of BE the Æquator, and then go on as before, making the Arch CG 44. 15′. that being the difference of the Latitudes of *Paris* and *Pernambouc*; and since the Arch FI is about 70 Degrees, if we reduce these 70 Degrees into Leagues, by multiplying 70 by 28, we have 1960 Parisian Leagues for the distance between *Paris* and *Pernambouc*.



When the distance of the two propos'd places is not very considerable, such as that of *Lions* from *Geneva*, the latter of which is 36 Minntes North of the first, the Latitude of *Lions* being 45. 46′. and that of *Geneva* 46. 22′. and likewise 6 Minutes of time East of *Lions*, which is equivalent to 1. 30′. upon the Æquator: In this Case, I say, the foregoing Method, tho' good in it self, will not succeed; and therefore the following will do better, which, tho' not Geometrical, will be liable to no sensible failure in a small distance.

Having drawn the Line AB divided into what equal Parts, and of what Magnitude you please, representing Leagues, draw perpendicular to it the Line AC of 17 Leagues taken upon the Scale AB, such being the difference

ference of the Latitudes, which we found to be 36 Minutes, and these reduced to Leagues, making about 17 Leagues.



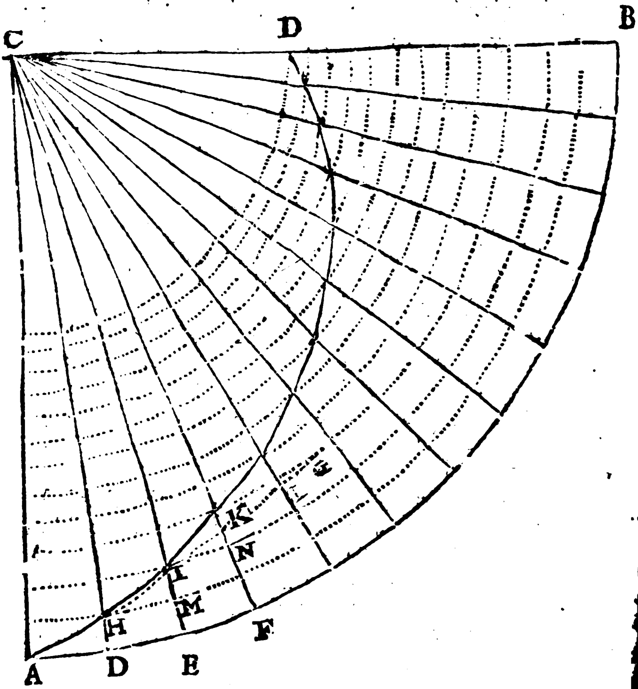
This done, add together the Latitudes of the two propos'd places, namely, 45. 46'. and 46. 22'. and take half their Sum 92. 8'. in order to have the mean Latitude 46. 4'. with respect to which you'll find by *Problem* 8. the quantity of an Arch of 1. 30'. that being the difference of the Longitudes of the two places proposed. Now this extent of the Arch comes to about 29 Parisian Leagues, and therefore you're to draw from the Point C, parallel to the Line AB, the Right Line CD containing 29 of the parts of the Scale AB; and then to take upon the same Scale AB the length of the Line AD, which proving to be 34 Parts, shews that *Lions* is, in a straight Line from *Geneva*, about 34 Parisian Leagues.

Forasmuch as the Triangle ACD is Right Angled at C, and the side AC is 17 Parts, and the other side CD 29, we compute the Hypothenuse AD or the distance inquir'd for, by adding 289 the Square of the side AC to 841 the Square of the side CD, and extracting the square Root of the Sum 1130, which brings us almost 34 Parisian Leagues for the length of the Line AD, or the distance of the two places, A being taken for *Lions*, D for *Geneva*, and the Line AD for the Arch of a great Circle that runs through both the Places; for the Line AC represents the difference of their Latitudes, or the distance of their Parallels, and the Line CD the mean difference of their Longitudes or Meridians.

PROBLEM X.

To describe the Curve-Line, that a Ship in the Sea would describe in steering its course upon the same Rumb of the Compass.

LEt's suppose the Arch AB, the Center of which is C, to be the Quadrant of the Circumference of the Terrestrial Æquator, so that C will represent one of the two Poles of the World, and all the straight



Lines drawn from the Center C to the Divisions of the Arch AB, as CD, CE, CF, &c. will represent so many Meridians.

Let

Let's suppose at the same time, that a Ship sets out from the Point A of the Æquator, the Meridian of which is AC, with intent to go to G by the Rumb AH, which makes with the Meridian AC an Angle CAH, supposed here to be 60 Degrees, which is call'd *the Inclination of the Loxodromy.* Now, 'tis evident that if the Vessel sets its Head always to the same Point, *that is*, if, when 'tis at H under the Meridian AD, it continue the same course by the Rumb or Vertical Point HI inclin'd to the Meridian AD to the extent of the same Angle of 60 Degrees, so that the Angle CHI will likewise be 60 Degrees; the three Points A, H, I, are not in a straight Line. In like manner if the same Ship continues its course from I, under the Meridian CE to K, which makes with the Meridian CE, the Angle CIK also of 60 Degrees; the three Points, H,I,K, are not in a straight Line, and so on till you come to L upon the last Meridian CB.

From hence we readily conclude, that the Line AHIKL, describ'd by the Ship in steering still to the same Point, which is call'd the *Loxodromick Line*, or barely the *Loxodromy*, is a Curve-line that always falls off from the Point G the intended Port, and imitates the figure of a Spiral Line, which, as you see approaches still nearer and nearer to the Pole C.

If you divide the *Loxodromick* Line AKL into several Parts, so small that they may pass for straight Lines, as AH, HI, IK, &c. and if you run through the points of Division as many Parallels or Circles of Latitude, all these Circles will be equidistant from one another, so that the Arches of the Meridians, DH, MI, NK, &c. will be mutually equal, as well as the Corresponding Arches AD, HM, IN, &c. not in Degrees, but in Leagues, by reason of the equality of the Rectilineal Right-angled Triangles, ADH, HMI, INK, &c. Remark.

When you know the time spent in running upon the same Rumb with a favourable Wind, a very small *Loxodromy*; and consequently know the Arch AD which is easily reduced to Leagues, allowing 28 to a Degree; and, if at H you take the Elevation of the Pole or the Latitude DH, which is also easily reduced to Leagues: You may easily compute how far you have run between A and H, by adding together the Squares of the Lines AD, DH, and extracting the square Root of the Sum.

'Tis visible that the *Loxodromy* is a straight Line when there's no Angle of Inclination, *that is*, when the Ship sails North and South, or keeps to the North and South Rumb mark'd upon the Compass, when the Needle do's not decline; for in that case the Vessel advancing upon the Meridian Line, must needs describe a straight Line, it being the common Section of the Meridian and the Horizon.

The same will happen, when a Ship under the Celestial Æquator, or one of its Parallels, sets its Head and Sails due East or West, so that the Inclination of the *Loxodromy* will be 90 Degrees; for in that case the Vessel describes either a Terrestrial Æquator or one of its Parallels which make with the Meridians right Angles.

In fine, 'tis visible, That, as we said before, a Vessel sailing upon the same oblique Rumb, so that the Inclination of the *Loxodromy* makes an oblique, (*i. e.* an acute or obluse) Angle; it describes upon the surface of the Sea, a Curve-line, such as AKL, in steering from A to G, in the oblique Course AH; for the terrestrial Meridians CA, CD, CE, CF, *&c.* are not parallel one to another; and certain it is, that if they were parallel, instead of describing the Curve-line AKL, which with these Meridians makes equal Angles, 'twould describe the straight Line AG and that would make with the same Meridians equal Angles.

This Curve-line AKL resembles that which would be describ'd by a heavy Body, as a Stone, falling from the surface of the Earth to its Center, if it be true that the Earth moves round its Axis from West to East; as I am now about to shew you.

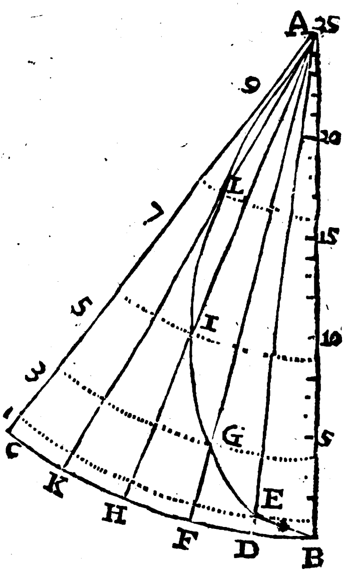
PRO-

PROBLEM XI.

To represent the Curve-line, that by vertue of the Motion of the Earth, a heavy Body would describe in falling freely from the upper surface to the Center of the Earth.

LET A be the Center of the Earth, and the Arch BC, part of its Circumference, which the Point B runs over by vertue of the Motion of the Earth in a certain space of Time, as going in equal portions of time thro' the equal Arches BD, DF, FG, HK, KC.

Upon this Supposition, the Semidiameter of the Earth will in the first portion of time take the Situation AD, and the Stone which was in B, will be de-



scended to E, when B arrives at D; in the second division of Time, B will arrive at F, and the Semidiameter AB taking its Situation AF, the Stone will be

got

got to G; and that in such a manner, that the part FG will be 4 when the part DE is 1, by the nature of heavy Bodies, which in falling freely from aloft downwards, acquire in equal portions of time equal degrees of Velocity, in running thro' the Spaces that increase as the Squares, 1, 4, 9, 16, 25, &c. of the natural Numbers, 1, 2, 3, 4, 5, &c. these Spaces rising gradually according to the odd Numbers, 1, 3, 5, 7, 9, &c. and therefore at the third Division of Time, when the Point B will be got to H, the Diameter AB will stand as AH, and the Stone will be got down to I, and the part HI will be 9; and at the fourth Division when the Point B is arriv'd at K, the Semidiameter AB will stand as AK, and the Stone will be got to L, the part KL being 16; as at the other Subsequent Division, the whole AC will be 25. Thus the Stone continuing its Descent, will make the Curve-line, BEGILA, which by consequence may be represented after the following manner.

Since the Sum of the first five odd Numbers, 1, 3, 5, 7, 9, is the square Number 25, the Root of which is 5, divide the Right Line AB into 25 equal parts of what Magnitude you will, from B to A. From A as a Center, distance B, describe the Arch of a Circle BC of what extent you will. Divide this Arch BC into five equal Parts at the Points D, F, H, K, and from these draw to the Center A the Radius's or Semidiameters, AD, AF, AH, AK; upon which you'll find the Points, E, G, I, L, of the Curve-line you want to describe, by taking the part DE equal to one of the parts of the Line AB, the Line FG equal to four of its Parts, HI to nine of its Parts, KL to sixteen, &c.

PROBLEM XII.

To know when a propos'd Year is Bissextile or Leap-year.

THO' the Solar Year, or the time that the Sun takes in going by its proper motion over the whole Zodiack, is about 365 Days, 5 Hours and 49 Minutes; yet we reckon only 365 Days (excepting the Leap-year) omitting the 5 Hours and 49 Minutes, which

which are but 11 Minutes short of 6 Hours. Thus it comes, that every common Year is about 6 Hours too short, which in four Years make almost a Day; and that Day we add or put in between the 23*d* and 24*th* of *February* in every fourth Year, which is stiled the Bissextile Year, by reason that it consisting of 366 Days, we are obliged to date *Sexto Kalendas Martii* for two successive Days, otherwise the Nones and the Ides would be put out of their usual places.

Now to know if the Year propos'd is Bissextile, divide the number of the Year by 4, and if there remains nought, 'tis a Leap-year, or a Year of 366 Days; if any thing remains after the Division, 'tis no Leap-year, and consists only of 365 Days. Thus we know that the Year 1693 is not Bissextile, for when we divide it by 4, there remains 3, which shews that the third Year after, *viz.* 1696. will be a Leap-year.

But after all, 'tis to be observ'd, that tho' the Division of the Years, 1700, 1800 and 1900 by 4, leaves no Remainder, yet we must not take them to be Bissextile Years. Now, this is occasion'd by the alteration of the Kalendar made by Pope *Gregory* XIII. in 1582, upon the Consideration that the six Hours added to every fourth Year, are eleven Minutes more than the due Addition, which in the space of four Centuries amount to three Days more than enough; and so the Compensation allotted for this Excess, is, to leave out the Leap-day in each of the three Years, 1700, 1800 and 1900; the year 1600 being reckon'd as Bissextile. The *Gregorian* Calendar.

This Reformation of the Calendar made in the last Century but one by Pope *Gregory* XIII, who in the year 1582 threw out ten Days, there being so many grown to a Surplusage from the time of *Julius Cæsar*, who instituted the Leap-year: This Reformation, I say, gave rise to the *Gregorian Calendar*, or the *New Calendar*, which the Church of *Rome* makes use of at present. Remark.

In the Sixteenth Century, it being found that thro' the growing Surplusage above-mention'd, the Vernal Equinox anticipated the 21*st* of *March* 10 Days; and that Equinox being the Period upon which depends the Regulation of *Easter*; these ten Days were not counted, but the 11*th* of *March* was call'd the 21*st*, this being the Day of the Vernal Equinox in the time of the Council of *Nice*: So by this Reformation the Equinoxes and Solstices are fix'd to the same Days and same Months. And 'tis objected against the old Style or *Julian Calendar*, that if it be continued for a long process of time, *Christmas* will fall to be celebrated at *Midsummer*, and the Festival of St. *John* the *Baptist* at the Winter Solstice.

PROBLEM XIII.

To find the Golden Number in any Year propos'd.

We acquainted you in the foregoing Problem, that the Solar Year consists of 365 Days, 5 Hours, and 49 Minutes; and now we come to tell you, that the Lunar Year or the Sum of twelve Revolutions of the Moon by its own proper Motion in the Zodiack, is 354 Days, 8 Hours, and 49 Minutes; which you see is about 11 Days shorter than the Solar Year, and consequently the New Moons come 11 Days sooner in one than in the preceding Year.

Thus you see the Sun and the Moon do not always finish their Periods at the same time; nor do they always meet in the same Disposition, *that is*, the New Moons do not return in the same Months, and on the same Days as in another Year, unless it be in the space of about 19 Years; I say, *about* 19 *Years*, because there wants of that number 1 Hour, 27 Minutes, and 32 Seconds; which is but inconsiderable, for the New Moons anticipated but one Day in 312 Years, which was one of the causes of the Reformation of the Calendar, and of the substitution of the Epacts in the room of the Golden Number, which is a period of 19 Years.

What we mean by the Golden Number.

This number of 19 Years, at the end of which the Sun and Moon return to the same Points they were joyntly in before, is what we call the *Golden Number*, so call'd by the *Athenians*, who received it with so much Applause, that they order'd it to be put in large Cha-

Characters of Gold in the middle of the publick place. It has likewise been call'd the *Lunar Cycle*, as being a Period or Revolution of 19 Solar Years, equal to 19 Lunar Years; twelve of which are *Common*, as having twelve Synodical Months a piece, and seven are *Embolismal*, *i.e.* consist of thirteen Moons each; which make in all 235 Moons, at the end of which the New Moons return on the same days of the same Months as before.

To find the Golden Number for the year 1693. (for instance;) add 1 to the number of the year 1693, and divide the Sum 1694 by 19, and neglecting the Quotient mind only the Remainder, *viz.* 3, which is the Golden Number for that year. The Reason of that addition of 1 to the number of years, is, that in the first year of Christ, 2 was the Golden Number.

'Tis evident that when the Golden Number for a year is once found, the addition of 1 will give the Golden Number of the year next insuing, as the Substraction of 1 will that of the immediately preceding year. Remarks.

'Tis equally evident, that all the years which have the same Golden Number, have the New Moons on the same days of the same Months. Thus it being New Moon *Aug.* 1. 1693, of which year 3 is the Golden Number, the New-Moon will happen on the same day of the same Month, in the years 1712, 1731, 1750, &c. which have also 3 for their Golden Number.

PROBLEM XIV.

To find the Epact for a propos'd year.

WE shew'd you in the foregoing Problem, that the Solar Year exceeds the Lunar by about 11 Days; which is the exact case, if you compare the common Solar Year, or what they call the *Egyptian* Year, *viz.* 365 Days, with the common Lunar or 354 Days; for here the exact difference is just 11 Days; and this difference of 11 Days is call'd the *Epact*, which being added to the common Lunar Year, (*i. e.* What an *Epact* is.

(*i. e.* the time of twelve Moons or Synodical Months, each of which is 29 days and a half) makes a common Solar Year.

What a Synodical Month.

By a *Synodical Month*, we underſtand the proceſs of time from one New-Moon to another, which, as we intimated above, is, 29 days and a half; or more preciſely, 29 Days, 12 Hours, and 44 Minutes, and which by conſequence is 2 days and ſeven hours longer than the *Periodical Month*, *i. e.* the Revolution or Period of the Moon by its proper Motion from a Point of the Zodiack to the ſame Point again; which Period extends to 27 Days, 5 Hours, and 44 Minutes, and indeed muſt unavoidably be leſs than the Synodical Month, by reaſon of the proper Motion of the Sun, by vertue of which it runs in a Periodical Month about 27 Degrees, which the Moon has ſtill to go before it can reach the Sun, after its return to the ſame Point where it was in conjunction with the Sun before. Now to travel theſe 27 Degrees, the Moon requires 2 Day, and 7 Hours, after finiſhing its Period or Revolution in the Zodiack.

What we mean by a Periodical Month.

The Synodical Months being each of 'em 29 days and a half, are found in the Calendar to be alternately 29 and 30 Days. Some begin the firſt Month from the New-Moon in *January*, as the *Jews* of old did from that in *September*; and the Church of *Rome* begins it with the *Eaſter* New-Moon, *i. e.* the next full Moon after the Vernal Equinox, or upon the day of that Equinox, which among them is fix'd to the 21*ſt* of *March*, becauſe the Vernal Equinox (as we intimated above) happened on that day when the Council of *Nice* ſat.

If the Moon is full before the 21*ſt* of *March*, that do's not begin the new year, but concludes the former year; for the firſt full-Moon or the fourteenth day of the Moon, muſt happen either upon or after the firſt of *March*, in order to adjuſt the feaſt of *Eaſter*, which the *Roman Catholicks* celebrate the next Sunday after the New-Moon: From whence it follows, that all the Moons beginning from the 8*th* of *March*, to the 5*th* of *April* incluſive, may be Paſchal Moons; and conſequently the Paſcha or *Eaſter* can't be celebrated before the 22*d* of *March*, nor after the 25*th* of *April*;

and

and so it may happen 35 days later in one year than in another.

To find the Epact of any year (which begins only in the Month of *March*) find by the foregoing Problem, the Golden Number of the year, and after multiplying that number by 11, (the difference of the Solar and Lunar year) divide the Product by 30, the number of days in a Synodical Month, and neglecting the Quotient, mind the Remainder for the Epact sought for, if the year in question was before the Reformation of the Calendar, or if you reckon by the old stile; but if you reckon by the new, and if the year propos'd came since the Reformation of the Calendar, you must substract from it the 10 days that Pope *Gregory* threw out; nay, if it comes after the year 1700, you must substract 11 days, because the Leap-day in the year 1700 is suppress'd for reasons abovementioned. If the number is so small as not to admit of that Substraction, add 30 to it, and then Substract.

Thus to find the Epact of the year 1693, (according to the *Gregorian* Calendar) I multiply its Golden Number, *viz*. 3 by 11, and divide the Product 33 by 30; the Remainder being 3, from which I can't yet substract 10, I add 30 to the 3, and from the Sum 33 substract 10, which leaves 20 for the Epact of the year.

The old Epact without regard to the *Gregorian* Emendation, may be found thus without the trouble of Division. Observe the top or end, the middle Joynt and the Root of the Thumb of your left Hand; and fix upon 'em these different Values, *viz*. Let the top of your Thumb be a place of 10, the middle Joynt of 20 and the Root of 0. Now reckon the Golden Number of the propos'd year upon your Thumb, beginning with the end or top, reckoning the end 1, the middle Joynt 2, the Root 3, and so go over again, the End 4, the middle Joynt 5, the Root 6, the End 7, &c. till you come to the Golden Number; and if it happens upon the Root, add nothing to it, the place of that being adjusted 0, if upon the middle Joynt add to it 20, or if upon the End add 10. The Sum is the Epact if under 30; if above 30 throw 30 out of it, and the remainder is the Epact.

'Tis

'Tis evident that when the Epact of a Year is once found, the addition of 11 will give that of the next, and 11 more the next after that, and so on; only you must take care still to throw out 30 when the Sum is above 30, and to add 12 instead of 11 when you have 19 or rather 0 for the Golden Number.

PROBLEM XV.

To find the Age of the Moon on a given Day of a Year propos'd.

To find the Age of the Moon, add to the Epact of the Year the number of the Months from *March* to the Month propos'd inclusive, and substract the Sum from 30 or from 60 if it surpasses 30; and the Remainder gives you the Day of the Month, on which 'twas New Moon; so having that, you may easily compute the Age of the Moon on the Day propos'd.

Or, without knowing the Day of the New Moon, you may find it thus: Add together these three, The Epact of the Year Current, the number of the Months from *March* inclusive, and the Day of the Month propos'd; the Sum is the Age of the Moon if not above 30; if it is, take 30 from it, and the Remainder is the Age. Thus if the Epact of the Year is 23, and the 18th of *April* is the Day propos'd, add 23 and 2 (for the Months of *March* and *April*) and 18; and from the Sum 43 substract 30; the Remainder 13 is the Age of the Moon.

Remark. Since the Epact of a Year does not begin but in *March*, the way of finding the Age on a certain Day of a Month of that Year preceding *March*, is this: Instead of the Epact of that Year, take the Epact of the preceding Year, and so proceed as above, reckoning the number of the Months from *March* inclusive, *January*, for example, 11, &c.

PRO-

PROBLEM XVI.

To find the Dominical Letter and the Solar Cycle of a propos'd Year.

SInce the common Year consists of 365 Days, which amount to 52 Weeks and a Day, and the Bissextile Year consists of 366 Days or 52 Weeks and 2 Days; since the seven Days of the Month, call'd *Feriæ*, are represented in the new Calendar, by the seven first Letters of the Alphabet, A, B, C, D, E, F, G, which are call'd *Dominical Letters*, because each of 'em is employed in their turn to represent the Lord's Day: 'Tis evident, that these Letters wou'd return in the same order every seventh Year, if the order were not interrupted every fourth Year by the additional Leap Day; from whence it comes that they do not return into the same order, till after four times seven Years, *i. e.* 28 Years; and that Period is what we call the *Solar Cycle* and the *Cycle of the Dominical Letter*. This Cycle was invented for the ready knowing of the Sundays any time of the Year by the Dominical Letter. What the Solar Cycle is.

To find the Dominical Letter of a Year propos'd, and withal the Letter for every Day of that Year: Divide the number of the Days elapsed from the first of *January* to the Day propos'd inclusive, by 7; and if nothing remains the Letter sought for is G; if any number remains, the Letter that corresponds to that number, beginning from A as 1, is the Letter sought for. Thus if 4 remains, D is the Letter for the Day propos'd. And if the Day propos'd be a Sunday, the Letter thus found is likewise the Dominical Letter of the Year.

To find the Dominical Letter for a propos'd Year since Christ, according to the new Calendar; add to the number of the Year its fourth part; or the next part less, if 'tis not exactly divisible by 4; and having substracted 5 from the Sum (the Year being within the 17th Century) divide the Remainder by 7 and neglecting the Quotient, mind the Remainder, which shews you the dominical Letter, reckoning from G the last Letter towards A; so that if nothing remains the Domi-

Dominical Letter is A, if 1 remains G is the Letter, if 2 F, and so on. Thus for the Year 1693 we add to it its fourth part 423, and after substracting 5 from the Sum 2116 we divide the Remainder 2111 by 7, and without regarding the Quotient, are directed by the Remainder 4 to the fourth Dominical Letter in the retrograde Order, *viz.* D.

I said above that 5 is to be substracted when the Year is within the 17th Century, *i.e.* between 1600 and 1700; for in the Century of 1700 we must substract 6, in that of 1800 7, in that of 1900 8, these Years being not reckon'd Bissextile by the new Calculation, as we intimated heretofore. Indeed the Year 2000 is reckon'd Bissextile, and so for that Century we continue to substract but 8; but for 2100, 2200, 2300 we must substract 9, 10, 11, the Bissextile Days being thrown out in these, and so on.

To find the Solar Cycle of a propos'd Year, add 9 to the number of the Year, divide the Sum by 28, and the Remainder is the Solar Cycle. Thus for the Year 1693, 9 added make 1702, and that divided by 28 leaves 22 remaining for the solar Cycle. The number 9 is here added, because the Solar Cycle immediately before the first Year of Christ was 9, and consequently the Cycle began 10 Years before Christ.

Remark. 'Tis evident that after finding the Solar Cycle of one Year since Christ, the addition or substraction of 1 gives the Cycle of the next ensuing or preceding Year.

'Tis equally evident, that after finding the Dominical Letter for a Year, the Letter for the next ensuing or next preceding Year is easily found by taking the next following Letter for the Dominical of the preceeding Year, and reciprocally the next preceding Letter in the order of the Alphabet for the Dominical of the following Year; which will serve for the whole Year if 'tis not Bissextile: Indeed if it is, the Dominical thus found will serve no longer than the 24th of *February*, at which time the other Letter next preceding in the order of the Alphabet is taken in for the rest of the Year: For a Bissextile Year having an additional Day, has two Dominical Letters.

PRO-

PROBLEM XVII.

To find on what Day of the Week a given Day of a given Year will fall.

IF the Year be since Christ, add to the number of the Year given, its fourth part, or the next lesser, if 'tis not exactly divisible by 4; to this Sum add the number of Days comprehended between the first of *February* and the propos'd Day, inclusive; then substract 2 for the *Julian* Calendar, or 12 for the *Gregorian* (if it be before 1700, otherwise it must be 13) and divide the Remainder by 7. The number remaining after this Division, is the number of the *Feria* in question; reckoning Sunday 1, Munday 2, Tuesday 3, and so on; and if nothing remains, Saturday is the Day.

PROBLEM XVIII.

To find the number of the Roman Indiction for a Year propos'd.

IN ancient Times the *Greeks* computed their Years by *Olympiads*, which is a Revolution of four Years, at the end of which they celebrated the *Olympick Games*, so call'd because they had been instituted by *Hercules* near *Olympus* in *Arcadia*; but after *Rome* brought *Greece* in subjection to them, they wou'd not reckon their Time by *Olympiads*, four Years being too short a term for them, but settled the Period of Computation to three *Lustrums* or fifteen Years, which they call'd an *Indiction*.

So that the Roman Indiction is a term of fifteen Years, at the end of which they begin their Computation with a continual Circulation: This Period of fifteen Years, was call'd *Indiction*, as some will have it, because it serv'd to point out (*indicare*) the Year of payment of the Tax or Tribute to the Republick, whence 'twas call'd the *Roman Indiction*, and since the *Pontifical Indiction* beginning the first of *January*, because the Court of *Rome* use it in their Bulls and Dispatches

What an Indiction is.

patches. Others take it from the summoning of the Provinces every fifteenth Year, to distribute Ammunition to the Soldiers; at which Period those who had serv'd so long in the Army were free to draw their Passports, and entitled to Immunities and Privileges.

However that be, the way of finding the number of the Roman Indiction for any Year since Christ is this. Add 3 to the number of the Year, and divide the Sum by 15, and the Remainder is the Year of Indiction. Here we add 3 because the Cycle of Indiction recommenced 3 Years before the Nativity of our Saviour. Thus for the Year 1700, add 3, and divide the Sum 1703 by 15, and the Remainder of the division is 8 for the number of the Indiction.

PROBLEM XIX.

To find the Number of the Julian *Period for a propos'd Year.*

The Construction of the *Julian* Period.

THE Roman Indiction has no connexion with the Celestial Motions, yet that Revolution of 15 Years is compar'd with the Period of the Solar Cycle of 28 Years, and the Period of the Golden Number of 19 Years; *viz.* by multiplying together these three Cycles, 15, 28, 19, the solid product of which gives that famous Period of 7980 call'd the *Julian Period*, from *Julius Scaliger*, who first invented it, and introduced by the modern Chronologers, as a Standard for adjusting all the difference of Times mentioned by Historians; it being certain, that this number of 7980 Years contains all the different Combinations of the three abovementioned Cycles, which in all that space of time can meet but once in the same disposition.

The number of this Period is easily found for any Year since Christ, if once we know its beginning, *that is*, the time when it would have begun before Christ, and even before the Creation of the World; for as this Period is great, so the time of its beginning when all the Cycles of which 'tis composed wou'd have been Number 1, surpasses by many Years not only the Christian Epocha, but even the time attributed in Scripture to the Crea-

Creation of the World. Now the way of finding its commencement is this.

Since the first Year of Jesus Christ corresponded to the 4th of the Indiction, the 10th of the Solar Cycle, and the 2d of the Lunar Cycle or the Golden Number; multiply 4 the number of the Indiction by 6916, 10 the number of the Solar Cycle by 4845, and 2 the number of theLunar Cycle by 4200; then add together the three Products, 27664, 48450, 8400, in order to divide their Sum 84514 by the Julian Period 7980: The Remainder of this Division shews that the beginning of the Julian Period is 4714 Years before the Nativity of Christ.

This done, if we want to know the number of this Period for any Year since Christ, as for 1693, we add 4714 to 1692 and the Sum 6406 is the Julian Year sought for. Or else you may follow the method above-mention'd iu multiplying 1 the number of Indiction for the Year 1693, by 6916; 22 the number of the Solar Cycle for the Year, by 4845; and 3 the number of the Lunar Cycle by 4200, and add together all the Products, *viz.* 6916, 106590, 12600, in order to divide their Sum 126106 by 7980; upon which the Remainder of the Division 6406 answers the Question: The Reason for chusing these Multipliers is contained in the Remark upon the next Problem, which see.

PROBLEM XX.

To find the number of the Dionysian Period for a Year propos'd.

THE Multiplication of 28 the Period of the Solar Cycle by 19 the Period of the Lunar, forms a Period of 532 called the *Dionysian Period*, from its Inventer. This Period serves to discover all the Differences and Changes, that can happen between the new Moons and the Dominical Letters in the course of 532 Yeers; after which the Combinations of one and t'other return in the same order, and repeat the former Series.

To find the number of this Period, for any Year since Christ, multiply the number of the Solar Cycle for the Year propos'd by 57, and the number of the

Lunar Cycle for the year by 476, and after adding their two Products, divide the Sum by 532 the Dionysian Period; the Remainder of this Division solves the Question.

Remark. The number 57 which here multiplies the number of the Solar Cycle, is such that being divided by 28 the Period of the Solar Cycle, it leaves 1 Remaining; and if it be divided by 19, the Period of the Lunar Cycle, there remains nought: And Reciprocally the number 476 (which here multiplies the number of the Lunar Cycle) divided by 19 the Period of the Lunar Cycle, leaves 1 remaining, and divided by 28 the Period of the Solar Cycle, nothing remains. Thus the first number 57 shews the Dionysian year, which has 0 or 19 for the Golden Number, and 1 for the Solar Cycle; and the second number 476 gives us to know the Dionysian Year, in which we have 0 or 28 for the Solar Cycle, and 1 for the Golden Number.

Now, to find this first number 57, which ought to be multiple of 19, that its Division by 19 may leave no Remainder; if we put, for Example, 38 the double of 19, for the number demanded; this 38 divided by 28 leaves 10 remaining, instead of 1; to help which, since 10 is less than the Divisor 28 by 18, if you add that 18 to 38 you have 56, which divided by 28 leaves nothing remaining; and therefore if you add to 38, 19, instead of 18, you have 57 the true number demanded, as being the exact multiple of 19, and but 1 above the multiple of 28.

If you substract this first number 57 from 532 the Dionysian Period, and add 1 to the Remainder 475, you have the second number 476; which may likewise be found directly and immediately by a Ratiocination not unlike the former; only you have more essays to make, as I am about to shew you.

To find this second number 478, which must be multiple of 28, that nothing may remain upon its Division by 28; put for the number propos'd 56 (for Example) the double of 28; this 56 divided by 19, leaves 18 remaining instead of 1, which the question requires; now, this Remainder 18 being less than the divisor 19 by 1, if you add that 1 to 56 you have 57, which divided by 19 leaves no Remainder; and

and therefore if instead of 1 you add 2 to 56 you have 58, which leaves 1 Remainder upon its Division by 19. But tho' 58 has one of the qualifications requisite, 'tis destitute of the other, *viz.* that of being the multiple of 28; and so can't be the number inquir'd for. This Tryal proving fruitless, we must e'en try again after the same manner, in taking the Triple, Quadruple or Quintuple of 28, and so on, till we find such a multiple of it, as leaves 1 remaining upon its Division by 19; and such a multiple we'll find to be the 17*th*, or the Product of 28 multiplied by 17, *viz.* 476 the number sought for. If you Substract this 476 from 532 the Dionysian Period, the Remainder is 56 which augmented by unity makes 57 for the first number.

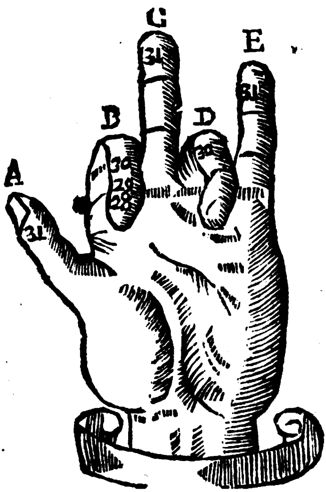
In like manner, the number 6916 by which you multiplied the number of Indiction in the foregoing Problem, is such, that being divided by 15 the Period of Indiction, it leaves 1 remaining; and when 'tis divided by 28 the Period of the Solar and 19 the Period of the Lunar Cycle, or, which is the same thing, by 532 the Product of these two Periods, there remains nothing. Again, the number 4845 by which we multiplied the number of the Solar Cycle in the foregoing Problem, is such, that being divided by 28 the Period of the Solar Cycle it leaves 1 remaining; and divided by 19 the Period of the Lunar Cycle, and by 15 the Period of Indiction, or, which is the same thing, by 285 the Product of these two Periods, it leaves no Remainder. And in fine, the number 4200 by which we multiplied the number of the Lunar Cycle in the foregoing Problem, is such, that being divided by 19 the Period of the Lunar Cycle, it leaves 1 remaining, and divided by 15 the Period of Indiction and by 28 the Period of the Solar Cycle, or, which is the same thing, by 420 the Product of these two Periods, it leaves no Remainder.

The first number 6916 gives us to know the *Julian* Year, in which we have 1 for the Indiction, and 0 for the Golden Number, and Solar Cycle, or 0 for the Dionysian Period; the second number 4845 shews the *Julian* Year, in which we have 1 for the Solar Cycle, and 0 for the Golden Number and Indiction; and the third number 4200 discovers the *Julian* Year, in which we have 1 for the Golden Number, and o for the Solar Cycle and Indiction. These three numbers were found after the same manner with the two numbers mention'd above.

PROBLEM XXI.

To know what Months of the year have 31 *days, and what have* 30.

HOld up your Thumb A, your Middle-finger C, and your Little-finger E; and lower or bend downwards your other two Fingers, *viz.* the Fore-finger B, and the Ring-finger or fourth Finger D. Then count the Months of the year upon your Fingers thus placed, beginning with *Mar.* upon your Thumb, then *April* upon the Fore-finger, *May* upon the Middle, *June* upon the Ring, and *July* upon the Little-finger; then count on, returning to your Thumb, and so round till you have reckon'd all the Months. When you have done, remember that all the Months that fell upon the Fingers held up A, C, E, have 31 days; and those upon the bended Fingers have but 30, excepting *February* upon the Fore-finger, which has 28 in a Common, and 29 in a Bissextile Year.



PRO-

PROBLEM XXII.

To find what day of each Month, the Sun enters a Sign of the Zodiack.

THE Sun enters the Signs of the Zodiack, about the 20*th* day of each Month of the Year; *that is*, the beginning of ♈ about the 20*th* of *March*, the beginning of ♉ about the 20*th* of *April*, and so on. But to know the time more precisely, you may make use of these two Artificial Verses;

Inclyta Laus Justis Impenditur, Hæresis Horret,
Grandia Gesta Gerens Felici Gaudet Honore.

Distribute the twelve words of these two Verses among the twelve Months of the Year, beginning with *March*, and ending with *February*, attributing to the first *Inclyta*, and to the last *Honore*. Then consider what number the first Letter of each word obtains in the order of the Alphabet, for that number substracted from 30, leaves remaining the day of the Month in question.

For Example; *Inclyta* answers to *March*, and to the Sign of *Aries*; and *I* its first Letter is the 9*th* of the Alphabet, which substracted from 30, gives us to know that the Sun enters *Aries* on the 21*st* of *March*. And so of the rest.

PROBLEM XXIII.

To find what degree of the Sign the Sun is in on a given day of the Year.

THE place of the Sun in the Zodiack, *i. e.* the degree of the Sign it is in, any day of any Month, is thus known. Suppose the day propos'd is *May* 18, *Justis* in the two Verses mention'd but now answers to that Month, and the Letter *I* being the 9*th* Letter of the Alphabet, we add 9 to 18 the number

ber of the day propos'd; and by the Sum 27 we are taught, that on the 18*th* day of *May* the Sun is in the 27*th* degree of Taurus, to which answers the word *Laus*.

This is the Method, when the Sum is under 30; for if it be above 30, we take the Sign that answers to the Latin word of the propos'd Month, and substract 30 from that Sum, the Remainder being the degree of the Sign. Thus, if *Aug.* 25. be the day propos'd, the word *Horret* answering to that Month, and the Sign being *Virgo*, we add 8 (the numeral value of the first Letter H) to 25, and substracting 30 from 33 the Sum, learn that on the 25*th* day of *August* the Sun enters the 3*d* degree of *Virgo*.

In this and the preceding Problem, we have taken it for granted, that the Reader is acquainted with the Order of the twelve Signs of the Zodiack, and their corresponding Months. The two following Verses shew the order of the twelve Signs.

> *Sunt Aries, Taurus, Gemini, Cancer, Leo, Virgo,*
> *Libraque, Scorpius, Arcitenens, Caper, Amphora, Pisces.*

In which we must observe, that the first Sign *Aries* corresponds to the Month of *March*; the second *Taurus* to *April*; and so on to the last *Pisces* which answers to *February*.

PROBLEM XXIV.

To find the place of the Moon in the Zodiack, on a given day of a given year.

FInd first the place of the Sun in the Zodiack by the foregoing Problem; and then the distance of the Moon from the Sun, or the Arch of the Ecliptick comprehended between the Sun and the Moon, by the following Method.

Having found by Problem 15 the Age of the Moon, and multiplied that by 12, divide the Product by 30, and the Quotient is the number of the Signs; as the remainder of the Division is the number of the De-

grees

grees of the distance of the Moon from the Sun. So if you count this distance upon the Zodiack, according to the order of the Signs, beginning from the place of the Sun, you'll end in the place of the Moon sought for.

For Example; I find the Sun on the propos'd day to be in the *17th* degree of Taurus, and the Age of the Moon to be 14. After multiplying 14 by 12 I divide the Product 168 by 30; and the Quotient 5 with the remainder of the Division 18, give me to know that the Moon is distant from the Sun 5 Signs and 18 Degrees. So if I reckon 5 Signs and 18 Degrees upon the Zodiack, beginning from the place of the Sun, the *27th* Degree of *Taurus*, I find the place of the Moon to be the *15th* Degree of *Scorpius*.

PROBLEM XXV.

To find to what Month of the Year a Lunation belongs.

IN the use of the *Roman* Calendar every Lunation is computed to belong to that Month in which it terminates, according to the ancient Maxim;

In quo completur Mensi Lunatio detur.

And therefore to solve the Problem, find by *Problem* XV. the Age of the Moon on the last day of the Month propos'd, and that will direct you whether the Moon terminates in that or in the succeeding Month, (the which last if it do's it belongs to that succeeding Month:) Or whether a prior Lunation did not terminate in the Month propos'd, and consequently belong to it.

PROBLEM XXVI.

To know which Lunar Years are Common, and which Embolismal.

THIS Problem is easily solv'd by the foregoing Problem, which gave us to know that one Solar Month may have two Lunations, or that two Moons may finish their Periods in the same Month, when 'tis a Month that has 30 or 31 Days; as *November*, on the first day of which one Lunation may terminate, and another on the 30*th*. In short, when we find any one Month of the year to have the termination of two Moons, we may conclude that *that* year has 13 Moons, and consequently is Embolismal.

PROBLEM XXVII.

To find the time of a given Night when the Moon gives Light.

HAving found by *Problem* XV. the Age of the Moon, and added 1 to it, multiply the Sum by 4 if it do's not exceed 15; but, if it exceeds 15, substract it from 30, and multiply the Remainder by 4; then divide the Product by 5, and the Quotient will give you so many twelfth parts of the Night, during which the Moon shines. These twelfth parts are call'd unequal hours, and must be counted after Sunset when the Moon Waxes, and before Sunrising when it Wanes.

For Example; 'tis demanded to know what time of the Night of *May* 21. *N. S.* the Moon will shine, its Age being then 17; we add 1 to 17, and after substracting the Sum 18 from 30, we multiply the Remainder 12 by 4, and divide the Product 48 by 5; the Quotient gives us 9 unequal hours and $\frac{3}{5}$ for the time of Moonshine before Sun-rise.

Remark. 'Tis an easy matter to reduce the unequal hours to equal or Astronomical hours, each of which is the 24*th* part of a natural day comprehending Day and Night;

Night; This Reduction I say, is easy, when once you know the length of the Night or Day propos'd. Thus in the foregoing Example, knowing that at *Paris* the Night of *May* 21 is 8 Hours 34 Minutes, I divide these 8 Hours 34 Minutes by 12, and so have 42 Minutes and 50 Seconds for the extent of an unequal Hour, which being multiplied by $9\frac{3}{5}$ (the number of unequal Hours from the rising of the Moon to Sunrise) gives in the Product 6 equal Hours and about 51 Minutes for the value of the said number of unequal Hours.

COROLLARY.

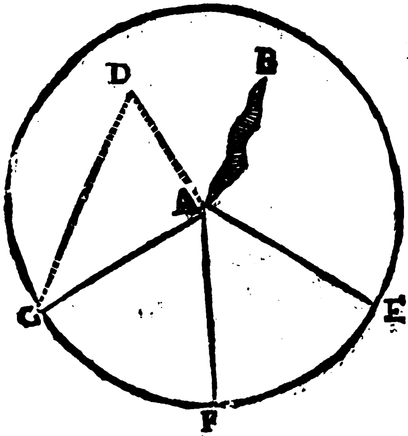
Here we see that if we know the time of the Rising of the Sun, we may from this Problem compute the time of the Moon's Rising; for, if to the hour of the Sun's Rise, *viz.* 4 Hours and 17 Minutes, we add 12 Hours, and from the Sum 16 Hours and 17 Minutes substract 6 Hours and 51 Minutes (the time between Moon's-rise and Sun's-rise) we have 9 Hours and 26 Minutes for the time of the rising of the Moon.

PROBLEM XXVIII.

To find the height of the Sun and the Meridian Line:

WHEN we shew'd in *Problem* III. the way of taking the Latitude of a Place, we then supposed the Altitude of the Sun and the Meridian Line to be known. So, we come now before we conclude to shew you how to find these.

First for the Altitude of the Sun any hour of the Day, Raise at Right Angles upon an Horizontal Plain the Stylus or Pin AB of what length you will, and mark a Point such as C at the extremity of the shadow of the Style AB, at the very time that you would know the Elevation of the Sun upon the Horizon. Then draw from the foot of the Style A to the Point



of the shadow C, the Line AC representing the Vertical of the Sun; and the Line AD equal to the Style AB, and perpendicular to the Point A. At last draw from the Point of the shadow C to the Point D the Line CD, representing the Radius of the Sun drawn from its Center to the Extremity B of the Style AB; which at the Point C, will make with the vertical of the Sun AC, the Angle ACD, and that Angle measured gives the degrees of the height of the Sun.

In the second place, to find the Meridian Line; mark upon any Horizontal Plain about two or three hours before Noon, the Point of the shadow C; and from the Root of the Style A, which represents the Zenith, draw thro' the Point C the Circumference of a Circle CFE, which shall represent the *Almicantarat* of the Sun; then mark after Noon, a second Point of the shadow, such as E, when the Extremity of the shadow of the Style AB is return'd to the Circumference CFE; and having divided the Arch CE into two equal parts at the Point F, draw from that Point

Point F to the Root of the Style A the Right Line FA, which is the Meridian Line demanded.

PROBLEM XXIX.

To know the Calends, Nones, and Ides of every Month of the Year.

THE Calends, Nones, and Ides, formerly in use among the *Romans*, are easily known by these three Latin Verses;

> *Principium Mensis cujusque vocato* Kalendas,
> *Sex Majus* Nonas, *October, Julius & Mars,*
> *Quatuor at Reliqui; dabit* Idus *quilibet Octo.*

The first of these Verses shews that the *Kalends* are the first day of each Month, the first day of the Month among the *Romans*, being the first day of the Apparition of the Moon at Night, on which they had a custom of calling in to the City the Country People to tell them what they were to do the rest of that Month.

The second Verse gives us to know that the *Nones* are the *7th* day of the four Months, *March, May, July,* and *October*; and the fifth day of the other Months: And from the third Verse we learn, that the *Ides* come eight days after the *Nones, that is,* on the fifteenth day of *March, May, July* and *October*, and the thirteenth of the other Months.

The *Romans* counted the other days backwards, still diminishing the Number; for the days between the Calends and the *Nones* of any Month, were denominated from the Nones; as in the Month of *March* the second day was *Sexto Nonas*, the third *Quinto Nonas*; the fourth *Quarto Nonas*; the fifth *Tertio Nonas*; and the sixth not *Secundo* but *Pridie Nonas*; the meaning of all which was, *six, five, four, &c.* days before the *Nones*, the Præposition *ante* being understood. In like manner the days between the *Nones* and the *Ides*, were denominated, *Septimo, Sexto, Quinto,* &c. *Idus*, the Præposition *ante* being still understood. The days between the *Ides* of a Month, and the Calends of the next, took their Denomination after the same manner from the succeeding Calends.

PRO-

PROBLEMS
OF THE
MECHANICKS.

MOST, if not all, the Problems of the Mechanicks are more useful than curious, in regard they commonly relate to the execution of the most necessary things in the way of Life, so that one might be very large upon that Subject: But, that this Volum may not exceed the due bounds, we shall here confine our selves to such Problems as seem to be the most useful, the most agreeable, and the easiest to be understood and practis'd.

PROBLEM I.

To keep a heavy Body from falling, by adding another heavyer Body to that side on which it inclines to fall.

A Table AB being set Horizontally, lay upon it a Key, (for instance) CD, which is like to fall because the part ED is suppos'd heavier than EC; add to its extremity D a crooked Stick DFG with a weight H made fast to the end of it G, and so posited as to answer perpendicularly to the Point E. In this case 'tis evident that the Key CD will not fall, upon the account, that in order to its fall EC which lies Horizontally must incline, and its Extremi-

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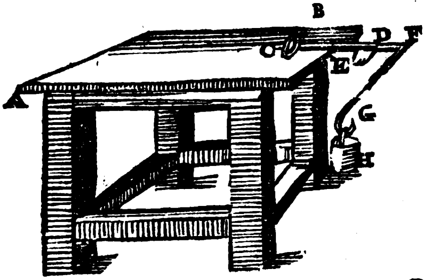
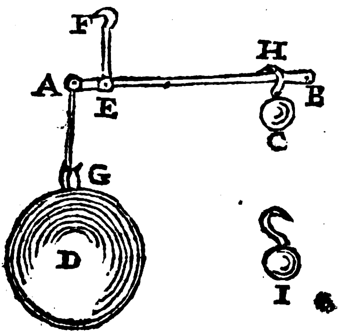


ty C make the Arch of a Circle, with its Center at the Point of rest E; but this can't be unless the weight H ascends instead of descending. And therefore the Point H and the Key CD will continue in repose.

PROBLEM II.

By means of a small Weight and a small pair of Scales, to move another Weight as great as you will.

I Suppose the Ballance AB is made fast at F above its Center of Motion E, by an unmoveable Hook EF, and that near its Extremity B there's a small weight C made fast at H; by vertue of which we



want to raise a huge weight D, which might represent the Earth if we knew its weight; and had a firm place to fix the Scales at.

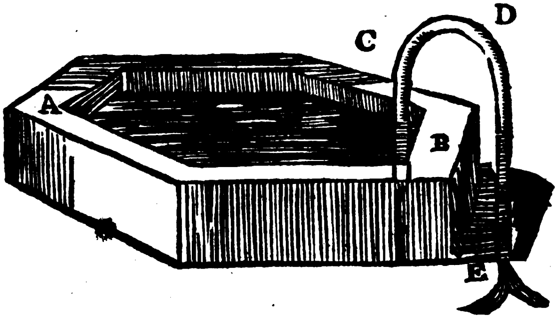
To find the distance EH of the Weight C from the Center of Motion E, at which the Weight D is to be promov'd by the smaller Weight C; see for a fourth proportional EH to the Weight I lesser than the Weight C, to

C, to the great Weight D, and to the Line AE which ought to be very small. By this means you have the Point H, from which the Weight I being suspended will hold the Point D in *Æquilibrio*, as appears from that general Principle of the Mechanicks, that two Weights continue in *Æquilibrio* about a fix'd Point, when their distances from that Point are in a reciprocal proportion to their Gravity. And therefore if instead of the Weight I, you put the greater Weight C at H, this greater Weight C will be able to move and cast the Weight D.

PROBLEM III.

To empty all the Water contain'd in a Vessel with a Syphon or Crane.

LET the Vessel AB, be propos'd to be emptyed without stooping the Vessel or piercing the Bottom. Take a crooked Syphon such as CDE full of Water, one of whose Extremities touches the bottom of the Vessel AB, and the end E stop'd close with your Finger is lower than the bottom of the Vessel AB. Then take away your Finger, and the Water of the Crane CDE running out at the extremity E, the



Water in the Vessel will enter at the other end, and supplying the place of that which is gone will continue to follow it, and run out till none, or very little is left in the bottom of the Vessel. This Experiment will succeed the easier, if the Syphon CDE be bigger

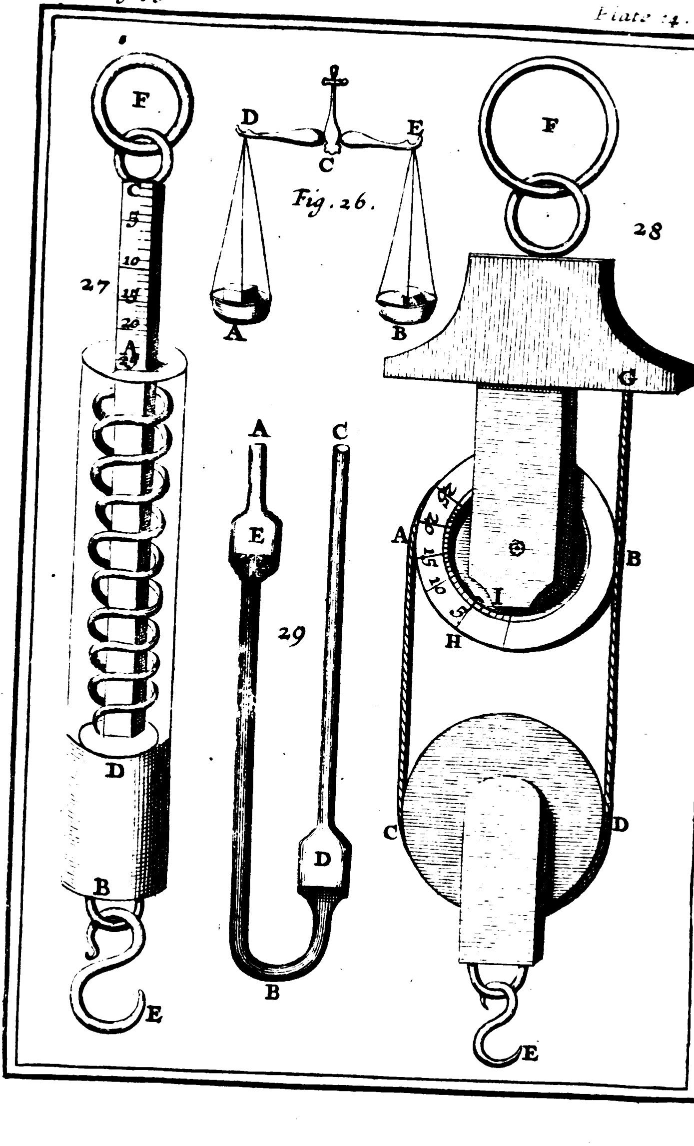


Fig. 26.
27
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29
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B
E
D
E
C
A
B
A
E
C
D
B
F
G
A
B
I
H
5
10
15
20
25
C
D
E

bigger in the middle than at the two ends, becauſe then the Water in the middle will weigh more, and have more force in ſucking or drawing the Water from the Veſſel. See *Probl.* XIV.

Thus 'tis that we eaſily empty a Cask of Wine by the Bung, without opening the Head; which may likewiſe be done by an empty ſtraight Pipe ſmaller at the two ends than in the middle, plunged in at the Bung, for then the Wine will enter it; and if with your Finger you ſtop the upper end of it, and ſo take the Pipe out, you'll find it full of Wine, which you may pour into a Glaſs, by taking off your Finger, which will make the Wine deſcend at the other end, becauſe the Air is free to ſupply its room.

By the ſame means we can make Water riſe from a low place in order to deſcend to a lower, provided the eminence over which 'tis to paſs is not higher than 32 Foot: For we know by many Experiments that the gravity of the Air, to which the Modern Philoſophers attribute what others call'd *fuga vacui*, can't make Water riſe higher than about 32 Foot.

'Tis likewiſe by means of a crooked Pipe, that, without an Aqueduct and with very little Charge, we can carry Spring-Water from the top of one Mountain to another of equal or little leſs height. For this end, we take a long Leaden Pipe which deſcends from the Spring to the Valley, and with a bend riſes again to the top of the adjacent Mountain; for the Water entring the Pipe aſcends about as far as it deſcends; I ſaid *about as far*, upon the conſideration that the Reſiſtance of the Air keeps the Water from riſing to the exact height.

PROBLEM IV.

To make a deceitful Ballance, that ſhall appear juſt and even both when empty, and when loaded with unequal Weights.

MAke a Ballance the Scales of which A,B, are of unequal Weight, and of which the two Arms CD, CE are of unequal length, and in reciprocal proportion to theſe unequal Weights; that is, the ſcale Plate 14. Fig. 26.

ſcale A is to the ſcale B, as the length CE is to the length CD; for thus the two ſcales A, B, will continue in *Æquilibrio* round the fix'd Point C; and the ſame will be the Caſe, if the two Arms CD, CE are of equal length and of unequal thickneſs, ſo that the thickneſs of CD is to that of CE, as the weight of the ſcale B is to that of A. This ſuppos'd, if you put into the two ſcales, A, B, unequal weights which have the ſame *Ratio* with the Gravities of the two ſcales, the heavier weight being in the heavier ſcale, and the lighter in the lighter ſcale, theſe two Weights and Scales will reſt in *Æquilibrio.*

We'll ſuppoſe that the Arm CD is three Ounces, and the Arm CE two Ounces, and reciprocally the ſcale B weighs three Ounces, and the Arm A two; in which caſe the ballance will be even when they are empty. Then we put a weight of two pound into the ſcale A, and one of three into B, or elſe one of four into A, and one of ſix into B, &c. and the ballance continues ſtill even, becauſe the weights with the gravity of the Scales are reciprocally proportional to the length of the Arms of the Beam. Such a pair of Scales is diſcover'd by ſhifting the weights from one ſide to another, for then the Ballance will caſt to one ſide.

PROBLEM V.

To make a new Steel-yard for carrying in one's Pocket.

Plate. 14. Fig. 27. THere has lately been invented in *Germany*, a new Steel-yard fit to be carried in one's Pocket, which is very convenient for weighing off-hand any indifferent big Weight, ſuch as Hay or Merchants Goods, from one to fifty pound weight and upwards.

This Machine is made of a Copper Pipe or Gutter AB, about ſix Inches long, and almoſt eight Lines broad, and within it is a Spring of Steel in the form of a Screw. At the upper end towards A there's a ſquare Hole, thro' which there paſſes a ſquare Rod of Copper CAD that runs thro' the Screw, and upon this Rod are the diviſions of Pounds mark'd, by hanging ſucceſſively to the Hook E a weight of one, of two, of of three Pound, *&c.* and running a Score upon the Rod where 'tis cut by the Square Hole A; which will fall upon different Parts or Points according to the different weight faſten'd to the Hook E, for theſe different weights extend the Spring, and ſo puſh out a greater or leſſer part of the Rod, according as they are more or leſs heavy. Here the Steel-yard is ſuppoſed to be ſuſpended by the Ring F, and the Rod is ſecured at the lower end by a Copper Ferrel.

The Sieur *Chapotot*, Ingeneer and Inſtrument-maker to the King of *France*, has invented another ſort of Peſon or Steel-yard in the form of a Watch, by which the gravity of any weight may be taken with great facility. Remark. Plate 14. Fig. 28.

This new Machine is compos'd firſt of two Pullies AB, CD, made faſt upon their Axletrees, and kept together by a String or Cord. The upper of theſe two, AB, is hollow like a barrel of a Watch, and contains within a Spring like that of a Watch, which being ſtop'd by the Axletree of the Pully, will have the ſame effect with that of a Watch.

The ſame Pulley AB contains the diviſion of Pounds, mark'd Mechanically as in the Steel-yard deſcrib'd but now, namely, by clapping ſucceſſively upon the Hook E a weight of one, of two, of three Pound, *&c.* the Machine being ſuſpended by the Ring F: For thus the gravity of the weight will turn the Pully AB, and ſo by vertue of the different gravities, the Point I will anſwer to different Points of the Pully AB, upon which theſe different Points are mark'd with the number of the reſpective Pounds hanging at E. Such is the new Machine with which any thing may be weigh'd, after the ſame manner, as with that laſt deſcrib'd.

One may eaſily perceive by the Figure, that the String or Cord BDCA keeps up and runs under the lower Pully CD, and is made very faſt at one end at the Point G, and at the other end at ſome Point of the other Pully, ſuch as H: Which contributes very much to turn the Pully AB round its Axletree when 'tis drawn or pull'd by the part AC of the Rope, by reaſon of the weight at the Hook E; which weight will then be mark'd upon the Pully AB by the Point I, the Machine being ſuſpended upon one's Thumb,

 or,

or, which is better, upon a Stick run through the Ring F.

PROBLEM VI.

To observe the various Alterations of the Weight of the Air.

THE Air being a Body must needs have Gravity; a proof of which we have in a Football or Bladder, which weighs more when blown than when 'tis not, and in an infinite number of other Experiments. *Torricelli* was the first that assign'd the gravity of the Air for the cause of all the effects that the Philosophers had till then imputed to a *fuga vacui*. As this Gravity is not infinite, the Sphere of the Air being limited, so its effect is limited, as we see in a Pump, where Water will not rise higher upon drawing up the Sucker, than 32 Foot, because the gravity of the Air can't force it beyond that height. In like manner in drawing up Quicksilver with a Syringe, 'twill rise no higher than two Foot and about three Inches, (at which height it weighs equally with a Column of Water of 32 Foot high) more or less according as the Air is fraighted with Vapours, or condensated with Cold.

Thus you see the gravity of the Air is not always equal at the same place; but varies, as 'tis more or less stuff'd with Vapours. Now, this difference of gravity is known by an Instrument call'd a *Barometer*, which is contriv'd after the following manner.

Of *Barometers*. Plate 14. Fig. 29.

Take a crooked or bended Tube of Glass, such as ABC, upon which are two Cylindrical Boxes E, D, mutually distant in height 27 Inches, that being much about the height to which the gravity of the Air can raise Quicksilver; that is to say, a Prism of Air from the Earth to the uppermost Surface of the Air is in *Æquilibrio* with about 27 Inches of Quicksilver in a Tube or Gutter perpendicular to the Horizon.

The Box D must be much bigger than the rest of the Tube CD, for a reason that you'll meet with in the Sequel; and the extremity A ought to be hermetically stop'd, that is, stop'd with its own proper Substance;

ſtance; but the other extremity C muſt be open; and there we muſt pour in as much Quickſilver as fills the Tube ABC, from the middle of the Box D to the middle of the other Box E, the capacity of which ſhould be almoſt equal to that of the firſt D.

At laſt you muſt fill the remainder of the Tube CD with ſome other Liquor that do's not freeze in Winter, nor yet diſſolve Quickſilver. Such is common Water mix'd with a ſixth part of Aquafortis.

If you place the Tube ABC thus fill'd with Air, and Water, and Mercury in the middle, perpendicularly againſt a Wall in a Room, where it may be conveniently ſeen and not hurt, you'll ſee the Quickſilver aſcend or deſcend in the two Boxes D, E, upon the leaſt alteration of the gravity of the Air. When the Air is heavier it preſſes the Water of the Tube CD, and makes it deſcend in the Box D, as well as the Quickſilver, which riſes as much in the Box E. If the Mercury deſcends thro' the gravity of the Air, for Example, a Line in the Box D, 'twill riſe a Line in the Box E, and the Water in the reſt of the Tube CD will deſcend into the Box D, ſo that if the Box D is ten times more capacious than the reſt of the Tube CD, 'twill require ten Lines of the Water of the Tube CD to fill one Line of the Box D; and thus the leaſt alteration of the gravity of the Air is very ſenſibly perceiv'd, eſpecially if the Boxes E, D, are made large. For the diſtincter perception of this Alteration, there is uſually a ſlip of Paper divided into Inches and Lines, paſted on along the Tube ABC; in order to obſerve the Diviſion at which the Mercury hangs; as we do in the *Thermometers*, which ſerve to diſtinguiſh the Degrees of Heat and Cold, as the *Barometer* do's the greater or leſſer gravity of the Air; which may likewiſe be done by a ſingle Tube of Glaſs three or four Foot long, ſhut at one end and fill'd with Quickſilver, after this manner.

What a Thermometer is.

Having ſtop'd with your Finger the open end of the Tube, to keep the Quickſilver from dropping out when the Tube is inverted, dip the open end into other Quickſilver in a Veſſel, then take off your Finger, and the Tube will not be quite empty, but the Quickſilver will hang in it to the height of 27 Inches and a half, more or leſs, according to the different temperature of the Air. Here the Mercury hangs by reason of the gravity of the whole Mass of Air, which gravitating upon the Mercury in the Vessel, presses it down and hinders its rise, so as to give place to that in the Tube, which by consequence can't descend.

PROBLEM VII.

To know by the Weight of the Air, which is the highest of two places upon the Earth.

THE gravity of the Air is not every where equal, for it gravitates less upon eminences and tops of Mountains, than in such places as lie lower, as Valleys; by reason that there's more Air over Valleys than over Mountains; just as the bottom of a Pit is more press'd by the gravity of Water when 'tis full, than when 'tis half full; for Liquid Bodies gravitate according to their height.

Thus we know by experience, that in all level places, or such as being equally high are equidistant from the Center of the Earth, Quicksilver rises in a *Barometer* to an equal height; and to a lesser height in places that lie lower. From hence we may conclude, that two Mountains, for example, are of equal height, if the Quicksilver rises equally upon both; and that one is higher than t'other, if the ascent of the Mercury is unequal.

Remark. To determine, as near as may be, the height of any place above the Plain of the Horizon, we must mind the following Experiments made by Mr. *Pascal* of the gravity of the Air upon the level of the Sea, and in places lying 10, 20, 100, 200 and 500 Toises higher, when the Air was indifferently charged with Vapours.

Upon the level of the Sea, the attracting Pump raises Water 31 Foot and about 2 Inches; and in places that are 10 Toises higher, it raises it 31 Foot and 1 Inch. Here you see 10 Toises Elevation causes 1 Inch Diminution. *(A Toise is 6 Foot.)*

By

By other Experiments we learn that in places that are 20 Toises higher than the Sea, the Water rises only 31 Foot; and in those of 100 Toises higher only 30 Foot 4 Inches; in the height of 200 Toises, only 29 Foot 6 Inches; and at 500 Toises about 27 Foot.

PROBLEM VIII.

To find the gravity of the whole Mass of Air.

WE found in *Problem* VII. *Cosm.* that the Surface of the whole Earth is 32356800 square Parisian Leagues, which amounts to 4659379200000000 square Feet. We must know likewise that a Cube foot of Water weighs about 72 Pound; and consequently that a Prism of Water having a square foot for its Base, and 32 foot for its height, weighs 2304 Pound, as appears by multiplying 72 by 32.

In fine, we must know, that considering that the gravity of the Air can't raise Water above 31 or 32 Foot, if we suppose all the places of the Earth to be equally loaded with Air, tho' indeed that is not absolutely true, since all places are not equally remote from the Center of the Earth, and the Air is not every where nor at all times equally pure; upon this Consideration, I say, we may suppose all the parts of the Earth to bear as great a pressure from the Air, as if they were cover'd with Water to the depth of 31 or 32 Foot.

Upon this Supposition, which may readily be receiv'd in Mathematical Recreations, 'tis manifest that if the whole Earth were cover'd with Water 32 Foot high, there would be as many Prisms of Water 32 Foot high, as there are square feet upon the Surface of the Earth, *viz.* 4659379200000000 Prisms of Water; which Number multiplied by 2304. (the weight of one of these Prisms in Pounds) yields 10735209676800000000 pounds for the weight of the whole mass of Air.

PROBLEM IX.

To find by the Gravity of the Air the Thickness of its Orb, and the Diameter of its Sphere.

BY the thickness of the Orb of the Air we understand the distance from its upper Surface where its gravitation ceases, to the Surface of the Earth, which we suppose to be in the Center of the Sphere of the Air, without farther enquiry into the precise truth of that Supposition, the discussion of which would be of little consequence in Mathematical Recreations.

To find in the first place this thickness, let's consider, that if 10 Toises (or 60 Foot) of height, cause an Inch diminution of the effect of the gravity of the Air, as we observ'd *Probl.* VII. and if the whole weight amounts only to 31 Foot 2 Inches, that is, 374 Inches, after a diminution of which the Air will cease to gravitate: We may find the thickness of the mass of Air, or the distance of its upper Surface from the Earth, by the Rule of Three Direct: If the diminution of one Inch arises from 10 Toises of height, what height must the diminution of 374 Inches proceed from? Here multiplying 374 by 10, you have 3740 Toises for the thickness in question, which doubtless is much greater.

In a second place, to find the Diameter of the Sphere of the Air, we take the Diameter of the Earth, which in *Probl.* VII. *Cosm.* we found to be 3210 Parisian Leagues, or 6420000 Toises; and add to it 7480 the double of 3740 the thickness of the Air, and the Sum gives 6427480 for the Diameter of the Sphere of the Air.

PRO-

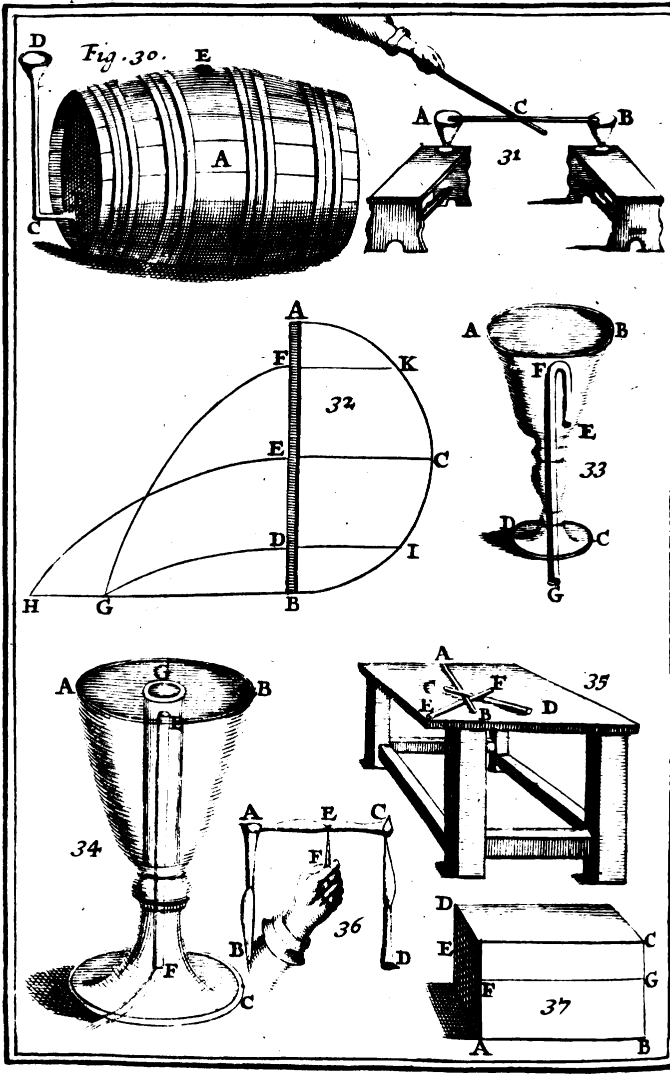
Fig. 30.
D
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C
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D
I
H
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A
B
F
E
D
C
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34
A
G
B
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C
35
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F
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B
D
36
A
E
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37
D
E
C
G
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A
B

PROBLEM X.

To fill a Cask with Wine or any other Liquor by a Tap in the lower part.

WE've intimated already that Liquid Bodies gravitate only according to their height, and so to fill the Cask A not by the Bung E, but by a lower Tap B in the lower part of it; we need only to put into that aperture a crooked Pipe, such as BCD, with a sort of Funnel in its upper end D, which ought to be as high as the Cask; and pour the Wine in at the Funnel D, which falling down the branch DC that ought to be very near Perpendicular, and entring the Cask by the other branch CB, which ought to be level, will assume an Horizontal Situation, and keep an equal height in the Cask with that in the Crane; and 'tis for that reason that we know the Cask to be full when the branch CD is full. Plate 15. Fig. 30.

PROBLEM XI.

To break with a Stick another Stick resting upon two Glasses, without breaking the Glasses.

THE Stick AB that is to be broken must not be very thick, nor yet lean much upon the Glasses; it ought as near as possible to be equally thick all over, for the easier finding of its Center of gravity C, which will then be in the middle. Plate 15. Fig. 31.

The stick AB being thus qualified, we lay its two ends, A, B, which ought to terminate in a Point, upon the brim or edge of two Glasses of equal height, so that the stick AB do's not lean to one side or end more than t'other, and the two pointed ends rest but lightly upon the edge of each Glass, to the end that when it bends a little thro' the violence of the stroak, it may easily slip off, and break at the same time. This done, we take another stick, and with that give a smart blow upon the middling Point C, which being the Center of gravity will receive all the force of the blow; thus, will the stick AB break, and that the more easily that the blow is violent, and fall clear of the two Glasses which remain unbroken, because the stick lay but very gently and equally upon the brim of each; for if it rests more upon one Glass than t'other, 'twill press that one most, and so may break it.

PROBLEM XII.

To find a Weight of a given number of Pounds, by the means of some other different Weights.

THIS Problem may easily be resolv'd by the double or triple Geometrical Progression, especially the Triple, 1, 3, 9, 27, 81, 243, &c. the property of which is such, that the last number contains twice all the rest and one more, when the Progression commences from Unity, as here. So that if the given number of Pounds is, for example, from 1 to 40, which is the Sum of the four first Terms, 1, 3, 9, 27; you may make use of four different Weights, one of which weighs 1 Pound, another 3, a third 9, and the fourth 27; and by them find the weight of any other number of pounds, for example 11 pounds.

Plate 14. Fig. 26. For, since the given number 11 is less than 12 by 1, and since 12 is the sum of the Weights 3 and 9 which you have; if you put into the Scale A the one pound weight, and into the other Scale the 3 and 9 pound weights, these two weights will then weigh only 11 pound, by reason of the one pound weight in the other Scale; and consequently if you put any substance into the Scale A along with the 1 pound weight, which stands in Æquilibrio with the 3 and 9 in the other Scale, you may conclude that Substance weighs 11 pound.

In like manner to find a 14 pound weight, put into the Scale A, the 1, 3, and 9 pound weights, and into the Scale B that of 27 pound, because this 27 ℔. weight outweighs the other three by 14. To find a weight of 15 ℔. put in one Scale 3 and 9, and in the other 27, which exceeds the other two by 15.

PROBLEM XIII.

A Pipe full of Water being perpendicular to the Horizon, to find to what distance the Water will flow thro' a hole made in a given Point of the Pipe.

DEscribe round the Pipe AB which is suppos'd to be full of Water and perpendicular to the Horizon, the Semicircle ABC, and bore the Pipe in several places, as at the Points D,E,F, for the Water to flow out at; In this case, the Water in flowing out will make the Semi-Parabola's DG, EH, FG; of which the Amplitudes BG, BH are double the corresponding Sines, *i. e.* the Lines DI, EC, FK, perpendicular to the Diameter AB; the Amplitude BG being the double of DI and of FK, and BH the double of EC: So that if the Point E is the middle of the Pipe AB, or the Center of the Semicircle ABC, EC being the greatest Sinus, the amplitude EH will likewise be the greatest; and since the Sines equally remote from the Center E, as DI, FK, are equal, so the two Semi-Parabola's DG, FG, found by the fall of the Water thro' the holes D and F equidistant from the Center E, have the same Amplitude BG. 'Tis evident that the greatest Amplitude BH is equal to AB the height of the Pipe, and that its extremity B is the focus of the Semi-Parabola EH, and by consequence if you broach the Pipe AB at its middle-point E, the Water will spout out to a distance equal to the length of the Pipe AB. Plate 15. Fig. 32.

But if you make a hole in the Pipe above or below the middle E as at F, you'll find the distance BG, to which the Water will then flow, by describing round the Pipe AB or round a Line equal to it, the Semicircle ABC, and drawing from the Point F to the Diameter AB the perpendicular FK, which will be half the distance sought for.

Or if the Pipe is so large, that you can't draw a Circle round it, do it by Arithmetick, multiplying the two parts AF, BF, into one another, the square Root of which Product gives the quantity of the Perpendicular FK, or half the distance BG. Thus, if AF is

2 In-

2 Inches, and BF 32 Inches, the length of the Pipe being 34. multiply 32 by 2, and from the Product 64 extract the ſquare Root 8, the double of which is 16 Inches for the diſtance BG.

PROBLEM XIV.

To contrive a Veſſel, which keeps its Liquor when fill'd to a certain height, but loſes or ſpills it all when fill'd a little fuller with the ſame Liquor.

Plate 15. Fig. 33.

TAKE a Glaſs, for example ABCD, and run thro' the middle of it a ſmall bended Pipe or Crane EFG open at the end E next the bottom of the Glaſs, and likewiſe at the other end G which muſt be lower than the bottom of the Glaſs; for then the Water or Wine pour'd into the Glaſs continues in it while the branch EF is filling, and till it comes to the bend F or the uppermoſt part of the Crane, which withal ſhould be a little lower than the upper edge of the Glaſs: But after that if you continue to pour more in, 'twill riſe higher in the Concavity of the Glaſs, and not finding place for a farther aſcent into the Crane by reaſon of its bending downwards at F, 'twill change its Aſcent into a Deſcent thro' the branch FG, and continue to deſcend and run out by the end G, as long as you continue to pour in; nay, when you have done pouring, you'll ſee that all that was in the Glaſs before is gone.

You may make the Water run out at the lower end G, tho' the Glaſs is not fill'd up to the top of the Crane, namely, by ſucking at the lower Aperture G the Air contain'd in the Crane, for then the Water will neceſſarily ſucceed in the room of the Air, and continue to deſcend thro' the branch FG till the Glaſs is empty, eſpecially if the Orifice touches the bottom of the Glaſs, as you ſaw in *Prob.* III.

Plate 15. Fig. 34.

Or elſe; run the ſmall Pipe EF perpendicular down thro' the Glaſs ABCD; let the Pipe be open at both ends, E and F, the uppermoſt of which, *viz.* E ought to be a little lower than the brim of the Glaſs, and the other end F a little lower than the bottom of the Glaſs. Put this ſmall Pipe EF in another larger Pipe

GI

GI ſtop'd at the upper end G, which muſt be a little higher than the end E of the firſt and ſmaller Pipe EF, and open at the lower end I, which muſt touch the bottom of the Glaſs if you would have all the Water to run out, which 'twill do when it riſes to G, for then paſſing thro' the Orifice I of the Pipe GI, 'twill enter the Pipe EF by the end E, and run out at the other end F.

PROBLEM XV.

To make a Lamp fit to carry in one's Pocket, that ſhall not go out tho' you roll it upon the Ground.

TO make a Lamp that never ſpills its Oil, and never goes out in any poſition whatſoever, make faſt the Veſſel that contains the Oil and the Match to an Iron or Braſs Ring, with two ſmall Pivots or Hinges diametrically oppoſite, that ſo the Veſſel may by its weight continue in *Æquilibrio* round the two Hinges, and turn with freedom within the Circle, ſo as to keep always to an Horizontal Poſition, as in your Sea-Compaſſes, which have two ſuch Circles to keep them Horizontally: And in like manner this firſt Circle ought to have two other Pivots diametrically oppoſite, which enter into another Circle of the ſame Subſtance; and that ſecond Circle has two other little Hinges inſerted in another Concave Body that ſurrounds the whole Lamp. Thus the Lamp with its two Circles may turn freely upon its ſix Hinges, which give to the Lamp when 'tis turn'd, ſix different Poſitions, *viz.* up and down, forwards and backwards, to the right and left, and which ſerve to keep the Lamp in an Horizontal Poſition, which being in the middle do's always reſt upon its Center of gravity, that is, its Center of gravity is always in the Line of Direction, which hinders the Oil to ſpill, turn it which way you will.

PROB-

PROBLEM XVI.

To place three sticks upon an Horizontal Plain, in such a manner, that each of 'em rests with one end upon the Plain, and the other stands upright.

Plate 15. Fig. 35.

TO make three Sticks, or three Knives, &c. keep one another up while each of 'em rests with one end upon a Table, even tho' a weight were laid upon 'em: Incline or slope one of 'em, as AB, raising the end B aloft, and resting the other end A on the Table; then put one of the other two Sticks as CD, a-cross over it, raising the end C, and touching the Table with the other end D; then take the third Stick EF and compleat the Triangle with it, making one of its ends E rest on the Table, and running it under the first AB so as to rest upon the second CD. The three Sticks lying thus a-cross one another, will mutually support one another, so that they cannot fall, through any weight upon 'em, unless they bend or break thro' the over-bearing Gravitation; which if moderate, will, instead of making them fall, strengthen them and keep them firmer in that Position.

PROBLEM XVII.

To make three Knives turn upon the point of a Needle.

Plate 15. Fig. 3

TO the end of the Haft of one Knife, as AB, fasten the point of another Knife AC, so as to make BAC a right Angle or thereabouts; then fasten to the end of the haft of the Knife AC the point of a third Knife CD, so as that the Angle ACD comes near to a right Angle; for thus the three Knives, AB, AC, CD, will be dispos'd in the form of a Ballance; the two Scales of which are represented by the two Knives that hang, AB, CD, and the Beam by the Knife AC, upon which by consequence you will find after several essays the Center of Motion, or the fix'd Point, from which the Ballance being suspended, will rest in *Æquilibrio* with its

its two Scales AB, CD. To this Point, such as E, put a Needle EF at Right Angles, so that the Knife AC, with the two other Knives, AB, CD, may remain in *Æquilibrio* round this the Center of their compounded Gravity. The Needle must be held very tight upon the Perpendicular, and then the least force, such as that of the blowing of one's Mouth, will make them turn and dance, as it were, round the point of the Needle without falling.

PROBLEM XVIII.

To take up a Boat that's sunk with a Cargo of Goods.

IF a Boat sinks in a deep River, you may bring her up again, by getting two other Boats, one empty, and the other deep loaded with some heavy Substance, as Stones, *&c.* You must tie these two Boats to the Boat that's sunk with two Ropes, and extending the Rope of the deep loaded Boat, unload her into the other that's empty; which will raise the first Boat a little, and make it draw along with it the Boat that's under Water, and at the same time make the second Boat swim so much deeper in the Water. The second Boat being thus loaded, you must bend her Rope and unload her again into the empty Boat, and thereupon she becoming lighter, will rise and draw the Boat under Water so far further up. Thus you continue to load and unload till you bring the Boat even with the Water, and then tow her to the side.

PROBLEM XIX.

To make a Boat go of it self up a rapid Current.

THE more rapid a River is, the easier 'tis to make a Boat go of it self up against the Current, by a Rope and a Wheel with its Axletree that has Wings like the Wings or Sweeps of a Mill-wheel.

Fix the Wheel with its Axletree at the place to which you would have the Boat conducted, and let its Sweeps be as deep in the Water, as there is occasion for

for turning it round; tie a Rope to the Boat and to the Axletree of the Wheel, which turning with its Axletree by vertue of the rapidity of the Water, will wind up the Rope on its Axletree, and so by the successive abbreviation of the Rope, drag it against the Current to the place propos'd; which 'twill reach so much the sooner that the Current is rapid, the rapidity quickening the motion of the Wheel.

PROBLEM XX.

To find the weight of a Cubical foot of Water.

WE intimated above *Prob.* VIII. that a Cubical foot of Water weighs about 72 Pounds; which is easily tried by filling a Vessel, the Concavity of which is just a Cubcical foot, and measuring the Water. But an easier way is this.

Plate 15. Fig. 37. Get a Rectangle Parallelepipedon, as ABCD, of some homogeneous Matter, the specifick Gravity of which is less than that of Water, such as Firwood, so that, when put into Water 'twill not sink quite: Take an exact account of the weight of this solid Body, which we shall suppose to be 4 pound.

Put it into Water, and make a mark where it ceases to sink, as EFG; for then the space taken up by it in the Water being ABGE, the Water that would fill that space, would weigh exactly 4 pound, that is, as much as the Body ABCD weighs in the Air, by this General Principle of the Hydrostaticks, that the weight of a Body is equal to that of a Column of Water equal to that the room of which is taken up in the Water.

This Column of Water, which is here represented by ABGE, may be measur'd Geometrically, by multiplying the breadth EF, which we shall suppose to be 4 Inches, by the height AF, which we suppose to be 3 Inches; and the product 12 by the length AB, or FG, which we shall call 8 Inches: For thus you have 96 Cubical Inches for the solidity of the Prism ABGE.

Thus we know that 96 Inches of Water weigh 4 pound; and to know the weight of a Cubical-foot of the

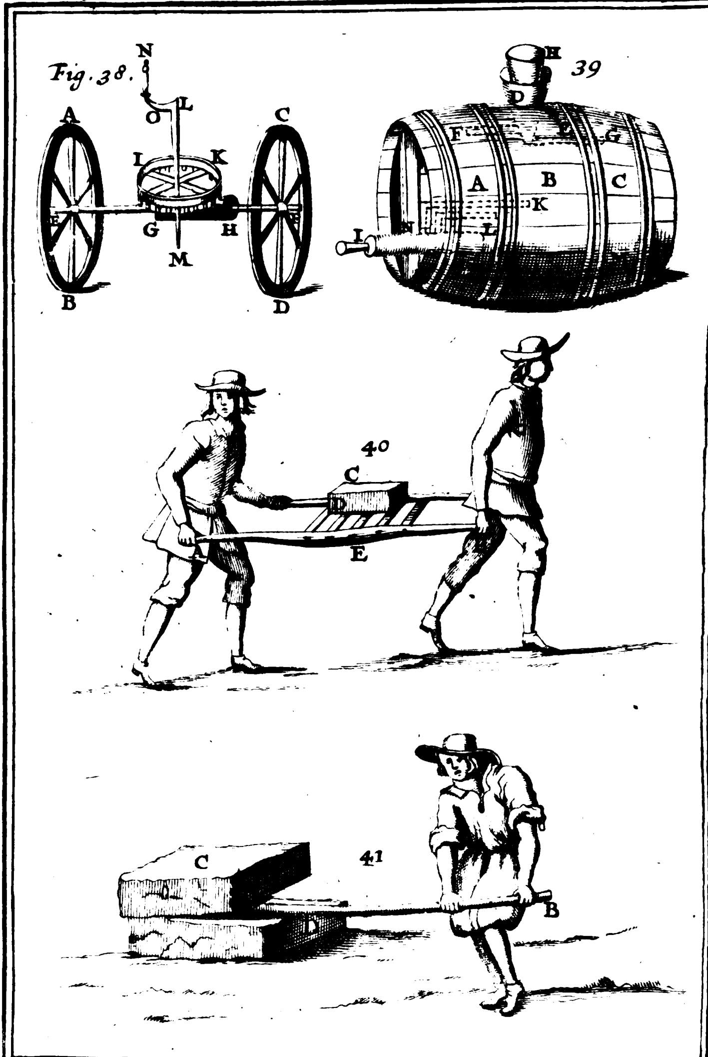
Fig. 38.
N
O
L
A
C
L
K
E
G
H
F
M
B
D
39
H
D
F
E
G
A
B
C
K
I
N
L
40
C
D
E
41
C
D
B

the same Water which is 1728 Cubical Inches (as appears by multiplying 12 by 12, and the Product by 12 again) we must say by the Rule of Three direct; If 96 Inches weigh 4 Pound, how much will 1728 Inches weigh; that is to say, we must multiply 1728 by 4, and divide the Product 6912 by 96, and so we'll find that a Cubical foot of Water weighs 72 Pound.

PROBLEM XXI.

To make a Coach that a Man may travel in without Horses.

THE two fore-wheels must be little, and moveable round their common Axletree, as in the ordinary Coaches; and the hinder Wheels must be large, as AB, CD, and firmly fix'd to their common Axletree EF, insomuch that the Axletree can't move, without the Wheels move along with it. Plate 16. Fig. 38.

Round the middle of the Axletree EF put a Trundlehead, with strong and close Spindles, and near to that fix upon the Beam a notch'd Wheel IK, the notches of which may catch the Spindles of the Trundlehead, and so in turning with the handle NOL, that Wheel round its Axletree LM, which ought to be perpendicular to the Horizon, it will turn the Trundle GH, and with that the Axletree EF, and the Wheels AB, CD, which will thereupon set forward the Coach, without Horses or any other Animal. I need not tell you that the Axletree must enter into the Beam, in order to turn within it.

There was invented at *Paris*, some years ago, a Coach or Chaise like that in Fig. 42. which a Footman behind the Coach makes to go with his two Feet alternately, by vertue of two little Wheels hid in a Box between the two Hind-wheels, as A,B, and made fast to the Axletree of the Coach. Plate 17. Fig. 42.

In short, the contrivance of the Machine is this. AA in Fig. 43. is a Roller, the two ends of which are made fast to the Box behind the Chaise, B is a Pully upon which runs the Rope that fastens the end of the Planks CD, upon which the Footman puts his Feet. Plate 17. Fig. 43.

E is

E is a piece of Wood that keeps fast the two Planks at the other end, allowing them to move up and down by the two Ropes AC, AD, tied to their two ends. F, F, are two little plates of Iron which serve to turn the Wheels, H, H, that are fix'd to their Axletree, which is likewise fix'd to the two great Wheels, I, I.

Thus, you will readily apprehend that the Footman putting his Feet alternatly upon C and D, one of the Plates will turn one of the notch'd Wheels; for Example, if he leans with his Foot upon the Plank C, it descends and raises the Plank D, which can't rise but at the same time the plate of Iron that enters the notches of the Wheel, must needs make it turn with its Axletree, and consequently the two great Wheels. Then the Footman leaning upon the Plank D, the weight of his Body will make it descend and raise the other Plank C, which turns the Wheel again; and so the Motion will be continued.

Fig. 42. 'Tis easy to imagine that while the two Hind-wheels advance, the two Fore-wheels must likewise advance; and that these will always advance straight, if the Person that sits in the Chaise manages them with Reins made fast to the Forebeam.

PROBLEM XXII.

To know which of two different Waters is the lightest, without any Scales.

TAke a solid Body the specifick gravity of which is less than that of Water, Dale or Firwood, for instance; and put it into each of the two Waters, and rest assured that 'twill sink deeper in the lighter than in the heavier Water; and so by observing the difference of the sinking you'll know which is the lightest Water, and consequently the wholsomest for Drinking.

PRO-

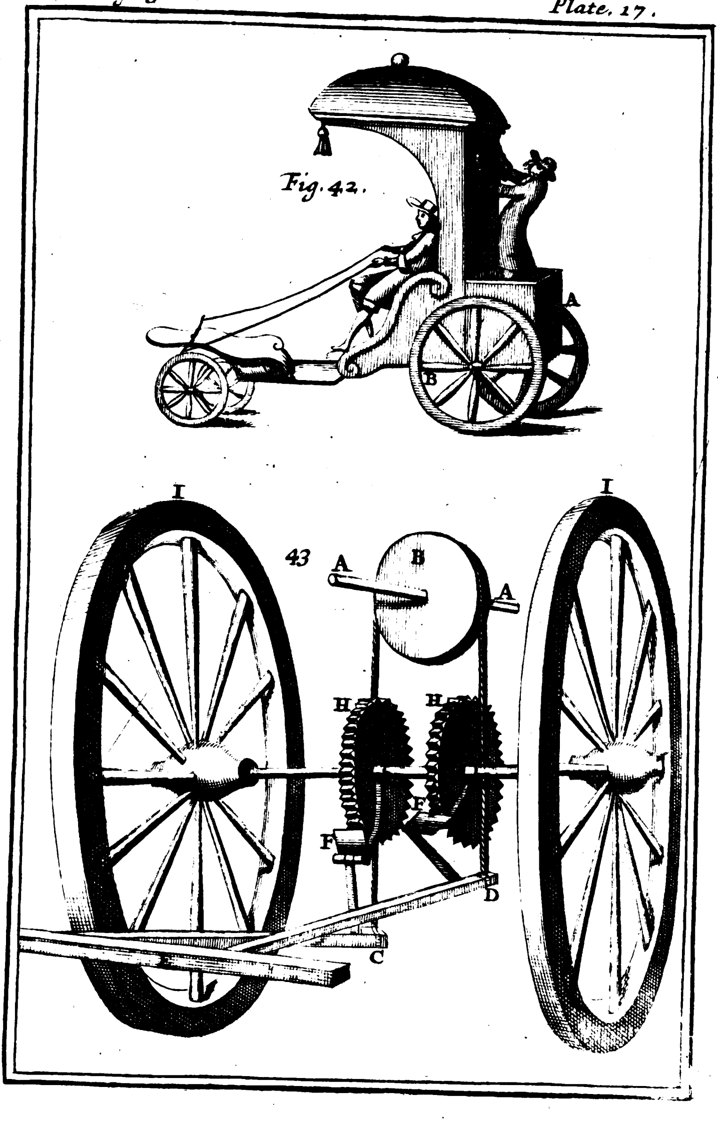
Fig. 42.
A
B
I
I
43
A
B
A
H
H
F
F
D
C

PROBLEM XXIII.

To contrive a Cask to hold three different Liquors, that may be drawn unmix'd at one and the same Tap.

THE Cask must be divided into three Parts or Cells, A, B, C, for containing the three different Liquors, as Red-Wine, White-Wine, and Water; which you may put into their respective Cells at one and the same Bung, thus; Plate 16. Fig 39.

Put into the Bung a Funnel D with three Pipes, E, F, G, each of which terminates in its respective Cell. Upon this Funnel clap another Funnel H with three Holes, that may answer when you will the Orifices of each Pipe; for thus, if you turn the Funnel H so as to make each Hole answer successively to its corresponding Pipe, the Liquor you pour into the Funnel H will enter that Pipe, it being still suppos'd that when one Pipe is open, the other two are shut.

Now to draw these Liquors without mixing, you must have three Pipes K, L, M, each of which answers to a Cell, and a sort of Cock or Spigot IN with three Holes answering the three Pipes, and so turning it till one of the Holes fits its respective Pipe, you draw the respective Liquor by it self.

PROBLEM XXIV.

To find the respective parts of a Weight that two Persons bear upon a Leaver or Barrow.

TO find the part of the Weight C, suppos'd to be 150 Pounds, which two Persons bear upon the Barrow AB, suppos'd to be 6 Foot long; we'll suppose that D is the Center of gravity of the Body C, and its line of Direction is DE, in which case we must consider the Point E, as if the Body C were hung; and then 'tis evident, that if the Point E be in the middle of AB, each Person will bear 75 pounds or half the weight C; but if 'tis not in the middle, but bears nearer to B for instance than to A, so that Plate 16. Fig. 40.

a heavier part of it falls upon B than upon A, that part may be determin'd, thus;

If you suppose the part AE of the Leaver or Barrow AB, to be 4 Foot, and consequently the other part to be 2 Foot (the whole length being suppos'd to be 6 Foot) multiply the given weight 150 by 4 the measure of the part AE, and divide the Product 600 by the length AB, *viz.* 6, and the quotient gives 100 pounds for the part of the weight born by a Power applied at B; so that consequently the Power at A must bear only 50.

PROBLEM XXV.

To find the Force necessary for raising a weight with a Leaver, the length and fix'd point of which are given.

Plate 16. Fig. 41.

WE'll suppose the weight C to weigh upon the Leaver AB 150 pounds; and the Power applied at its extremity B to be distant 4 Foot from the fix'd Point D, so that the remaining part AD of the Leaver is 2 Foot, the whole Leaver AB being suppos'd 6 Foot long. Multiply the weight C, 150, by 2 the part AD, and divide the Product 300 by 4 the other part BD; and the Quotient 75 will be the Force requisite for sustaining the weight C by a Power at B; from whence you will readily conclude, that the Power applied at B must have a force somewhat greater than that of 75 pounds, for moving and raising the weight C.

PROBLEM XXVI.

To contrive a Vessel that holds its Liquor when it stands upright, and spills it all if it be inclin'd or stoop'd but a little.

Plate 18. Fig. 44.

YOU may easily resolve this Problem by observing *Problem* 3. and 14. for if you put within the Vessel AB, a Syphon or bended Tube CDEF, the Orifice of which C touches the bottom of the Vessel, the other Mouth F being lower than the bottom of the Vessel

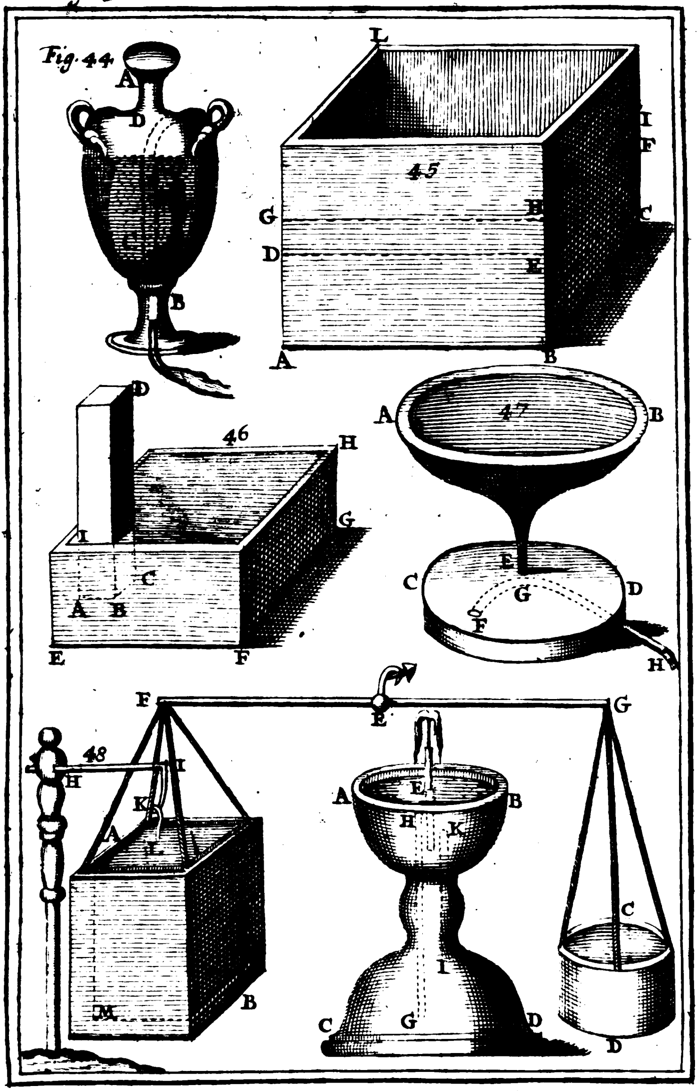

Fig. 44
45
46
47
48

Veſſel, ſo that the Leg or Branch CD is ſhorter than the other DEF: And then, if you fill the Veſſel with Water to about the upper part D, the Water will not run out; but if you incline the Veſſel AB never ſo little towards A, as if you were going to drink, the Water will go from the Branch CD into the Branch DEF, and run all out at the Mouth F, even tho' the Veſſel be ſet upright again, upon the account that the Air can ſucceed into the room of the Water when it deſcends thro' the Branch DEF.

PROBLEM XXVII.

To find the weight of a piece of Metal or Stone without a pair of Scales.

IN the firſt place get a Concave Veſſel in the figure of a Priſm, of what Baſe you will, tho' a ſquare or oblong Baſe is moſt convenient, as ABC, the length of which AB is ſuppoſed to be 6 Inches, the breadth BC 4 Inches, ſo that the Baſe ABC is 24, as appears by multiplying 6 by 4. Plate 18. Fig. 45.

This Veſſel muſt be fill'd with Water to a certain part, for example to DEF; in which you're to put the piece of Metal taking care that it be all cover'd, for if 'tis not quite cover'd, you muſt pour more Water in: When the Metal is in, the Water will riſe to the part GHI, for example, ſo that the Priſm of Water GEI will be equal to the ſolidity of the piece propos'd.

Now, the ſolidity of the Priſm of Water GEI is found by multiplying the Baſe DEF, which is equal to the Baſe ABC, *i. e.* 24 ſquare Inches, by its height EH or FI, which we ſuppos'd to be 2 Inches; for the Product gives 48 Cubical Inches for the ſolidity of the Priſm of Water GEI; by which you may find its weight, ſuppoſing a Cubical Foot of the ſame Water to weigh 72 Pounds, and ſaying by the Rule of Three Direct; If a Cubical Foot or 1728 Ounces weigh 72 Pounds, what will 48 Inches weigh? Thus multiplying 72 by 48, and dividing the Product 3456 by 1728, you find the weight of the Priſm GEI to be 2 Pounds.

The weight of the Water being thus found, you will easily find the weight of the piece of Metal or Stone, by multiplying the weight found 2, by 3 if the piece is Flint or Rock-Stone, by 4 if 'tis Marble, by 8 if Iron or Brass, by 10 if Silver, by 11 if Lead and by 18 if Gold. Thus you'll find the propos'd Piece, to weigh 6 pounds if it be hard Stone, 8 pound if Marble, 16 if Iron, 20 if Silver, 22 if Lead, and 36 if Gold.

Remark. An easie way of finding the Solidity of Irregular Bodies.

'Tis true the weight thus found is not very exact but it may serve for Mathematical Recreations. 'Ti to be observed that by this Problem you may fin with great facility the solidity of a Body, that can' be taken exactly by common Geometry without di ficulty, that is, when a Body is very irregular, as rough Stone, or any other unpolish'd Body. Fo hereby you may find the solidity of a Prism of Wat to which the rough Body must needs be equal.

PROBLEM XXVIII.

To find the solidity of a Body, the weight of which known.

THIS Problem may easily be resolved by t following Table, which shews in Pounds a Ounces the weight of a Cubical foot of several dif rent Bodies; and in Ounces, Drams, and Grains, weight of a Cubical Inch of the same Bodies, Pound containing 16 Ounces, the Ounce 8 Dra and the Dram 72 Grains.

A Table of the weight of a Cubical Foot, and of a Cubical Inch of several different Bodies.

	A Cubical Foot		A Cubical Inch.		
Of	*Pounds.*	*Ounces.*	*Oun.*	*Drams.*	*Grains.*
Gold	1326	4	12	2	52
Mercury	946	10	8	6	8
Lead	802	2	7	3	30
Silver	720	12	6	5	28
Copper	627	12	5	6	36
Iron	558	0	5	1	24
Pewter	516	2	4	6	17
White Marble	188	12	1	6	0
Free-Stone	139	8	1	2	24
Water	69	12	0	5	12
Wine	68	6	0	5	5
Wax	66	4	0	4	65
Oil	64	0	0	4	43

You learn by this Table, that a Cubical foot of Iron, for instance, weighs 558 Pounds, and so if a piece of that Metal weighs, for example, 279 Pound, you find its Solidity by the Rule of Three Direct, *viz*. If a weight of 558 Pounds gives a Cubical foot, or 1728 Inches of Solidity, what will a weight of 279 Pounds yield? Thus multiplying 279 by 1728, and dividing the Product 482112 by 558, you have in the Quotient 864 Cubical Inches for the solidity of the piece propos'd.

If on the other hand you have a piece of Silver, for example, and want to know the weight of it, find first its Solidity with Water as in the foregoing Problem; and if that Solidity, is, for example, 48 Cubical Inches, multiply the number 48 by 6 Ounces, 5 Drams, and 28 Grains, which is the weight of a Cubical Inch of Silver, as you see in the foregoing Table, and you have in the Product 20 Pounds, 2 Drams, and 48 Grains for the weight of the Piece of Silver propos'd. And so in other cases. Remark.

PROBLEM XXIX.

A Body being given that's heavier than Water, to find what height the Water will rise to, in a Vessel fill'd to a certain part with Water, when the Body is thrown into it.

Plate 18. Fig. 45.

WE'll suppose a Vessel in the form of a Rectangle Parallelepipedon, as ABCL, in which there is Water to the height AD: We throw into it a Ball of Iron, the Specifick Gravity of which is greater than of Water; and want to know what height the Water will then rise to. We measure the Area of the Rectangular Base ABC or DEF, in multiplying the length ED by the breadth EF; and the solidity of the Ball by multiplying the Cube of its Diameter by 157, and dividing the Product by 300: And if the Solidity, is, for example, 96 Cubical Inches, and the Area DEF 48 square Inches, in dividing the solidity 96 by the Area 48, you have in the Quotient two Inches for the height EH or DG, to which the Ball makes the Water rise, as taking up a Place or Room equal to that of the Prism GEI, the solidity of which is consequently 96 Inches, as well as that of the Ball.

Another way is as followeth. Take with an exact pair of Scales the weight of the propos'd Body, which we shall suppose to be 31 Pounds; and from thence find the solidity of the same Body by Problem 28, where you will find it to be 96 Cubical Inches if it be Iron. For this reason, the solidity of the Prism GEI will likewise be 96 Cubical Inches, and consequently that Prism being divided by the Base DEF which we supposed to be 48 square Inches, the height EH will be found 2 Inches.

PRO-

PROBLEM XXX.

A Body being given of less Specifick Gravity than Water, to find how far 'twill sink in a Vessel full of Water.

TAke a piece of Deal, for example, the Specifick Gravity of which is less than Water, and you'll find 'twill not sink quite in the Water, but only to such a depth, till it takes up in the Water a certain extent of space answerable to a Bulk of Water of equal weight with the piece. Now to find exactly what part of it will be under Water, you must find the weight of it, and the measure of a quantity of Water of the same weight, by the foregoing Problems; and then you'll see the Body sink until it hath taken up the space of that quantity of Water.

Supposing the piece of Deal ABCD to weigh 360 Pounds, and a Cubical foot of the Water contain'd in the Vessel EFGH to weigh 72 Pounds; divide 360 by 72, and you have in the Quotient 5 for the Cubical foot of Water that weighs likewise 360 Pounds; so that the Prism ABCD will sink in the Water till it fills the space of 5 Cubical feet; and to know how far that will be upon the Prism, take upon it at its lower end a Prism of 5 Cubical feet of the same Base with the Base ABCD, which we here suppose to be 4 square Foot, and divide the 5 Cubical feet by the Base 4, for so you have 1 Foot 3 Inches for the height or depth AI, to which the Prism ABCD will sink in the Water. Plate 18. Fig. 46.

PROBLEM XXXI.

To know if a suspicious piece of Money is good or bad.

IF it be a piece of Silver that's not very thick, as a Crown or half a Crown, the goodness of which you want to try: Take another piece of good Silver of equal ballance with it, and tie both pieces with

Thread or Horse-hair to the Scales of an exact Ballance (to avoid the wetting of the Scales themselves) and dip the two pieces thus tied in Water; for then if they are of equal goodness that is, of equal purity, they will hang in *Æquilibrio* in the Water as well as in the Air: but if the piece in question is lighter in the Water than the other, 'tis certainly false, that is, there's some other Metal mix'd with it that has less Specifick Gravity than Silver, such as Copper; If 'tis heavier than the other, 'tis likewise bad, as being mix'd with a Metal of greater Specifick Gravity than Silver, such as Lead.

If the piece propos'd is very thick, such as that Crown of Gold that *Hiero* King of *Syracusa* sent to *Archimedes* to know if the Goldsmith had put into it all the 18 pounds of Gold that he had given him for that end; take a piece of pure Gold of equal weight with the Crown propos'd, *viz.* 18 pounds; and without taking the trouble of weighing them in Water, put them into a Vessel full of Water, one after another, and that which drives out most Water, must necessarily be mix'd with another Metal of less Specifick Gravity than Gold, as taking up more space tho' of equal weight.

PROBLEM XXXII.

To find the Burden of a Ship at Sea, or in a River.

FRom what has been said in Problem 30. one may easily find the burden of a Ship, *i. e.* what weight 'twill carry without sinking. For 'tis a certain truth, that a Ship will carry a weight equal to that of a Quantity of Water of the same Bigness with it self; substracting from it the weight of the Iron about the Ship, for the Wood is of much the same weight with Water; and so if 'twere not for the Iron a Ship might sail full of Water.

The Consequence of this is, that, however a Ship be loaded, 'twill not sink quite, as long as the weight of its Cargo is less than that of an equal bulk of Water. Now to know this Bulk or Extent, you must measure the Capacity or Solidity of the Ship, which we here suppose to be 1000 Cubical feet, and multiply that

that by 73 pounds the weight of a Cubical foot of Sea Water; for then you have in the Product 73000 pounds for the weight of a bulk of Water equal to that of the Ship.

So that in this example we may call the burden of the Ship, 73000 Pounds, or 36 Tun and a half, reckoning a Tun 2000 Pounds, that being the weight of a Tun of Sea-water. If the Cargo of this Ship exceeds 36 Tun and a half she will sink; and if her Loading is just 73000 ℔. she'll swim very deep in the Water upon the very point of sinking; so that she can't sail safe and easie, unless her Loading be considerably short of 73000 pounds weight. If the Loading comes near to 73000 pounds, as being, for example, just 36 Tun, she will swim at Sea, but will sink when she comes into the Mouth of a fresh Water River; for this Water being lighter than Sea-water will be surmounted by the weight of the Vessel, especially if that weight is greater than the weight of an equal Bulk of the same Water.

PROBLEM XXXIII.

To make a pound of Water weigh heavier, or as much more as you will.

WE know by Experience, that if you hang a great Stone by a Cord, the Stone hanging within a Vessel so as not to touch it, leaving room for a pound of Water round it; and if you fill that void space with VVater, the Vessel that with the VVater alone weighs but about a pound, as containing but a pound of VVater, will weigh above an hundred pounds if the Stone in the Vessel fills the space of an hundred pounds of Water. Thus, you see a pound of Water in this Case weighs above an hundred pounds; and if the Stone takes up the space of a thousand pounds, the one pound of Water will weigh above a thousand; and so on.

For the same end you may make use of a Ballance, the two Scales of which AB, CD, gravitate equally round the Center of Motion E, which shall be, if you will, at the middle of the Beam E, as in the common Ballances; for having fix'd with an Iron Hook HIK, at the point H of a Nail or any other firm thing, the Plate 18. Fig. 48.

Body

Body LM, equal for example, to 99 pounds of VVater, you need only to put the Scale AB round the Body LM, so as to leave space for a pound of VVater; for then 100 pounds of VVater pour'd into the Scale CD, will be in *Æquilibrio* with one pound of VVater in the other Scale AB.

PROBLEM XXXIV.

To know how the Wind stands, without stirring out of one's Chamber.

FIX to the Cieling of your Room a Circle divided into 32 equal parts, with the Names of the 32 Rumbs or Wind-points, the points of North and South being upon the Meridian Line. The Circle or Dial thus divided, must have a Needle or Hand moveable round its Center, like the hand of a VVatch or Clock; and that Hand must be fix'd to an Axletree that's perpendicular to the Horizon, and may move easily upon the least VVind, by vertue of a Fane on its upper end above the roof of the House; and then the VVind turning the Fane, will at the same time turn its Axletree, and the Hand that's fix'd to it, which will accordingly point to the Rumb from whence the VVind blows.

Plate. 19. Fig. 50. Upon the *Pont Neuf* at *Paris*, and likewise in the *French* King's Library, there's such a Dial, not upon a Cieling, but against a VVall; which shews the VVind-point by the Motion of a Fane, AB, the Axletree of which CD, which is likewise perpendicular to the Horizon, is sustain'd above by an Horizontal Plain EF, thro' which it runs at Right Angles, and below by the Plain GH, upon which it rests with its extremity D, which ought to be sharp pointed, for the resting a'most upon a Point contributes to facilitate its Motion upon the least air of VVind; and at the same time that of the Cop IK, which has eight equal VVings or Gutters that the notches of the VVheel LM catch upon; whence it comes, that the Motion of the Fane turning the VVheel LM, turns likewise the Axletree PQ, which being parallel to the Horizon, passes thro' the VVall at Right Angles, and like-

wise

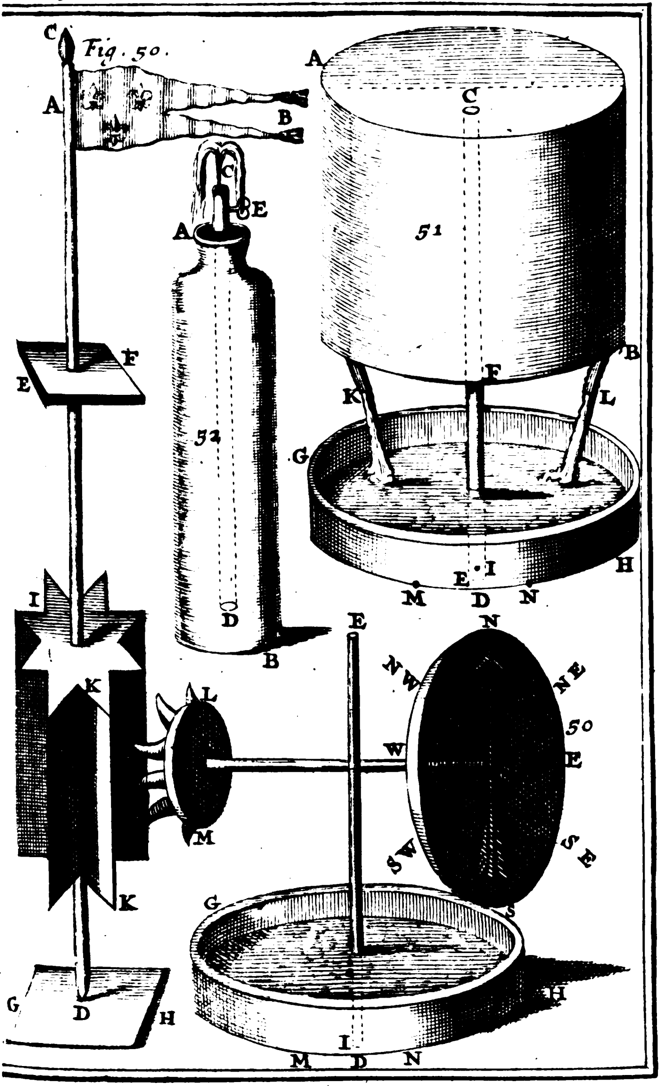
Fig. 50.
C
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B
E
F
I
K
K
L
M
G
D
H
C
E
A
52
D
B
A
C
51
B
F
K
L
G
I
E
H
M
D
N
E
N
NW
NE
50
W
E
SW
SE
S
G
H
I
M
D
N

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Plate. 19. Fig. 50. Upon the *Pont Neuf* at *Paris*, and likewise in the *French* King's Library, there's such a Dial, not upon a Cieling, but against a VVall; which shews the VVind-point by the Motion of a Fane, AB, the Axletree of which CD, which is likewise perpendicular to the Horizon, is sustain'd above by an Horizontal Plain EF, thro' which it runs at Right Angles, and below by the Plain GH, upon which it rests with its extremity D, which ought to be sharp pointed, for the resting a'most upon a Point contributes to facilitate its Motion upon the least air of VVind; and at the same time that of the Cop IK, which has eight equal VVings or Gutters that the notches of the VVheel LM catch upon; whence it comes, that the Motion of the Fane turning the VVheel LM, turns likewise the Axletree PQ, which being parallel to the Horizon, passes thro' the VVall at Right Angles, and like-

wise

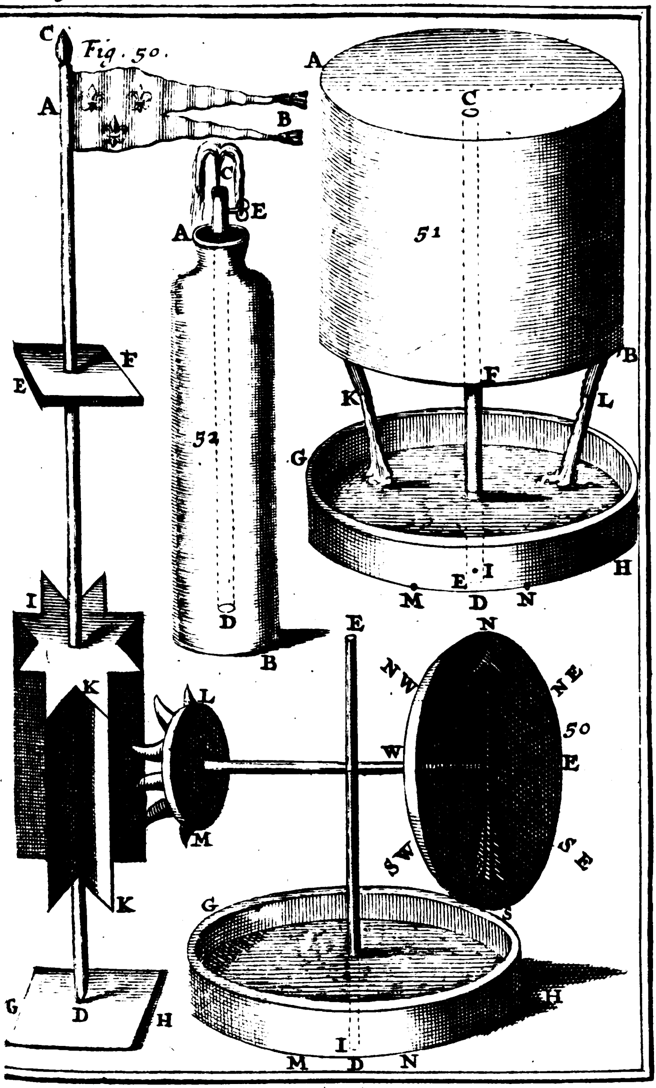
Fig. 50.
C
A
B
E
F
I
K
L
M
K
G
D
H
C
E
A
52
D
B
A
C
51
B
F
K
L
G
E
I
H
M
D
N
E
N
NW
NE
50
W
E
SW
SE
G
S
H
I
M
D
N

wise the Hand NR, fix'd to its extremity P, which passes thro' the Dial on which the Rumbs are mark'd.

PROBLEM XXXV.

To contrive a Fountain, the Water of which flows and stops alternately.

PRovide two unequal Vessels, AB, CD, of white Iron or some such Matter, the greatest being the uppermost AB, which communicates with the lesser CD by the Orifice E; that so the VVater pour'd into the greater AB may run from it into the lesser CD, and from thence out at the Extremity H of the Crane GH, the other Extremity of which, F, is open, and placed not far from the bottom of the Vessel. Plate 18. Fig. 47.

VVhen the VVater of the Vessel CD rises thro' the open end F of the Crane to the upper part G, 'twill descend thro' the other Orifice H, if it be lower than the aperture F, and if the Crane FGH is so large or thick that it discharges more VVater at H than there enters into the Vessel CD at E, the Vessel CD will soon be empty, and the Fountain give over running: But the VVater will recommence its flux thro' H, when it reascends thro' the Branch FG to G; and so on alternately.

You may contrive this Fountain of what figure you will, as well as the following which runs likewise alternatly by Intervals, and is made thus;

Take a Vessel AB which has two Bottoms, that is, is close on all sides like a Drum; thro' the middle of it run a long Pipe CD soldered to the lower bottom at F, with its two ends open, C, D; the first of which C must not quite touch the upper Bottom, but leave passage for the VVater, when one has a mind to fill the Vessel AB; which is done by turning up the Vessel AB with its Pipe CD, so that the Hole D will then be uppermost, and pouring in the VVater at D. This done stop up the Pipe CD with another and a very little smaller Pipe ED, that can just enter it, and is fix'd in the bottom of a Case or Cistern that's a little longer than one of the two bottoms of the Vessel AB, Plate 19. Fig. 51.

The

The two Pipes CD, DE, ought to have at an equal height two Apertures or Holes *I, I*, and the smallest DE ought to be moveable within the greater CD, that so you may turn the smaller with its Case GH when you will, till the two Holes *I, I*, meet. Farther, the Vessel AB ought to have several little Holes in its lower Bottom, as KL, for giving egress to the VVater; and the Case or Receptacle GH ought likewise to have two smaller Vents, M, N, for the VVater to run out.

Now, the Vessel AB being fill'd with VVater, as we directed but now; and the Pipe CD being stop'd by the Pipe DE, which we suppos'd so thin that it could just fill it, without any necessity of the Extremity E its reaching to the end C, provided the two other ends, D, D, do but fit: This done, I say, turn the Vessel again to its first Position, in which 'twill stand as in the Figure, the Case GH being its Base, and being turn'd together with its Pipe E till the two Vents *I, I*, meet and make but one Orifice; for then the VVater contain'd in the Vessel AB will run out at the Vents KL, as long as the Air can pass thro' the aperture *I* to supply the room of the VVater that runs from AB into the Case GH; but when the VVater in the Receptacle GH rises above the Vent *I* (which will infallibly happen, since more VVater runs at the Vents K, L, than at M, N, the former being suppos'd larger than the latter) the Air not finding access at *I*, the VVater in the Vessel AB, will give over running thro' the Vents K, L, but the VVater in the Receptacle GH will continue to run at the Vents M, N, so that this VVater will grow lower by degrees, till the Vent *I* is uncover'd again, and then the Air having access at *I* will renew the flux of the VVater thro' K, L; which in a small time will raise the VVater in the Case GH, so as to cover the Vent *I* again, upon which the Stream from A, B, will stop, and so on alternately till there's no VVater left in the Vessel AB.

Remark. This is call'd the *Fountain of Command*, because it runs at a word given, when the VVater is near the renewal of its flux thro' the Vents KL, which is easily known; for when the Vent *I* begins to get clear of VVater in G, H, the Air struggling for access at that

Vent

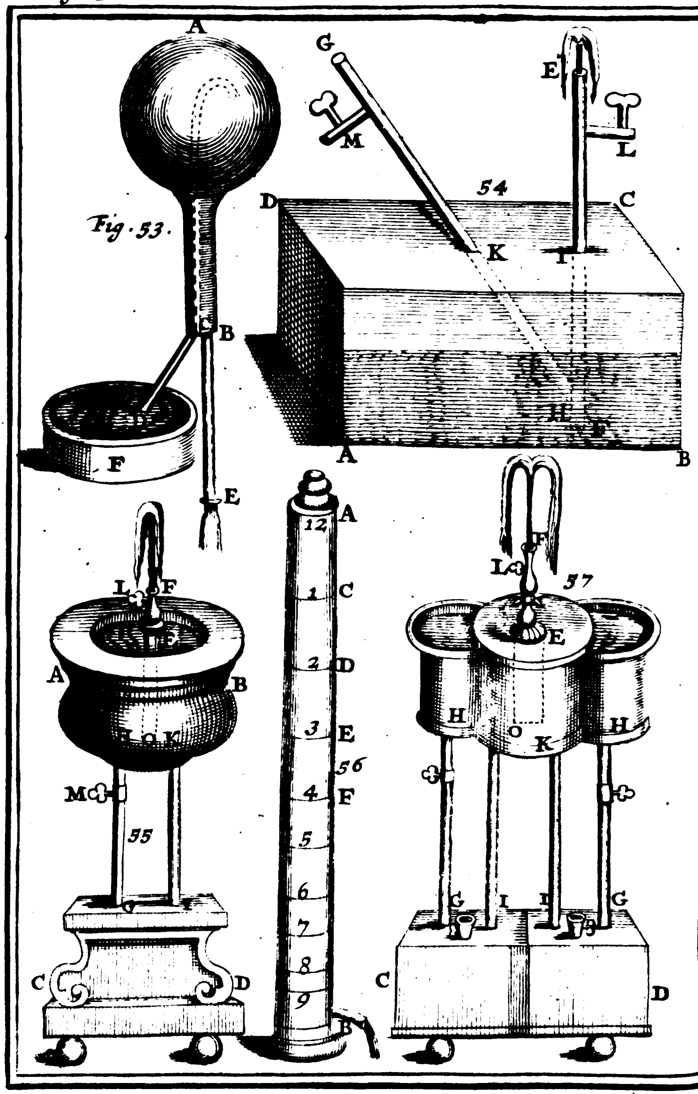
Fig. 53.
A
B
C
E
F
54
G
M
D
K
I
E
L
C
H
F
A
B
55
L
F
A
B
M
C
D
56
A
12
1
C
2
D
3
E
4
F
5
6
7
8
9
B
57
L
F
E
H
O
K
H
G
I
I
G
C
D

Vent makes a little noise, and so gives notice that the Fountain is about to run.

PROBLEM XXXVI.

To make a Fountain by Attraction.

TO the Mouth B of the Phiol or Glass Matrass AB, adjust two Pipes, CD, CE, inclining the one to the other in the form of a Syphon or Crane, and soldering them together at the Extremities C, which ought to be open as well as the other Extremities, D, E; and then stopping the remaining part of the Mouth B so as to keep the Air quite out. Plate 20. Fig. 52.

Turn this Machine upside down, and fill it either quite, or to a certain part by one of the two Pipes CD, CE, the first of which CD ought to be smaller and shorter than the second CE, for a Reason to be given in the Sequel.

This done, put the Phiol AB in its first Situation, as you see it in the Figure, placing it perpendicular upon a Table with a hole in it, thro' which the big Pipe CE must pass; then place under the other lesser Pipe CD a Vessel full of VVater, as DF, so that the Pipe CD may touch the bottom of the Vessel; and you'll see the Water of the Phiol AB run out at the greatest Pipe CE; but when it has run out to C, the weight of the Water that flows out at the Mouth E of the greater Pipe CE will draw or suck the Air of the Matras AB, and that so much the more forcibly, as it is bigger and longer than the Pipe CD; upon which the Water of the Vessel DF will mount up thro' the Pipe CD, and spout out at the Mouth C with an impetuous force into the Phiol; and continue the spout so much the longer the more Water there is in the Vessel DF, for the Water cast up into the Phiol will continually fall down and find an egress in the greatest Pipe CE.

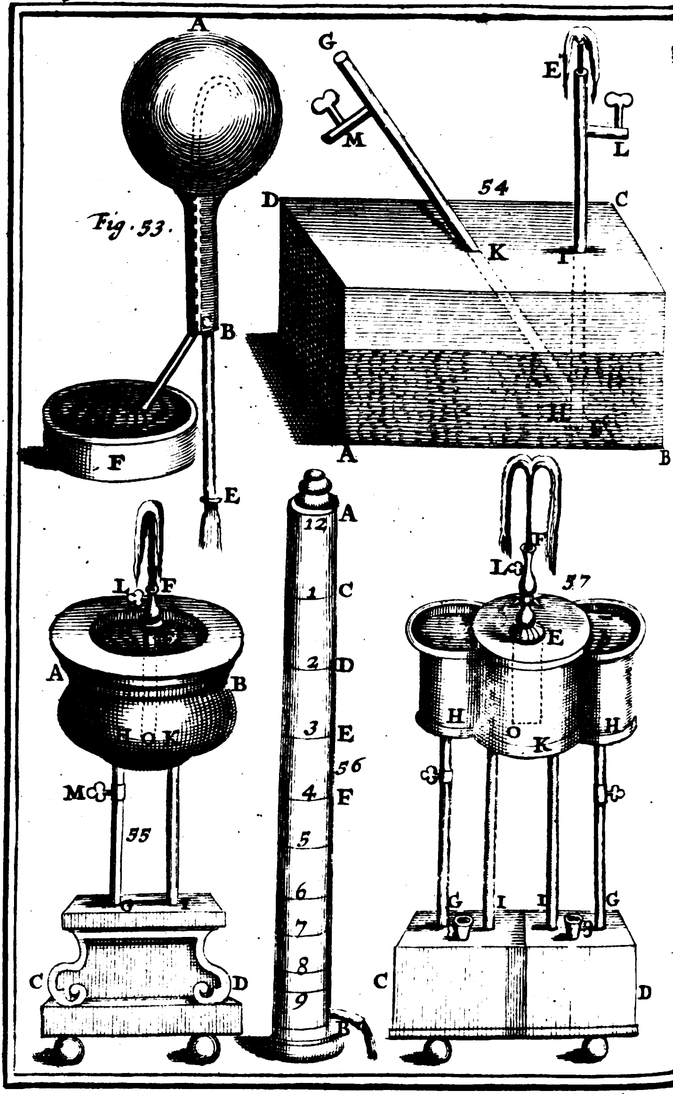
Fig. 53.
A
B
E
F
54
G
M
E
L
D
C
K
I
A
B
A
12
C
1
D
2
E
3
56
F
4
5
6
7
8
9
B
L
F
A
B
M
55
C
D
C
D
57
F
L
E
H
O
K
H
G
I
I
G
C
D

Vent makes a little noiſe, and ſo gives notice that the Fountain is about to run.

PROBLEM XXXVI.

To make a Fountain by Attraction.

TO the Mouth B of the Phiol or Glaſs Matraſs AB, adjuſt two Pipes, CD, CE, inclining the one to the other in the form of a Syphon or Crane, and ſoldering them together at the Extremities C, which ought to be open as well as the other Extremities, D, E; and then ſtopping the remaining part of the Mouth B ſo as to keep the Air quite out. Plate 20. Fig. 53.

Turn this Machine upſide down, and fill it either quite, or to a certain part by one of the two Pipes CD, CE, the firſt of which CD ought to be ſmaller and ſhorter than the ſecond CE, for a Reaſon to be given in the Sequel.

This done, put the Phiol AB in its firſt Situation, as you ſee it in the Figure, placing it perpendicular upon a Table with a hole in it, thro' which the big Pipe CE muſt paſs; then place under the other leſſer Pipe CD a Veſſel full of VVater, as DF, ſo that the Pipe CD may touch the bottom of the Veſſel; and you'll ſee the Water of the Phiol AB run out at the greateſt Pipe CE; but when it has run out to C, the weight of the Water that flows out at the Mouth E of the greater Pipe CE will draw or ſuck the Air of the Matras AB, and that ſo much the more forcibly, as it is bigger and longer than the Pipe CD; upon which the Water of the Veſſel DF will mount up thro' the Pipe CD, and ſpout out at the Mouth C with an impetuous force into the Phiol; and continue the ſpout ſo much the longer the more Water there is in the Veſſel DF, for the Water caſt up into the Phiol will continually fall down and find an egreſs in the greateſt Pipe CE.

PRO-

PROBLEM XXXVII.

To make a Fountain by Compression.

Plate 18. Fig. 49. THIS Fountain is compos'd of two equal Vessels or Basins, AB, CD, joyn'd together; the bottom of the lowermost being flat to serve for a base to the Machine, and that of the upper being somewhat Concave to receive the Water that's pour'd into it, when we mean to fill the Vessel CD with Water, and make the Fountain run. The Vessel AB ought to have in the middle of its Concavity an Orifice with a small Pipe EF, the Extremity of which O must be near the bottom of the Vessel, the other end being rais'd a little above the side of the Vessel AB, that so the Water contain'd in the Vessel may run out with facility.

Besides this, there are in the Machine two hidden Pipes, GH, IK; the first of which GH is solder'd to the bottom of the Vessel AB about H, where the Orifice or Hole is, thro' which the Water pour'd into the Concavity of AB passes into the lower Vessel CD, making its egress from the Pipe GH at the lower extremity G, which for that reason ought not to touch the bottom of the Vessel. The second hidden Pipe IK is solder'd to the upper part of the Basin CD, where there is likewise a Vent or Mouth as well as at the other extremity K, which must not touch the bottom of the Vessel AB, to the end that when the Machine is inverted, the Water of the Basin CD may enter the Pipe IK, and fill the Basin AB, the Capacity of which is suppos'd equal to that of the Basin CD.

This done, set the Machine in its first Situation, as you see it in the Figure, and pour Water a second time into the Concavity of AB; upon which the Water will enter the Pipe GH at H, and so repair to the Basin CD, where 'twill make a strong pressure upon the Air, as well as upon that in the Pipe IK; and the Air thus compress'd will press the Water in the Basin AB, and so force it to spout out impetuously at the Mouth F. This agreeable Waterwork will continue to play a long time, because the Water still falling

back

back into the Basin AB, 'twill re-enter the Basin CD by the Pipe GH, and so continue the pressure of the Air, till all the Water of the Basin AB is gone, and the Air can have free access at the Mouth F of the small Pipe EF.

One may readily apprehend, that the two Vessels AB, CD, ought to have no other mutual Communication, but what they have by the two Pipes GH, IK, as you see in the Figure; and that the two Pipes GH, IK, ought to be so soldered at H and I, that no Air can either enter or get out.

In Figure 55. you have another Model of a Fountain, by the Cock L of the Pipe EF, and the Cock M of the Pipe GH, the Mouth of which H enters the lower bottom of the upper Vessel AB, giving vent to the Cock L, and turning or stopping the Cock M, you fill the Vessel AB with Water, pouring it in at the Mouth F; and then by opening the Cock M, the Water of AB will pass thro' the Pipe GH and fill the Vessel CD. Again, stopping the Cock M and opening L, you fill AB, as before; and then if you give vent to the Cock M, the Water of the Basin AB will make a pressure upon that of CD, and the Water of CD thus compress'd will push out with Violence the Water of AB at the Mouth F, and so will make a Water spout like that last describ'd. Plate 20. Fig. 55.

To make this *Jet* or Water spout twice as high, divide the Basin AB into three Cells, and the Basin CD into two, and double the Pipes GH, IK, as you see in Fig. 57. for then the pressure of the Air being double, will have a double effect, that is, the Water will rise twice as high as before. Plate 20. Fig. 57.

Another Fountain by Compression may be made with only one Vessel AB, and one Pipe in the middle CD, open at its two ends C, D; the lowermost of which D ought not to come close to the bottom of the Vessel. At the Mouth A the Pipe ought to be so solder'd that no Air can pass; and above the Mouth A the Pipe CD ought to have a Spigot or Cock, E, for stopping or giving vent to the Pipe CD as there is occasion; and that after this manner. Plate 19. Fig. 52.

Put into the Vessel AB as much Air and Water as is possible, with a Syringe, at the Mouth C, stopping the Cock E as you Syringe to prevent the exit of the Air

Air that's extreamly compress'd in the Vessel AB; in this case, the Water being heavier than the Air will remain at the bottom of the Vessel, and bear a strong pressure from the Air, which is likewise mightily compress'd it self; and for that reason, if you open the Pipe CD by opening the Cock E, the Air will make the Water spout out with Violence at the Mouth C, and that pretty high. This agreeable Water-Spout will continue so much the longer, that the Mouth C is small, and the Air in the Vessel AB much compress'd; and that Compression of the Air will be considerably greater if you heat the Vessel but a little.

Plate 20. Fig. 54. We shall mention yet another Method of contriving a Fountain by Compression, with only one Vessel or Basin; *viz*. Take the Vessel ABCD close stop'd on all sides, with two Pipes EF, GH, communicating mutually at H where they are soldered, and open at the ends, E, F, G. The end F must not touch the bottom of the Vessel ABCD; and each of the two Pipes must have a Cock out of the Vessel, as L, M, and withal must be so soldered at I, K, as to deny all passage to the Air.

Now, to set this Fountain in going, turn or stop the Cock L, and open the Cock M, in order to force with a Syringe as much Water as you can into the Vessel ABCD; then stop the Cock M to prevent the egress of the Air that's extremely compress'd in the Vessel ABCD: But open the Cock L, and the Water will spout impetuously out at E, which ought to be but a small vent that the Water-Spout may continue the longer.

PROBLEM XXXVIII.

To contrive a Fountain by Rarefaction.

Plate 21. Fig 58. HAving joyn'd two unequal Vessels AB, CD, close on all sides, by two equal Pipes, EF, GH, solder'd to the lower bottom of the upper Vessel AB, at F and H, and to the upper bottom of the lower Vessel CD at E and G; so that the Air can have no passage but by the Mouth of these two Pipes, which are suppos'd to be open at the ends E, F, G, H; put in

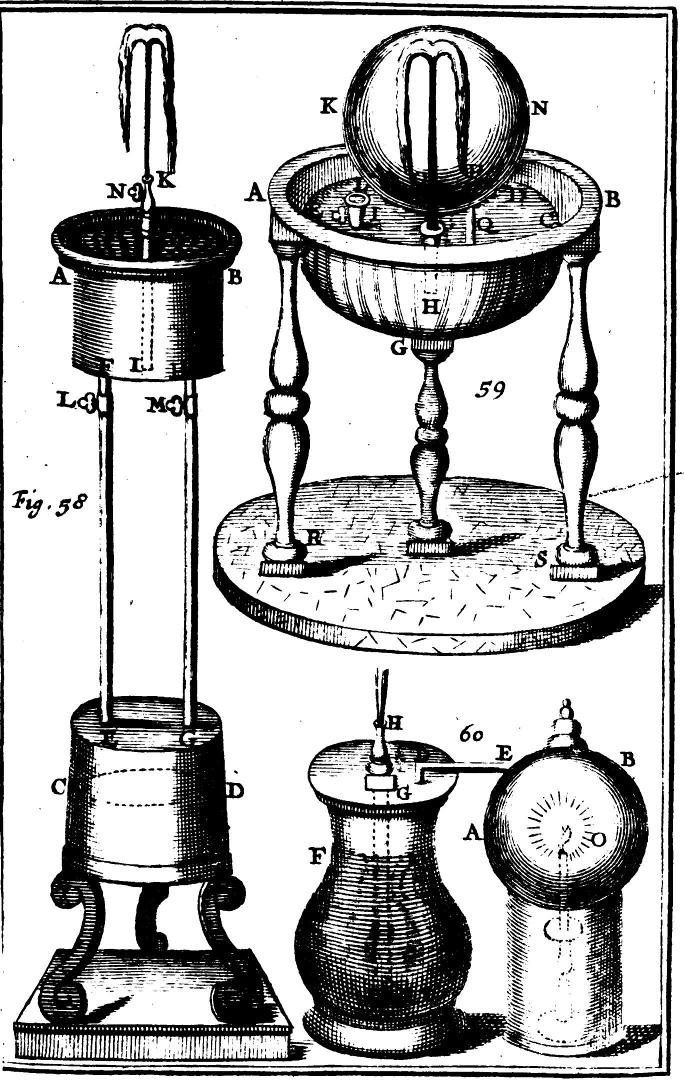
Fig. 58
N
K
A
B
L
M
C
D
59
K
N
A
B
H
G
R
S
60
H
E
B
G
A
O
F

in the middle of the upper Veſſel AB a third ſmaller Pipe IK, the lower end of which I, muſt not come cloſe to the bottom of the Veſſel AB, and the upper end K muſt be ſomewhat higher than the upper End of the Veſſel AB. This aperture at K ought to be ſmall, and each of the three Pipes, EF, GH, IK, ought to have a Cock, as L, M, N.

Having ſhut the two Cocks L, M, open the Cock N, and at the Mouth K fill the Veſſel AB with Water. Then open the two Cocks L, M, that the Water of the Veſſel AB may deſcend thro' F and H into the Veſſel CD, and fill it but part full, the capacity of CD being ſuppos'd greater than that of AB. Then ſtop the two Cocks L and M, and fill the Veſſel AB with freſh Water. This done, ſtop the Cock N, and put hot burning Coals under the Veſſel CD, which will rarifie the Air and the Water in the Veſſel CD; and ſo if you open the Cock N, the Water in the Veſſel AB will fly out at K, and make a pleaſant Water-Spout.

Another way is as followeth. Get a Veſſel of Copper or any other Metal, as AB divided into two parts, the uppermoſt of which CDE is open, and the other GH ſhut cloſe on all ſides, but at I, where it has a little Pipe in the form of a Funnel IL with a Cock M, in order to pour in at that Funnel, the Cock being open, as much Water as will fill part of the part GH. Plate 21. Fig. 52.

In the middle of the Veſſel AB place a Pipe HO, with its lowermoſt end H not quite touching the bottom of the Veſſel, and the upper end O a little ſmaller, and rais'd above the Veſſel to receive a Sphere of Glaſs KN, thro' which and thro' the upper ſide of the Veſſel AB you're to run another Pipe PQ, open at its two ends, that the Water that riſes from AB into the Sphere KN thro' the Pipe HO, may return by the Pipe PQ into the Veſſel AB, and ſo make a continual Water-Spout.

But to make the Water in the Veſſel AB riſe of it ſelf into the Sphere KN, by the Pipe HO, you muſt ſtop the Cock M, and heat the Air and Water in the Veſſel AB, by putting under the Plain RS a Grate cover'd with red hot Coals, the heat of which will rarifie the Air and make the Water aſcend, &c.

Remark. There's no question, but these two sorts of Fountains will succeed, when the Machine is duly made; but I can't promise so much of a third sort of Fountains, which you see represented in Fig. 60. and which is presently apprehended by only looking upon the Figure; for perhaps the Candle O may go out, when 'tis put into the Concave Sphere AB, at the aperture C, which is design'd for rarifying by its heat the Air in the Sphere, that the Air thus rarified passing from the Sphere thro' the Pipe DE, may press the Water contain'd in the Vessel DF, and so force it to spout out at the upper end of the Pipe GH.

Plate 21. Fig. 60.

PROBLEM XXXIX.

To make a Clock with Water.

As heavy Bodies in descending freely thro' the Air continually increase their Celerities, and in equal times pass thro' unequal Spaces, which rise or increase in the proportion of the Squares, 1, 4, 9, 16, *&c.* of the natural Numbers, 1, 2, 3, 4, *&c.* beginning from the point of Rest: So, on the Contrary, liquid Bodies running into any Vessel thro' the same Orifice, continually lessen their Celerities, and the upper surface of the Liquor, as Water contain'd in the Glass Cylinder AB, falls lower, in running continually at the Orifice B, in the proportion of the same square Numbers, 1, 4, 9, 16, *&c.* in equal times.

Plate 20. Fig. 56.

For this Reason; if the Tube of Glass AB full of Water empties it self in 12 Hours, the way to know how much the Water sinks every Hour, and to mark the Hours upon the Tube AB, is this. The Square of 12 being 144, we divide the length AB into 144 equal Parts, and then take 121 the Square of 11 for the first Hour from B to C; 100 the Square of 10 from B to D for the Point of 2 a Clock, supposing A to be the Noon-Point; 81 the Square of 9 from B to E for the Point of 3; 64 the Square of 8 from B to F for the Point of 4; and so on.

Remark. If the Tube AB do's not empty it self exactly in 12 Hours thro' the Orifice B, you must make it so to do by lessening or increasing the Orifice B, as you see occasion.

Now

Now, to find this Diminution or Augmentation, that is, to find the measure of B or the Diameter of a Hole thro' which all the Water in the Cylinder AB will pass in just 12 Hours: We'll suppose the Diameter of the Orifice B to be two Lines, and all the Water of the Cylinder AB to run out thereby in 9 Hours; in this case we multiply 9 by 2 the number of the Diameter, and divide the Product 18 by 12, the time allotted for the due flux of the Water; and thus you'll find that the Diameter of the Hole B ought to be a Line and a half, to give passage to all the Water in the Prism AB just in 12 Hours.

If you would know the quantity of Water that runs each Hour thro' the vent B, measure the height AB, suppos'd to be 6 Foot, and the Area of the Base of the Cylinder by multiplying 144 the Square of 12 its Diameter (suppos'd to be an Inch or 12 Lines) by 785, and dividing the Product 113040 by 1000; the Quotient will give about 113 square Inches for the Area of the Base of the Cylinder AB. Plate 20. Fig. 56.

This Area being common to all the Cylinders of Water, the heights of which are AC, CD, DE, &c. will lead us to the knowledge of their Solidities, *viz.* by multiplying the Area's by the heights when known; and these Solidities are the quantity of Water that issues each Hour thro' the Orifice B. Now, the Method of finding the heights, AC, CD, DE, &c. is this:

The height AB being suppos'd 6 Foot which is equivalent to 864 Lines, and which we have divided into 144 equal Parts, each of these Parts will be 6 Lines, as appears by dividing 864 by 144; and the height BC which is 121 of these Parts will by consequence be 726 Lines, as appears by multiplying 121 by 6; so that the part AC will be 138 Lines, as appears by Subtracting 726 from 864. Therefore, if you multiply 113 the Base of the Cylinder by 138 or the height AC, you have 15594 Lines for the Solidity of the Cylinder AC, or the quantity of Water that will run thro' the Orifice B in the first Hour, that is, from Noon to one a Clock.

In like manner, the height BD being 100 Parts, Subtract it from the height BC, which was 121, and the Remainder is 21 for the Height CD of the second Cylinder;

linder; and each part being 6 Lines, the part CD will be 126 Lines, as appears by multiplying 121 by 6. So if you multiply 126 by the common Base 113, you have in the Product 14238 Cubical Lines for the solidity of the second Cylinder CD, or the quantity of Water that will issue thro' the Aperture B from 1 to 2 a Clock. And so of the rest.

COROLLARY.

Plate 22. Fig. 61. This directs us to the way of adding to this Water-Clock another that shews the Hours by its ascent in the Prism GHI, the Base of which is known, for example 226 Square Lines; in making the Water of the Cylinder AB fall into this Prism, which for that end should be placed lower than the Orifice B, and be at least as wide or large as the Cylinder AB; and in marking the Hours upon the Prism, thus.

The quantity of Water that answers to the first Hour, being 15594 Cubical Lines, we divide that Solidity 15594 by 226 the Area of the Base of the Prism GHI, and find in the Quotient 69 Lines for the Height GK of the first Hour in the Prism GHI.

In like manner, the quantity of Water corresponding to the second Hour, or to the Cylinder CD, being 14238 Cubical Lines, we divide that Solidity 14238 by the same Base 226, and find in the Quotient 63 Lines for the height KL of the second Hour in the Prism GHI. And so of the rest.

'Tis evident, that, if the Base of the Prism GHI were equal to that of the Cylinder AB, the divisions of the Hours in the Prism GHI, would be equal to those of the Cylinder AB; only the Order would be inverted, the height GK being equal to the height AC, the height KL to the height CD, and so on.

PROBLEM XL.

To contrive a Water Pendulum.

Plate 22. Fig. 62. BY a Water Pendulum, we mean a Water-watch or Clock in the figure of a Drum or round Box of Metal well solder'd, as ABCD, in which there's a certain

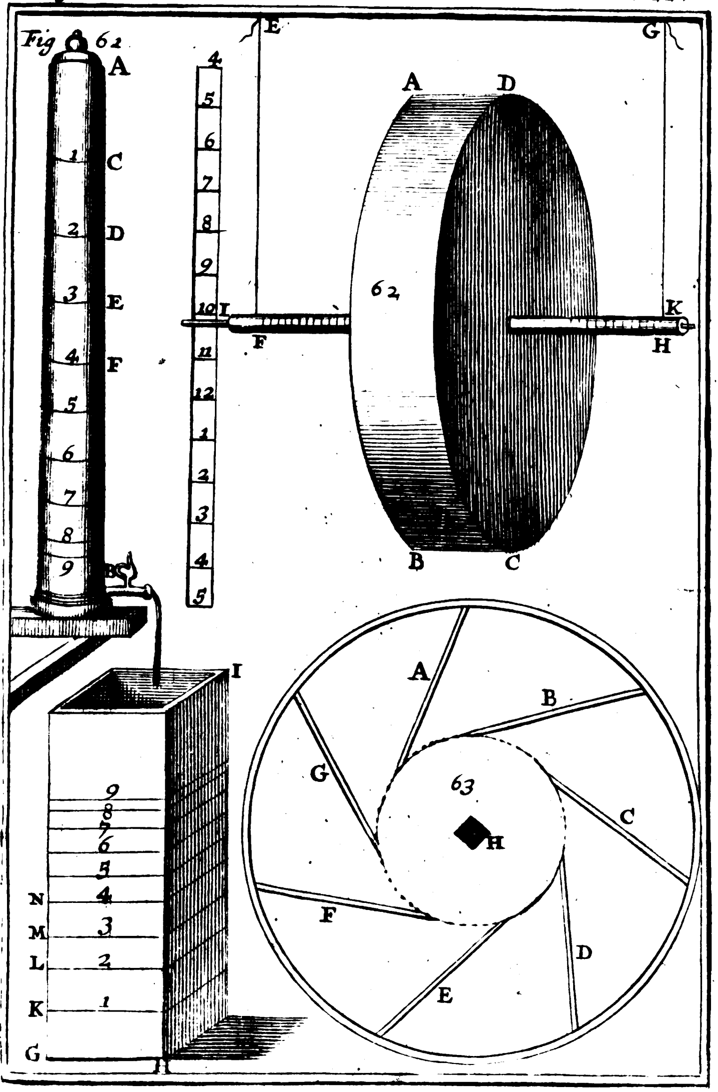
Fig 61
A
C
D
E
F
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B
I
9
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7
6
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4
3
2
1
N
M
L
K
G
4
5
6
7
8
9
10
11
12
1
2
3
4
5
E
G
A
D
62
I
F
K
H
B
C
A
B
G
63
H
C
F
D
E

certain quantity of prepar'd Water, and several little Cells communicating one with another near the Center, which gives passage to no more Water than just what is necessary for causing the gradual and gentle descent of the Watch by its own weight, which is suppos'd to hang by two fine and equal Threads or Cords, EF, GH, winded round an Iron Axletree IK that is equally thick, which passes thro' the middle of the Box at Right Angles, and descending along with it shews without any noise, by one or both its Extremities, I, K, the Hours mark'd upon an adjacent Vertical Plain, with the Divisions taken from a good Wheel-Clock.

Who was the first Inventer of these, I do not know, but I have seen one of 'em, made of Pewter, the Measures and Proportions of which I shall here lay down as a Rule for making of others, whether larger or smaller.

The Diameter AB or CD of the two Heads of the Drum or Barrel ABCD was about five Inches; and the breadth AD or BC, or the distance between the two Heads, which were equal and mutually parallel, was two Inches. The inside of the Barrel was divided into seven Cases or Cells by as many small plains inclin'd, or Tongues of Pewter solder'd to each Head, and to the Circumference or Concave Surface, These Tongues were each of 'em two Inches long, as A, B, C, D, E, F, G, and, as you see in Figure 63. did so slope that they graz'd upon and touch'd the Circumference of a Circle describ'd round the Center H at an Inch and a half Interval. These shelving Tongues serve to make the Water pass from one Cell to another as the Machine turns and descends, and points to the Hours with the extremity of the Axletree, which was run at Right Angles thro' the middle of the Drum, or the Hole H, that Hole being square that the Clock might rest the firmer upon the Axletree.

Plate 22. Fig. 62.

Plate 22. Fig. 63.

In fine, there were in this little Machine seven Ounces of purified, that is, distill'd and prepar'd Water, put in thro' two Holes in the same Head at an equal distance from the Center H, which were afterwards stop'd up to hinder the egress of the Water, when the Clock turns with its Axletree, continually changing its situation, in descending insensibly by the unwinding of the two Cords that hold it always perpendicular and are winded round the Axletree, which by that means is always parallel to the Horizon.

Remark. 'Tis evident that if this Clock had been suspended by its Center of Gravity, as 'twould be if the lower surface of the Axletree pass'd exactly thro' the middle of each Head, it would not move at all; and the cause of its Motion is its being hung off of the Center of Gravity by the two Cords winded round its Axletree; the thickness of which ought not to be very considerable with respect to the bulk of the Clock and the quantity of Water therein contain'd, that so the Clock may roll moderately by vertue of the passage of the Water from one Cell to another. 'Tis equally evident that the Machine must not descend all on a sudden, because the force of its Motion is counterballanced and lessen'd by the weight of the Water it contains.

To wind up this Clock, when it has run down to the end of the two Cords, you need only to raise it with your Hand, and make it turn the contrary way, on the same two Cords, which may be as long as you will, provided they are equal, and fix'd at equal heights above the Horizon, that so the Axletree may be always Horizontal.

The Pendulum's of this kind, that are now made at *Paris*, are of Copper, and commonly go 24 Hours from the top to about two Foot below. The Division of the Hours is regulated, as we said before, by a Clock that goes true.

This Clock is liable to the change of Air, *i. e.* its Driness or Humidity, as well as other Clocks; but it has this conveniency that it makes no noise, and so do's not disturb one in the Night, and when one wakes the Hours may be distinguish'd by little Buttons or Pegs fix'd upon 'em.

Besides, this sort of Clocks do's not often want mending; you need only to change the Water once in two or three Years; because it soils and grows thick in time, and so for want of due Fluidity makes the Clock go slower. This fresh Water, which ought to be distill'd Spring Water, is put in at a Hole made in one of the two Heads, and afterwards stop'd up with Wax, the Barrel being first clear'd of its foul Water,

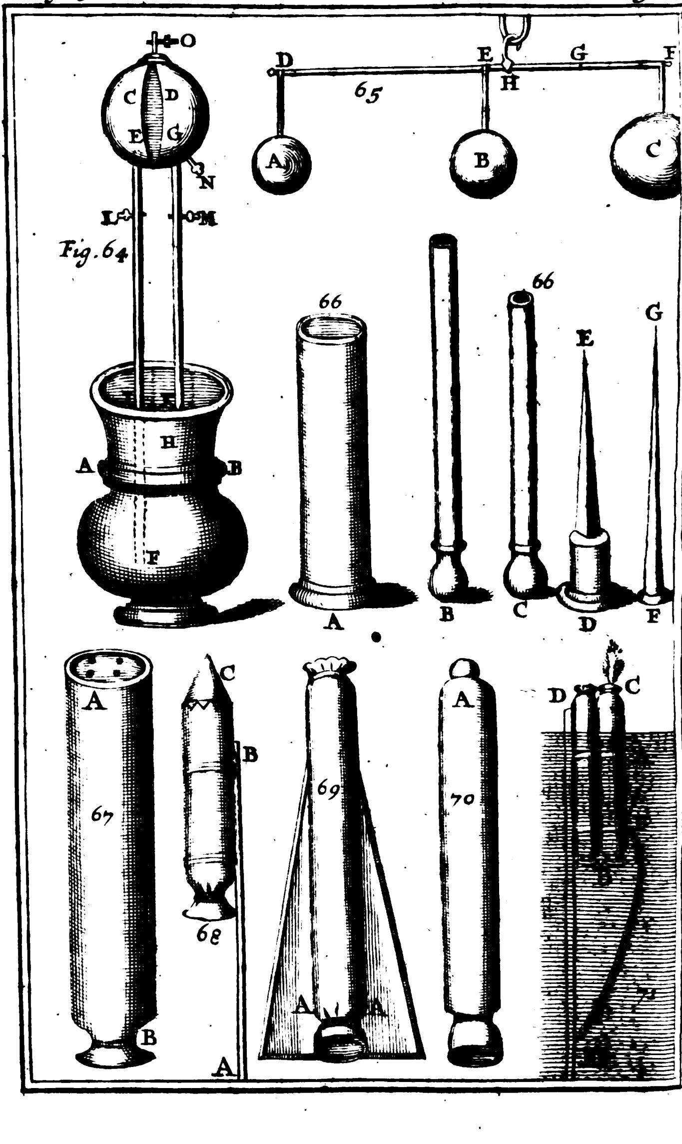
O
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Fig. 64
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A
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70
D
C

ter, and wash'd five or six times with warm fair Water.

Father *Timothy* the *Barnabite* has made one of these Clocks 5 Foot high, that wants winding but once a Month; and shews not only the hours of the Day upon a Dial-Plate, but the day of the Month, the Feasts of the Year, the Sun's place in the Zodiack, its time of Rising and Setting, the length of Day and Night, by means of a small Sun that moves and descends imperceptibly, and at the end of every Month is rais'd up to the Head of the Barrel, after finishing its Monthly Course.

PROBLEM XLI.

To make a Liquor ascend by vertue of another Liquor that's heavier.

WE'll suppose there's Wine in the Vessel AB, which we want to raise to the part DG of the Concave Sphere CD, suppos'd to be separated into two parts, C, D, which have no other Communication one with another, but what they have by the Orifice O. At this Orifice O we suppose a Funnel so contriv'd that the Water pour'd into it may enter (when we will) the part CE, and fill it quite full. This Funnel must have a Cock for opening and stopping upon occasion. Plate 23. Fig. 64.

The Concave Sphere CD is supported by two Pipes EF, GH, open at both ends, the greatest of which EF is soldered at E and I, and has its lower end F, near the bottom of the Vessel AB, which is shut close on all sides, and the other Mouth E near the lower bottom of the Sphere CD. The smallest Pipe GH, is soldered at G and K, and its lower Mouth H terminates near the upper side or Head of the Vessel AB, and its upper end G at the inferior side of the Sphere CD. Each of these two Pipes, EF, GH, has a Cock, as L, M; and the part DG, has a Cock below at N.

Open the Cock O, and stay the other three, L, M, N; and pour Water in at O till the part CE is full; then open the two Cocks, L, M, and the Water

ter contain'd in the part CE, will descend thro' the Pipe EF, and press the Wine contain'd in AB, so as to make it rise thro' the Pipe GH into the part DG, by reason that the Pipe CF being larger than the GH, has more weight. So if you stop the Cock M and open N, you may draw the Wine at N and drink it.

PROBLEM XLII.

When two Vessels or Chests are like one another, and of equal weight, being fill'd with different Metals, to distinguish the one from the other;

THIS Problem is easily resolv'd, if we consider that two pieces of different Metals of equal weight in Air, do not weigh equally in Water; because that of the greatest Specifick Gravity takes up a lesser space in Water, it being a certain Truth, that, any Metal weighs less in Water than in Air, by reason of the Water the room of which it fills. For example, if the Water weighs a Pound, the Metal will weigh in that Water a pound less than in the Air. This Gravitation diminishes more or less according as the Specifick Gravity of the Metal is greater than that of the Water,

We'll suppose then two Chests perfectly like one another, of equal weight in the Air, one of which is full of Gold, and the other of Silver; we weigh 'em in Water, and that which then weighs down the other must needs be the Gold Chest, the Specifick Gravity of Gold being greater than that of Silver, which makes the Gold lose less of its Gravitation in Water than the Silver. We know by experience, that Gold loses in Water about an eighteenth parth only, whereas Silver loses near a tenth part: So that if each of the two Chests, weighs in the Air, for Example 180 Pounds, the Chest that's full of Gold will lose in the Water ten pounds of its weight; and the Chest that's full of Silver will lose eighteen; that is, the Chest full of Gold will weigh 170 Pounds, and that of Silver only 162.

Or,

Or, if you will, considering that Gold is of a greater Specifick Gravity than Silver, the Chest full of Gold tho' similar and of equal weight with the other, must needs have a lesser bulk than the other. And therefore, if you dip separately each of 'em into a Vessel full of Water, you may conclude that the Chest which expells less Water, has the lesser Bulk, and consequently contains the Gold.

PROBLEM XLIII.

To measure the depth of the Sea.

TIE a great Weight to a very long Cord, or Rope; and let it fall into the Sea till you find it can descend no farther, which will happen when the Weight touches the bottom of the Sea, if the Quantity or Bulk of Water the room of which is taken up by the Weight and the Rope weighs less than the Weight and Rope themselves; for if they weigh'd more, the weight would cease to descend, tho' it did not touch the bottom of the Sea.

Thus one may be deceiv'd in measuring the length of a Rope let down into the Water, in order to determine the depth of the Sea; and therefore to prevent mistakes, you had best tie to the end of the same Rope another Weight heavier than the former, and if this Weight do's not sink the Rope deeper than the other did, you may rest assured that the length of the Rope is the true depth of the Sea: If it do's sink the Rope deeper, you must tie a third Weight yet heavier, and so on, till you find two Weights of unequal Gravitation that run just the same length of the Rope, upon which you may conclude that the length of the wet Rope is certainly the same with the depth of the Sea.

PRO-

PROBLEM XLIV.

Two Bodies being given of a greater Specifick Gravity than that of Water, to distinguish which has the greatest Solidity.

IF the two Bodies propos'd were of the same Homogeneal Matter, 'twere easie to distinguish that of the greatest Solidity, by weighing them in a pair of Scales, and adjudging the greater Bulk, *i. e.* in this case Solidity, to the heavier.

But if they consist of different Homogeneal Matters, of different Specifick Gravity, but greater than that of Water; put them separately into a Vessel full of Water, and rest assured, that that which expells most Water, is most bulky, as taking up most Room.

Or else weigh them both in Air and Water, and observe how much the weight found in the Air decreases in the Water; for questionless that of the greatest Bulk or Extent, will lose most of its Weight, as filling the room of a greater Bulk of Water.

'Tis by this Problem that we know whether a suspicious piece of Gold or Silver is good or bad, by comparing it with a piece of pure Gold or Silver, as we shew'd Prob. 31.

PROBLEM XLV.

To find the Center of Gravity common to several Weights suspended from different points of a Ballance.

Plate 23. Fig. 65.

TO find the Center of Gravity, of three Weights, for example, A, B, C, suspended from three Points, D, E, F, of the Ballance DF, to which we shall attribute no Weight, nor to the Strings, DA, EB, FC, which hold up the Weights: We'll suppose the Weight A to be 108 Pounds, the Weight B 144 Pounds, and the Weight C 180 Pounds; the distance DE 11 Inches, and the distance EF 9 Inches, so that the whole length of the Beam DF is 20 Inches.

Upon

Upon this Suppoſition, we find firſt of all the Center of Gravity G common to the two Weights, B, C, by finding a fourth proportional to their Sum, to the Weight C, and to the Diſtance EF, that is, to the three Numbers 324, 180, and 9; for in this fourth Proportional we have 5 Inches for the Diſtance EG, and conſequently 16 for the Diſtance DG, and ſo find the Point G about which the two Weights, B, C, continue in *Æquilibrio.*

In the next place we look for a fourth Proportional, to the Sum of the three Weights, A, B, C, to the Sum of the two former Weights, B, C, and to the Diſtance DG, *i. e.* to the three Numbers 432, 324, 16; for this fourth Proportional gives 12 Inches for the Diſtance DH, and conſequently one Inch for the Diſtance EH; and ſo the Point H is the Center of Gravity ſought for, about which the three weights given A, B, C, will remain equally poiſed.

PRO-

PROBLEMS OF PHYSICKS.

PROBLEM I.

To represent Lightning in a Room.

THE Room in which you're to represent Lightning must not be large, but quite dark, and so very close, that the Air can't readily enter it. The Room being thus in order, take a Basin into it with Spirit of Wine and Camphyr, which must boil there till 'tis all consum'd and nothing left in the Basin. This will rarifie the Camphyr, and turn it into a very subtile Vapour, which will disperse it self all over the Room; insomuch that if any one enters the Room with a lighted Flambeau, all the imprison'd Vapour will in a Moment take fire, and appear as Lightning, but without hurting either the Room or the Spectators.

Remark. Camphyr is of a nature so proper to retain and keep an unextinguishable Fire, that 'twill burn entirely, and that very easily upon Ice or among Snow, which it melts notwithstanding their coldness; and if it be reduced to Powder and thrown upon the Surface of any still Water, and then lighted, 'twill produce a very pleasant sort of Fire, for the Water will appear all Fire and Flame; the Reason of which I take to be, because the Camphyr is of a fat Nature which resists Water, and of a light and fiery Substance, which the fire

fire grasps so keenly, that 'tis impossible for this Substance to disengage it self when once 'tis intangled.

PROBLEM II.

To melt at the flame of a Lamp a ball of Lead in Paper, without burning the Paper.

TAKE a very round and smooth leaden Ball, wrap it up in white Paper, that is not rumpled, but clings equally about the Ball without Wrinkles, at least as far as is possible; hold the Ball thus wrapt up over the flame of a Lamp or a Flambeau, and 'twill grow hot by Degrees, and in a little time melt, and fall down in drops through a hole in the Paper, without burning it.

PROBLEM III.

To represent an Iris *or Rainbow in a Room.*

EVery one knows that the Rainbow is a great Arch of a Circle, that appears all on a sudden in the Clouds before or after the Rain, towards that part of the Air that's opposite to the Sun, by vertue of the resolution of the Cloud into Rain; This Arch is adorn'd with several different Colours, of which the Principal are five in Number, namely, Red which is outtermost, Yellow, Green, Blue, and Violet and Purple which is interiour.

This *Iris* seldom appears alone, and is call'd the *First* and the *Principal* Rainbow, to distinguish it from another that commonly appears along with it, and for that Reason is call'd the *Second* Rainbow, the Colours of which are not so lively as those of the *First*, tho' they're disposed after the same manner, but in a contrary order, upon which account a great many take it for a Reflection of the *First*.

If you want to represent at one time, two such *Iris*'s in your Room, put Water into your Mouth and step to the Window (upon which the Sun is suppos'd to shine) then turn your Back to the Sun, and your Face to to the dark part of the Room; and blow the Water, which is in your Mouth, making it spurt out with Violence, into little Drops or Atoms; and among these little Atoms or Vapours, you'll see by the Rays of the Sun, two Rainbows resembling the two that appear in the Heavens in Rainy Weather.

Oftentimes we see Rainbows in Water-works or Spouts, when we stand between the Sun and the Fountain, especially when the Wind blows hard, for then it disperses and divides the Water into little drops. Which is full evidence, that the Rainbow, which the Philosophers admire as much as the ignorant People do Thunder, is form'd by the Reflexion and Refraction of the Rays of the Sun, darted against several little drops of Water, that fall from the Clouds in time of Rain.

A Rainbow may likewise be very easily Represented, in a Room with a Window that the Sun shines upon, by a Triangular Prism expos'd to the Rays of the Sun, which in passing thro' the Glass, will by their different Reflexions and Refractions produce upon the Wall or Cieling of the Room, a very agreeable *Iris*, or at least a texture of several different Colours resembling those of the Rainbow; and the further the Cieling or Wall is distant, and the more 'tis dark, the Colours will appear the more Charming and Lively. You may likewise imitate the Colours of the Rainbow by exposing to the Sun a Sphere of Crystal or Glass, or a Glass full of clean Water.

PROBLEM IV.

Of Prospective Glasses or Telescopes.

Telescopes are long and light Pipes or Tubes, which contain in their Concavities two or more Spherical pieces of polish'd Glass Perpendicular to the Axis of the Pipe, and placed at such a distance one from another, that when one or two Eyes look thro' these Glasses they see remote Objects, as if they were near at hand. They are likewise call'd *Prospective Glasses*, and *Dioptrical Ocular Glasses*. When they are made only

only for one Eye, as they are most commonly, they are call'd *Single Ocular Glasses*; and on the other hand they are call'd *Double Ocular Glasses*, or *Binocles*, when they're compos'd of two single Ocular Glasses, so adjusted in one Pipe, that both Eyes may see through 'em at once. Father *Cherubin* the *Capuchine*, has writ a particular Treatise of them, and pretends that remote Objects are better discern'd by them, than by the single Prospective Glasses.

The small Prospective Glasses that People carry in their Pockets, and those which are larger and are made use of for discovering remote Terrestrial Objects, and even the greatest of all which are used for Celestial Observation, have commonly only two Glasses at the extremities of the Prospective which are call'd *Lens's*, and of which that nearest the Eye, call'd the *Ocular Glass*, is Concave, and that at the other end nearest the Object, call'd the *Objective Glass*, is Convex.

In a Prospective that's a Foot long, the Diameter of the *Lens*, that's Convex on both sides, may be four Inches, and that of the Concave as much; and in a Prospective that's five Foot long these Diameters may, each of 'em, be twelve Inches. The Telescopes for the Stars, which are *Astrocopes*, are made with two Convex Glasses, and the larger they are they are the better; those made for observing the spots of the Sun call'd *Helioscopes*, are made like the ordinary Telescopes, only the Glasses are colour'd to prevent the Rays of the Sun from annoying the Eyes.

These Prospective Glasses, are said to have been first invented in *Holland*, and first made use of for Celestial Observations by *Galileus*. They are of great use, for reading a piece of Writing at a Distance, for descrying at Sea, Ships, Capes, and Coasts, and in an Army by Land for taking a view of the Officers, Cannon, March, &c. of the Enemy. The use of Telescopes.

By the use of them several remarkable things in the Heavens, unknown to the Ancients, have been discover'd. In ancient times they reckon'd only seven Planets in the Heavens, namely, the Moon, Mercury, Venus, the Sun, Mars, Jupiter and Saturn; but the Moderns have found many more. By Telescopes they've discover'd four round Jupiter, which *Galileus* who first descry'd 'em call'd *Stellæ de Medicis*, and which

which turn regularly round Jupiter at unequal Distances, without ever quitting it, and for that Reason they're call'd the *Satellites* of Jupiter. The first of these *Satellites* or that next to Jupiter, compleats its Period in 1 Day, 18 Hours, and 29 Minutes, and the last or that which is remotest from Jupiter, finishes its Circumvolution in 16 Days, 18 Hours, and 5 Minutes.

By the same means they've discovered five Planets round Saturn, which are likewise call'd the *Satellites* of Saturn; and of which the first or that nearest to Saturn finishes its course in 1 Day, 21 Hours, and 19 Minutes; and the last or that remotest from Saturn in 79 Days, and 21 Hours.

They've likewise observed round the same Saturn a Ring of Light, that's flat and thin, which declines from the Ecliptick about 31 Degrees, and turns continually round Saturn, as is gather'd from its appearing sometimes in a straight Line, *viz.* when 'tis seen *Profil-ways* which happens every fifteenth Year, and at other times in an Oval form when 'tis turn'd Obliquely, and again quite round when 'tis seen in the Front.

Aristotle took the *Galaxie* or Milky way for a Meteor, but our Telescopes give us to know that 'tis a Collection of several little Stars which form a broad Circle like the Zodiack, that passing from North to South thro' the Constellation of Orion towards the Æquator, cuts the Zodiack at almost Right Angles. 'Tis true indeed that according to the testimony of *Plutarch*, *Democritus* did utter some such thing, but then 'twas only by Conjecture.

Several Discoveries made by Telescopes.

Besides these, there's an infinite number of other Stars hid to the natural infirmity of the Eyes, which are easily brought to light by Telescopes. Monsieur *Cassini* informs us, that some Stars appear to the naked sight like the rest, but when view'd by a Telescope appear double, triple and quadruple. The first of *Aries* appears to be compos'd of two equal Stars, distant from one another the length of one of their Diameters. The same thing is observ'd of that at the head of *Gemini*; and in the *Pleiades* there are some which appear to a Telescope Triple and Quadruple.

In

In fine, by the means of Telescopes, we have observ'd considerable inequalities in the Moon, particularly, Mountains casting their Shadow to the side opposite to the Sun, Concavities, Plains and Valleys. Likewise *Maculæ* or Spots, *i. e.* dark Bodies turning round the Sun, which in appearance blacken and darken it. Monsieur *Tarde* took these for Stars, and call'd them the *Stars of Bourbon*, which have regulated Periods round the *discus* of the Sun, from East to West, with respect to the Inferior Hemisphere of the Sun, and finish these their Periods in 26 or 27 Days.

We have likewise remark'd upon the surface of Jupiter, not only several dark Girdles, like unto the spots observ'd in the Moon, which move in Parallel Lines round that Planet from East to West, almost according to the Eclíptick; but likewise Spots of different sizes among these Girdles, which have their Regulated Periods. The same thing is observ'd in *Venus*, which gives us reason to presume that these Planets turn round their *Axis*'s variously inclin'd, excepting the Moon which do's not seem to turn, in regard its Spots appear always turn'd to the Earth after the same manner.

Ptolemy believ'd, as appears by his System, that Venus and Mercury were always under the Sun, upon the account that he had sometimes seen 'em eclipse that glorious Star; but since the use of Telescopes we've discover'd that these two Planets have, like the Moon, two different *Phases*; which gives us to know, that *Venus* and *Mercury* not only borrow their Light from the Sun, as the Moon do's, but likewise turn round it like *Satellites*; and so we discover that *Ptolemy's* System is absolutely false with respect to these two Planets,

Since we have not found different *Phases* in the three other Planets, *Mars*, *Jupiter* and *Saturn*, which are call'd the Superior Planets, we readily infer from thence, that, they are higher than the Sun, for they borrow their Light from it, as well as the *Satellites* of *Jupiter* and *Saturn*: For with respect to the *Satellites*, for instance, of *Jupiter*, we observe by a Telescope, that they cast their Shadows against its *Discus*, when they are between the Sun and Jupiter, and in like manner Jupiter darkens them, when 'tis between them and the Sun: And with respect to *Mars* we find by a Telescope, that 'tis always of a round Figure in its Opposition, and crooked between its Conjunction and Opposition, as it happens to the Moon a little before and a little after its Opposition.

Remark. If instead of applying the Eye to the Ocular Glass of a Telescope, we apply it to the Objective Glass, 'twill produce a quite contrary effect, that is, in stead of augmenting the Object or bringing it nearer, 'twill make it appear less and more remote by an agreeable sort of Perspective. This we offer upon the Supposition that the two Glasses are well placed, for otherwise the Object will appear confused, and without any distinction of Parts. These Glasses are put into Tubes for the better gathering of the Species, and keeping off the dazzle of too much surrounding Light; for to see an Object well, the Object ought to be surrounded with Light, and the Eye with Darkness. And for this reason, the Eye placed at the bottom of a very deep Well, may see the Stars at Noon time of Day; and 'tis by this Contrivance that in the Royal Observatory at *Paris* one may see in the Day time the Stars that are near the Zenith.

Of Multiplying Glasses. Some Prospectives are made of Crystal cut with the point of a Diamond to several Angles, which serve to multiply the appearances of Objects to the Eye looking thro' the Crystal; the occasion of which is the various Refraction, which sends to the Eye as many different Images of the Object, as there are different Plains in the Crystal; and these are call'd *Multiplying Glasses*, and *Polyedron Glasses.* Thro' this sort of Prospectives, a Tree appears as a Forest, a House as a City, and a Company of Soldiers like a numerous Army.

Of Microscopes. We have likewise Ocular Microscopes, which are call'd barely *Microscopes*, and are compos'd of one or more lenticular Glasses, that are parts of a very small Sphere, and magnifie the Objects prodigiously, so that by their means one may easily and distinctly see the smallest and otherwise Invisible Objects, when they are near at hand.

These Microscopes, which are likewise call'd Engyscopes, are made after several different ways, which 'tis needless here to repeat. I shall only take notice, that

that some are made only of one lenticular Glass convex on both sides, and done up in a little Box, in which is a small Hole for one's Eye to see thro' the Glass a Flea, or any other Insect placed on the other side of the Bottle or Box, upon which occasion all its otherwise invisible Parts are distinctly and wonderfully magnifyed.

If you put into such a Microscope a Flea or a Louse, you'll see a sort of a Fight between these two monstrous Animals. The Flea will resemble a Grass-Hopper, or rather a Lobster, by reason of the Scales observed upon its Body, and its pointed Tail, with which these Animals prick Men. The Louse will resemble a hideous Monster with a transparent Body, which gives the opportunity of seeing the Circulation of the Blood in its Heart, which sensibly beats and boils, thro' the passion excited in it by its Enemy. *Of several Insects.*

In these and several other Insects, we observe commonly two Eyes; among which those of Flies and of several other Insects that creep upon the Earth, appear intersected with several little Squares, like Fishers Nets. I said, we observe commonly two Eyes; because in a Spider we find six and sometimes eight Eyes, six of which are placed in an Arch of a Circle, and the other two in the middle.

An Ant has likewise Eyes, tho' several are of another Opinion who have not observ'd them, by reason of their black Colour like that of their Eyes. These Eyes are easily perceiv'd in the small Ants that we find in the largest Eggs, for these little Ants are white, which contributes much to the discovering of their black Eyes. *Several Discoveries made by Microscopes.*

To a Microscope the smoothest Skin of Mankind appears frightful, and full of Wrinkles; and the smoothest best polisht Glass appears rough, full of chinks, and as compos'd of several uneven irregular Pieces. In like manner the finest Paper appears rough and uneven, and full of Cavities and Eminences. The same thing is observ'd in the hardest and best polish'd Bodies, such as a Diamond; and therefore when we would choose a good Diamond, we ought to look upon it with a Microscope, and take that which is least ragged.

By a Microscope we discover in the powder or dust of Cheese, and even in the Cheese it self, an infinite number of Animals colour'd very agreeably, with very large clear black Eyes, Claws on their Feet, Horns on their Head, and three remarkable Points in their Tail. In Milk, Vinegar, and Fruit ready to spoil thro' long keeping, we find Animals in the form of Worms and Serpents. In the Noses of several Men we find Worms with a black Head, resembling Lizards and Spiders; as well as in the Scab, the small Pox, Ulcers, and generally in all Corrupt Bodies.

In fine by the means of a Microscope, we find that a Mite has its Back cover'd with Scales, that it has three Feet on each side, and two black Spots on the Head. We likewise find that the least spot of Mouldiness upon the cover of a Book is a little Parterre cover'd with Plants, which have their Stems, their Leaves, their Buds and their Flowers. We discover in Common Salt the figure of a Cube, in Salt of Nitre the figure of Pillars with six Faces, in Sal Armoniack an Hexagon, in Salt of Urine a Pentagon, in Allum an Octagon, and in Snow a Sexangular Form.

PROBLEM V.

To make an Instrument by which one may be heard at a great distance.

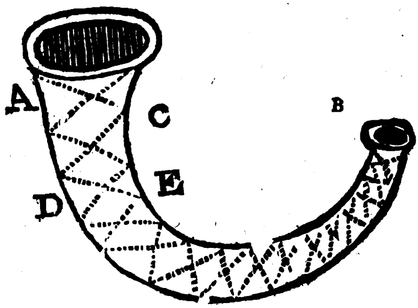
As Prospective Glasses serve the Eyes, so an Instrument may be made to serve the Ear: For certain it is, that the long Tubes call'd *Sarbacanes* will make one to hear very distinctly at a good distance: For Pipes serve generally to inforce the activity of Natural Causes. Of this Experience is sufficent Evidence; for by it we find that with a *Sarbacane* we can shoot to a great distance, and with a great force a little Ball placed in the Pipe, only by blowing upon it; and that the longer the Pipe is, the greater is the force: Tho' after all, as I take it, it ought not to be extravagantly long, but proportion'd to the force of the blowing. Thus, we see Cannons of the same bore, and different length, increase their force from eight to twelve Foot long; but beyond that length their force diminishes; which proceeds undoubtedly

doubtedly from this, that the length of the Cannon is no longer proportion'd to the force of the Powder, which pushes out the Ball.

Since every thing that's mov'd thro' the Cavity of a Pipe, has so much the more Violence, the longer the Pipe is, provided the length of the Pipe is proportional to the moving Force; we may easily gather from thence the Reason, why a Voice thro' a long Pipe is heard at a great distance, the Air being push'd with Violence thro' the Pipe; and 'tis for much the same Reason, that Fire confin'd within a Tube burns very fiercely, what it would scarce heat in the Air; and Water runs impetuously when confin'd to a long Canal, as we see in Waterworks and Spouts of Fountains.

Some *Sarbacanes* are made of fine Metal, as Silver, Copper, or any other Sonorous Matter, in the form of Funnels, or at least wider at one end than at the other; and these are made use of for hearing at a Distance a Preacher or any other Person that speaks publickly, by clapping the narrowest end to the Ear; and turning the wide end to the Speaker, in order to collect the sound of his Voice.

Experience shews that Horns and Trumpets, which are almost of the same form, contribute very much to fortifie the Sound, and make it to be heard at a Di-



stance; especially those Trumpets which are bended to an Arch of a Circle, as AB; for the Air makes a stronger Reflexion in a crooked than in a straight Pipe, as as is evident from the Figure, in which the Lines AC, CD, DE, *&c.* represent the different Reflexions of the Air push'd out by him who blows at B.

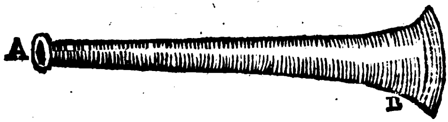
Father *Kircher* the Jesuit, in his Treatise *De arte magna lucis & umbræ, l.* 2. *Part.* 1. *cap.* 7. *Prop.* 3. speaks of a certain Horn with which *Alexander* the Great spoke to his whole Army though numerous and widely dispers'd, and by which his Orders were heard by all his Soldiers, as well as if he had been just by every one of 'em. He adds, that according to what he had read of it in the Vatican at *Rome*, 'twas seven foot and a half in Diameter, and might be heard at the distance of an hundred *Stadia*, the extent of which makes about five Leagues.

Thus you see that the Invention of the Speaking Trumpet is very Ancient; and of this its Antiquity you will be more fully perswaded if you believe *Theodorus*, who speaking of the Oracle of *Delphos*, says, they sometimes made use of the Speaking Trumpet, for the more dexterous gulling of those who came to consult the Oracle, for this Instrument made them hear a more than human Voice. This Instrument has been reviv'd in our days by Sir *Samuel Morland*, who call'd it *Tuba Stentereophonica*; and tho' that Tuba do's not carry so far as *Alexander's*, yet it raises a Man's Voice with a greater distinction of the Syllables and Words.

This Author made several of different Sizes, the Reach of which was likewise different. One of 'em which was four Foot and a half long, was heard at the distance of 500 Geometrical Paces: Another that was sixteen Foot and eight Inches long, was heard at the distance of 1800 Geometrical Paces; and a third of four and twenty Foot above 2500. He tells us, that if these Trumpets be good, they must widen gradually by little and little, and as it were insensibly like AB, and not all on a sudden. See the following Figure.

That

That Author has not given us a very exact Figure of the Trumpet, he only tells us that the Aperture A of the narrow end, ought to be equal to the Aperture of the Mouth of the Speaker; otherwise the Voice dwindles considerably, there being a great deal



of Air lost. So that the small end ought to be so adjusted to the Mouth as to lose no Air; and at the same time the Mouth must have liberty to open and shut, that the Articulation may be form'd and preserv'd entire.

We have here represented the Trumpet straight, like the ordinary Trumpets; but you may give 'em any other Figure, for example, a Circular or Ellyprical Figure, like that of *Alexander's*. For the winding, instead of doing any harm, serves rather to fortifie than to weaken the Voice, as we have said already. A Pistol shot off in one of these Trumpets makes a noise like a Cannon. 'Tis now high time to come to the Uses, and the advantage of this Speaking Trumpet.

In the first place, the Speaking Trumpet is of good use at Sea, in a Storm or a dark Night, when one Ship dare not come within reach of speaking nakedly to the other. For by this Trumpet they may speak to another at the distance of a Mile or more, especially if they take the advantage of the Wind, which forwards the Voice very much. The uses of the Speaking Trumpet.

An Admiral may, in imitation of *Alexander* the Great, make use of it in a Calm, to convey his Orders to his whole Fleet, tho' dispers'd to the extent of two or three Miles round him.

In fine, if a Ship is all alone in a great Storm, he who commands the Ship, may by a Speaking Trumpet, make his Voice to be distinctly heard by all the Seamen. And in case of a great Expedition, it may be used on Shoar, to give speedy Orders to all the Ships in a Road; and if Secrecy be requir'd, the Orders

ders may be conveyed in obscure Terms previously concerted.

In the second place; The Speaking Trumpet may be of great use at Land; for by it a General may, like *Alexander*, speak to his whole Army at once, tho'. forty or fifty thousand strong; both for giving the necessary Orders, for rallying dispers'd Troops, and for raising the Courage of the Soldiers; and by the same Instrument, a Herald at Arms may be distinctly heard by several Millions of Souls, whereas without it his Voice could not be heard by above thirty or forty Persons.

'Tis likewise very convenient for an Intendant or Overseer of Works, in giving Orders to all his Workmen at once, without shifting his place; as also for giving the Alarm to the adjacent Country, when a House is rob'd.

In fine, 'tis of great use, when a Town is Besieged, for acquainting the Besieged when they may expect Succour, for keeping the Officers to their Duty, and scaring the Inhabitants from Mutinies.

Remark. The Speaking Trumpet ought to be made of some resounding Substance, such as white Iron, for that contributes much to the fortifying of the Voice. 'Tis said that a Monk happening one day to sing thro' a single Cornet of Pastboard, observed his Voice to be very much heighten'd by that Instrument, and so took up the fancy of filling a Chorus of Musick with it, a moderate Voice so imployed surpassing the force of the Base Hoboys and Violins generally made use of in Musick.

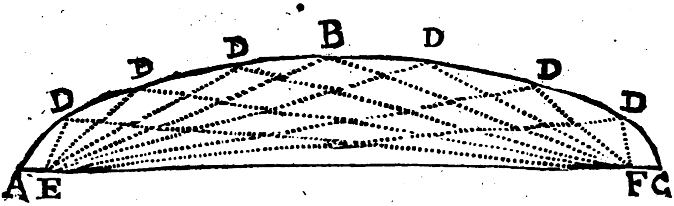
As this Trumpet inlarges the Sound, and fortifies the Voice; so 'tis very useful for a help to the Ear; for if you fix to its Mouth or small end a little Cornet of Pastboard, and put that to the Ear, it fortifies the Sense of Hearing, and will make one hear the least noise made at a great distance; for the width of the other end of the Trumpet serves to gather and fetch in the Sound, and the Cornet to convey it to the Ear. 'Tis upon this Principle that *Vitruvius* mentions certain Vessels or Pipes, that were used in Plays for inforceing the Voice of the Actors; and 'twas by the same Vessels and Pipes that an *Italian* Prince heard from his Parlour, the Voice of those that were walking in an adjoining Flower-Garden.

The

The Hearing may likewise be assisted, and the Sound augmented, by a long Beam of some light resounding Wood, such as Fir, as AB; for we know by Experience, that if a Man lays his Ear to one Extremity A, he will hear the least noise at the other Ex-

tremity B, tho' the Beam were 200 Foot long; for by reason of the quantity of the Pores of which the Wood is compos'd, it may be consider'd as a Canal or hollow Pipe, the property of which is to convey the Sound as far as 'tis long.

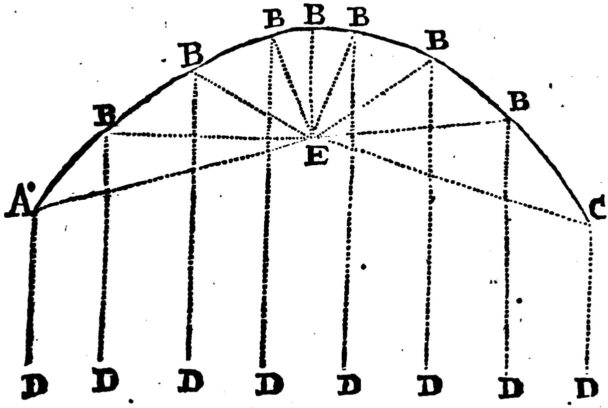
Experience teaches, and Geometry demonstrates that one laying his Ear to one of the two *Focus*'s of an Elliptick or Oval Vault, will readily hear another Person speaking very low at the other *Focus*; and at the same time People standing in the middle between 'em shall hear nothing. Let the Elliptick Arch-roof be ABC, the two *Focus*'s of which are E and F; he who speaks very low at E, will be readily heard by another at F; tho' those who are in the middle be-



tween E and F, shall hear nothing. Now, the cause of this, is the Air, which being push'd on all hands from E towards D against the Arch-roof, by the Voice at E, reflects in an infinite number of straight Lines which terminate at the other *Focus* F, with Angles of Reflexion equal to those of Incidence; for the property of these two *Focus*'s EF is such, that if from the same Point of the Ellipsis ABC, such as D, you draw the two straight Lines DE, DF, these two straight Lines will make with the same Ellipsis, on one side and t'other, equal Angles.

The

The case is almost the same in a Parabolick Arch-Roof or Dome, ABC, the *Focus* of which is E, where a Person standing may easily hear another speaking very low at D, for the Air which the Voice



pushes from D against the Roof at B, by the Line DB parallel to the Axis of the Parabola, reflects in the Line BE, which by the property of a Parabola repairs to the *Focus* E.

PROBLEM VI.

To make a Consort of Musick of several parts, with only one Voice.

Of Eccho's.

THE Sound conveyed distinctly to the Ear, by remote Bodies, against which the Air is driven by the Voice of an Animal or otherwise, and then reflected, is what we call an Eccho; which is sometimes double, triple, *&c.* when the Voice is strong enough to make several Bodies, at different Distances, beat back at several times the parts of the Air to our Ears, so that one Eccho is no sooner ended than another begins.

Tho' most Eccho's make us hear only the last words of the Voice, because the Air, tho' strongly impress'd, has not the same force at the end that it had at the beginning; yet it may be so contrived as to make a Consort of Musick of several Parts, that is, a Consort of of several Songs tun'd together, by only one Voice or one Instrument, to the sound of which the Eccho Answers.

For if the Eccho answers only once to the Voice or the Sound of the Instrument, he who Sings or Plays may make a *Duo*, that is, a Musick of two Parts; and again a *Trio* or Musick of three Parts, if the Eccho answers twice. But indeed he must be an expert Musician, and one that's well vers'd in varying the Tune and the Note.

Thus commencing, for example at *Ut*, he may begin *Sol* a little before the Eccho answers, so as to finish the Pronunciation of *Sol* by that time that the Eccho has compleated its Answer, and then he will have a *Fifth*, which is a perfect Consonance in Musick; and in like manner, if at the same time with the Eccho's answering to the second Note *Sol*, or a little before, he repeats it upon a higher or lower Note, he will make a Diapason or Eighth, which is perfect Harmony in Musick. And so on, if he has a mind to continue the chace with the Eccho, and sing alone the two Parts.

To this purpose we see by Experience in several Churches, when they're singing, that there seems to be many more parts in the Chorus than there really are, the quantity of Eccho's making the Air to resound on all sides, and so multiplying the Voice and redoubling the Chorus.

PROBLEM VII.

To make the String of a Viol shake without touching it.

CHoose at pleasure three Strings in a Viol, or any other Instrument of that sort, without any Intermediating String, and tune the First and the Third to the same Note, without touching that in the Middle; then strike one of the two Strings thus tun'd pretty hard with a Bow, and you'll find that when it shakes the other will tremble sensibly and visibly, and the middle String tho' nearer, shall not stir no manner of way.

This

This Problem may likewise be resolved by two Stringed Instruments of the same sort, as two Viols, two Lutes, two Harps, two Spinettes, &c. by putting the two in the same Tune, and then placing them at a convenient Distance, and in a proper Position; for one of the two Instruments being touch'd with a midling force, will move the other, *that is*, the Strings of the other, which are suppos'd to be in Unison, will produce such another Harmony, especially if the Strings in one and t'other Instrument are equally long and equally thick. For this I can assign no other Reason but Experience.

PROBLEM VIII.

To make a Deaf Man hear the Sound of a Musical Instrument.

IT must be a String'd Instrument, with a Neck of some Length, as a Lute, a Guitarre, or the like; and before you begin to play, you must by Signs direct the Deaf Man to take hold with his Teeth of the end of the Neck of the Instrument; for then if one strikes the Strings with the Bow one after another, the Sound will enter the Deaf Man's Mouth, and be conveyed to the Organ of hearing thro' the Hole in the Palate: And thus the Deaf Man will hear with a great deal of Pleasure the sound of the Instrument, as has been several times Experienced. Nay, those who are not Deaf, may make the Experiment upon themselves, by stopping their Ears so as not to hear the Instrument, and then holding the end of the Instrument in their Teeth while another touches the Strings.

PROBLEM IX.

To make an Egg enter a Vial without breaking.

LET the Neck of the Vial be never so strait, an Egg will go into it without breaking, if it be first steep'd in very strong Vinegar, for in process of time the Vinegar do's so soften it, that the Shell will bend

bend and extend lengthways without breaking. And when 'tis in, cold Water thrown upon it will recover its primitive hardneſs, and, as *Cardan* ſays, its primitive Figure.

PROBLEM X.

To make an Egg mount up of it ſelf.

MAke a little Hole in the ſhell of the Egg, and ſo take out the Yelk and the White, and fill the Egg-ſhell with Dew; then ſtop up the Hole and expoſe it to the Rays of the Sun at Noon-day; for then the Dew not being able to bear the Light, nor too great Heat, will riſe up with the Egg-ſhell, eſpecially if it leans againſt a little Stick or piece of Wood, that ſlopes never ſo little, and if the Hole is well ſtop'd. *May* Dew is ſaid to be beſt; and 'tis obſerv'd by the Farmers, that the more *May* abounds in Dew, the more plentifully do's the Earth bring forth; for Dew being a ſubtile Vapour, produced in the Morning by a weak Heat, and preſerv'd by a moderate Cold, 'tis very well diſpoſed for the Reception of Celeſtial Vertues; and when it inſinuates it ſelf into Vegetables, it communicates to them the Vertues it retains; and hence it comes that Plants moiſten'd with it thrive better, than when they are nouriſh'd with Spring, Well, or River Water.

PROBLEM XI.

To make Water freeze at any time in a hot Room.

FIll a Vial with warm Water, the Neck of which is ſomewhat narrow, and having ſtop'd it cloſe, put it in a Veſſel full of Snow mix'd with common Salt and Saltpetre, ſo as to leave the Vial cover'd all over with Snow; and in a little time the Water will be quite frozen, tho' in the Summer time, and in a very hot Room,

If you throw cold Water with Snow upon a Table, and upon the Snow ſet a Platter full of Snow with a ſufa sufficient quantity of Salt and Saltpetre pounded; the Salt and the Saltpetre will make the Snow so cold, that in a little time the Water under the Platter will be turn'd to Ice, and make the Platter stick so fast to the Table, that you can't move it without some difficulty.

Remark. The Saltpetre and Sal-Armoniack are likewise possess'd of the vertue of making Water so extremely cold, that if you put a sufficient quantity of 'em in Common Water, 'twill become so cold that your Teeth can scarce bear it. They might therefore be very usefully imployed in Summer for cooling Wine or any other Liquor, by setting the Wine Bottles in Water thus refrigerated.

If you dissolve a pound of Nitre in a pail of Water, the Water will be excessive cold, and so very proper for the uses above-mention'd. 'Tis well known that Wine is likewise cool'd with Ice; and in regard Ice can't always be had in Summer, I shall prescribe a way of making it.

How to make Ice in Summer. To make Ice in Summer, put two Ounces of refin'd Saltpetre, and half an Ounce of Florentine Orris, into an Earthen Bottle fill'd with boiling Water; stop the Bottle close, and convey it forthwith into a very deep well, and there let it steep in the Well-Water for two or three Hours, at the end of which you'll find the Water in the Bottle all Ice; so you have nothing to do but to break your Bottle and take out your Ice.

PROBLEM XII.

To kindle a Fire by the Sun-beams.

THIS Problem may be resolv'd either by *Refraction* in using lenticular Glasses thicker in the middle than in the sides, call'd *Burning-Glasses*, thro' which when the Rays pass they refract and unite in one Point call'd the *focus*, at which you may light a Match or any other combustible Matter: Or else by *Reflexion*, in using a concave Looking-Glass of Metal well polish'd in its Concavity, which may be either Spherical or Parabolick, and is likewise call'd a *Burn-*

Burning-Glass, but much better than the former sort; for by it you may in a Moment set fire to a piece of Wood, and in a short time melt Lead, and even Iron, and vitrify Stone, as we intimated above at large in *Probl.* 16. *Of the Opticks*, which see.

PROBLEM XIII.

To make a Fowl roasting at the Fire, turn round of it self with the Spit.

TAke a Wren and spit it on a Hazel Stick, and lay it down before the Fire, the two ends of the Hazel Spit being supported by something that's firm; and you'll see with Admiration the Spit and the Bird turn by little and little without discontinuing, till 'tis quite roasted. This Experiment was first found out by Cardinal *Palotti* at *Rome*, who shew'd it Father *Kircher*, in order to know the Physical Cause of it; which to my Mind is easily discover'd, for the Hazel Wood is compos'd of several long and porous Fibres, into which the heat insinuates it self, and so makes it turn round when the Wood is hung right.

PROBLEM XIV.

To make an Egg stand on its smallest end, without falling, upon a smooth Plain such as Glass.

PLace a Looking-Glass quite Level, or Horizontally, without inclining to either side; toss the Egg with your Hand till the Yelk bursts, and the matter of it is equally dispers'd thro' all the parts of the White, so that the White and the Yelk make but one Body. Then set the end of the Egg upon the Horizontal Plain, holding it till 'tis upright, and then 'twill continue in that situation without falling, by reason of the *Æquilibrium* made on all sides by the parts of the Yelk equally mix'd with the White, so that the Center of gravity in the Egg continues in the Line of Direction.

PRO-

PROBLEM XV.

To make a piece of Gold or Silver disappear, without altering the position of the Eye or the Piece, or the intervention of any thing.

PUT the piece of Gold in a Porringer full of Water, or a Vessel that's broader than 'tis deep, and let the Eye be in such a Position, as just barely to see the piece at the bottom over the Brim of the Vessel; then take out the Water, and tho' the Porringer continues in the same Position as well as the Eye, the Piece which appear'd before by vertue of the Refraction made in the Water, will then be cover'd from the sight by the sides of the Porrenger.

PROBLEM XVI.

To make a Loaf dance while 'tis baking in the Oven.

PUT into the Dough a Nutshell fill'd with Live Sulphur, Saltpetre and Quicksilver, and stop'd close; as soon as the Heat comes to it, the Bread will dance in the Oven; which is occasion'd by the nature of Quicksilver, for it can bear no Heat without being in a continual Motion. Thus, by the means of Quicksilver put into a Pot where Pease are to be boil'd, all the Pease will leap out of the Pot as soon as the Water begins to heat. In like manner Quicksilver put into hot Bread, will make it dance up and down the Table.

PROBLEM XVII.

To see in a dark Room what passes abroad.

MAke your Room so close and dark, that the Light can come in no where but through a little Hole left in a Window upon which the Sun shines; over against this Hole, at a reasonable distance from it, place

place some white Paper, or a piece of Linnen; and you'll see every thing that passes by the outside of the Window appear on the Paper or Linnen, only their Figures are inverted.

For your further Satisfaction in the Resolution of this Problem, look back to Problem 18 of the Opticks,

PROBLEM XVIII.

To hold a Glass full of Water with the Mouth down, so as that the Water shall not run out.

TAKE a Glass full of Water, cover it with a Cup that's a little hollow, inverting the Cup upon the Glass; hold the Cup firm in this Position with one Hand, and the Glass with the other, then with a Jerk turn the Glass and the Cup upside down, and so the Cup will stand upright, and the Glass will be inverted, resting its Mouth upon the interior bottom of the Cup. This done, you'll find that part of the Water contain'd in the Glass will run out by the void space between the bottom of the Cup and the brim of the Glass; and when that space is fill'd, so that the Water in it reaches the brim of the Glass, all passage being then denied to the Air, so that it can't enter the Glass, nor succeed in the room of the Water, the Water remaining in the Glass will not fall lower, but continue suspended in the Glass.

If you would have a little more Water descend into the Cup, you must with a Pipe or otherwise draw the Water out of the Cup, to give passage to the Air in the Glass; upon which part of the Water will fall into the Glass till it has stopt up the passage of the Air afresh, in which case no more will come down; or, without sucking out the Water in the Cup, you may incline the Cup and Glass so that the Water in the Cup shall quit one side of the brim of the Glass, and so give passage to the Air, which will then suffer the Water in the Glass to descend till the passage is stopt again.

This Problem may likewise be resolved by covering the brim of the Glass that's full of Weter, with a leaf of strong Paper, and then turn the Glass, as above; and without holding your Hand any longer upon the Paper, you'll find it as it were glewed for some time to the brim of the Glass, and during that time the Water will be kept in the Glass.

PROBLEM XIX.

To make a Vessel or Cup that shall throw Water in the face of the Person that drinks out of it.

Plate. 24. Fig. 72. GET a Cylindrical Vessel of Metal or of what other Substance you will, such as ABCD; and another Conical Vessel EFG, the Mouth or Aperture of which EF, is larger than the Mouth AB; and so the Conical Vessel being put with its *Vertex* down into the Cylinder it exactly fills the Aperture AB, but its Point G at which there's another Aperture do's not touch the bottom CD, and that for a reason to be given in the Sequel. Tho' this Conical Vessel do's by its roundness exactly stop the Mouth or Aperture AB, yet 'tis difficult to hinder the Air to enter in between 'em, and therefore to cut off all manner of passage for the Air, the Conical Vessel should be neatly glewed to the brim AB.

This done pour Water or Wine into the Conical Vessel, at its Mouth or Aperture EF, and the Liquor will descend thro' the Aperture G, into the Cylindrical Vessel, and will there rise to about the height of the Aperture G; for 'twill scarce be able to rise higher by reason of the Air inclosed in the Vessel, which will be there very much compress'd. Now, the Liquor not being able to rise higher in the Cylindrical Vessel ABCD, will rise in the Conical Vessel EFG, and fill it if you continue to pour Liquor into the Vessel EFG.

After this Preparation, if you present the Vessel to any one, to drink out of it when the whole Conical Vessel EFG is empty, the Water remaining in the Cylindrical Vessel ABCD being press'd by the Air, which is likewise compress'd it self, will impetuously fly out thro' the Aperture G, and wet all the face of the Person that's a Drinking.

PRO-

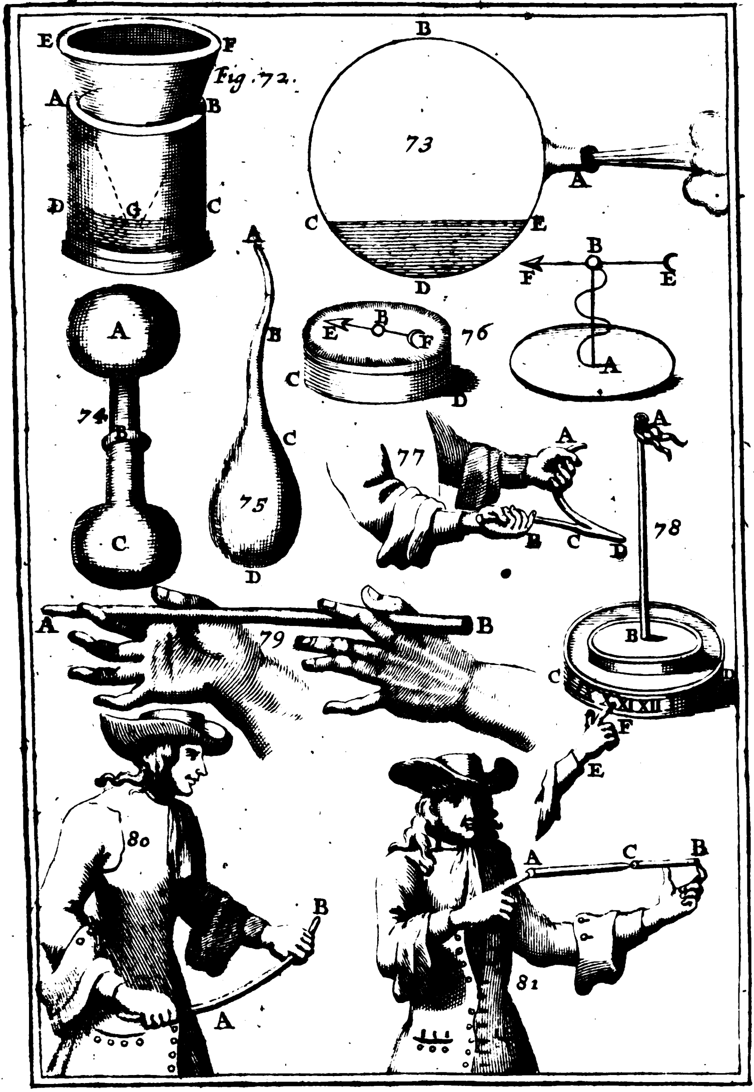


Fig. 72.
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PROBLEM XX.

To make a Vessel that will produce Wind.

THE Vessels that produce Wind are call'd *Æolipilæ*, being compos'd of Metal, such as Brass, in the form of a hollow Ball, as ABCDE, which at first is fill'd only with Air; and then being brought to the Fire, the Air is rarified, so that a considerable part of it gets out at the Aperture A which ought to be very small. This Aperture is so made, that, Water may by it enter the Æolypile, when the neck A is dip'd into cold Water, which will condensate the Air and give passage to the Water, and force it to enter to fill the *Vacuum*. Plate 24. Fig. 73.

Having thus fill'd part of the Æolipile with Water, as far, for example, as CE, set it upon hot burning Coals in a situation like that represented in the Figure; and the Water in the lower part CDE upon the approach of the Heat, will gradually rarefy, and by little and little rise up in Vapours, which fly into the space CBE, where there's nothing but Air; and then the Vapours and the Air pursuing one another, strive to get out in a Croud at the Aperture A; upon which occasion, those which are next the Aperture fly out with great Velocity, and produce such an Impetuous Wind and Whizzing, that 'twill cause a wind Instrument, such as a Flagelet, to sound if applied to the Aperture.

Remark. To render this Machine more agreeable, they commonly make it in the form of a Head, with the hole at the Mouth, which will continue to blow till all the Water is evaporated, which may hold long enough, for, as we intimated above, it evaporates but by little and little. If in stead of Common Water, you put into the Æolypile Spirit of Wine, and set fire to the Vapour that comes out, you'll see with Pleasure a continual Fire, which will last as long as the Vapour continues its violent egress.

Of the cause of Winds. This Wind having all the properties of the Winds that blow on the surface of the Earth, some Philosophers pretend to demonstrate from thence the Origin of Winds, by comparing the Cavities of Mountains to the Cavity of an Æolypile; the Water convey'd from the Sea to theſe Cavities by ſeveral Subterraneous Paſſages, to the Water contain'd in the Æolypile; the Heat in the Bowels of the Earth which reduces that Water into Vapor, to the heat that rarifies and dilates the Water in the Æolypile; and in fine the various chinks of the Earth, thro' which the Vapours riſe, to the hole of the Æolypile.

PROBLEM XXI.

To make Glaſs-Drops.

Plate 24. Fig. 75.

GLaſs-Drops are thick little pieces of Glaſs, made almoſt like a Drop, which have a long ſlender end, as ABCD, which being broken at its Extremity A, the Drop CD breaks preſently with a Crack, and flies into white Powder and little Fragments to two or three foot round.

Theſe Drops, which have excited the Curioſity, and perplex'd the Reaſon of moſt Philoſophers, are made by letting a little of the melted Matter of which the ordinary Glaſſes are made, fall into a Veſſel full of cold Water; for then this melted Matter which is very glutinous while 'tis red, makes a long String, by which they hold the Drop in the middle of the VVater, where it cools and hardens in a little time; after which they ſeparate the String which is out of the Water, ſo that the remaining part in the VVater do's not break, commonly call'd a Glaſs Drop. To this Drop there ſticks a ſmall end, part of which may be ſeparated, by making it red at the flame of a Candle, without breaking the Drop; nor will this Drop break if you lay it upon VVood, and with a Hammer ſtrike upon its thickeſt part D, for its External Parts are very hard, and ſupport one another like a Vault. And they only break, upon bending the ſlender end A till it breaks, by vertue of the Spring rais'd by that effort in all its parts, which ſhake and tremble like an extended String, put into Motion by forcing it to bend; whence it comes that theſe parts do in a little time return with very great velocity to their firſt Diſpoſition; and

and that the parts which are less united, and only contiguous, as it were, disunite and separate, and that occasions the Disunion and Separation of all the rest, and their flying all about with a Noise. See upon this Head Mr. *Mariotte's* Discourse of the Nature of the Air publish'd in 1679, in which he has in my Opinion wrote more pertinently of this Subject, than any one besides.

PROBLEM XXII.

To make new Wine keep its sweetness for several Years.

MR. *Lentin* informs us, that if you let New Wine heat by it self, it loses in a little time all its Sweetness, especially if the Casks are left open; but if you boil it upon a Fire immediately after the Grapes are pressed, most of the Volatile Principles of the Sweetness concentrate, and link themselves with the more fix'd parts of the VVine, which preserves its Sweetness for several Years.

A sweet and new VVine may preserve its Sweetness at least a whole Year, if you pitch the Cask well both within and on the outside, to hinder the Water to penetrate into it, and so spoil the VVine, which ought to be put into it before it boils; and keep the Cask well stop'd in a Cistern of VVater, so as to be cover'd all over for a Month or thirty Days; and then take out the Cask and place it in a Cellar. Remark.

In the year 1692, I had a Cask full of *Burgundy* VVine brought me in the Summer to *Paris* by VVater, which immediately upon its Arrival was clap'd into my Cellar; and after a few days standing, I found it boiling as if it had been quite New, and that it had reassumed its former Sweetness, which continued about a Month; and after that it prov'd extraordinary good VVine. Some tell you that a piece of Cheese or Pumice-stone thrown into the Cask, will break the violence of fermenting VVine.

VVhen the New VVine has lost its Sweetness, it may be recover'd by Casking it up immediately, and putting in the bottom of the Cask half a pound of To Recover the Sweetness of New Wine.

Mustard-Seed, less or more, according to the size of the Cask.

PROBLEM XXIII.

To know when there is Water in Wine, and to separate it from the Wine.

IF the VVine is neither sweet nor new, but fine and clear of its Lee, you may know (according to *Porta* and Father *Schott*) whether 'tis mix'd with VVater or not, by throwing into it Apples or Pears, for if the VVine is unmix'd they'll sink to the bottom, if 'tis mix'd they'll swim above, because the Specifick Gravity of VVater is greater than that of VVine.

Some order wild Apples or Pears, and if these can't be had, ripe Apples or Pears. Others make use of an Egg, and alledge, that when the VVine is pure the Egg falls swiftly to the bottom, but if 'tis mix'd with VVater, the Egg descends more slowly, the VVater having by vertue of its Gravity more force to bear up the Egg than the VVine has.

Now the contrary will happen, if the VVine be Sweet and New; that is, when such VVine is unmix'd, the Egg will descend slower than when 'tis mix'd; by reason that New VVine unmix'd is by vertue of its Lee heavier than VVater, and consequently becomes lighter by the addition of Water.

VVhen you have discovered that the VVine is mix'd, you may separate the VVater by a dry Bulrush, according to *Mizauld*; for the Rush being a Plant that grows and thrives in watry marshy Places, if it be dryed, and one end of it put into mix'd VVine, the VVater will insinuate it self into the Rush, and so the VVine will be left alone. By the same Reason, the Rush may serve to discover whether the Wine is mix'd with VVater or not.

Remark. On the other hand, some pretend you may separate the VVine from the VVater, by putting in a long narrow piece of Linnen, VVoollen, or Cotton Cloth, one end of which hangs out of the Vessel, as if the Wine being lighter would rise and flow out upon the Cloth while the VVater stays behind; but this and several

othe

other ways for the same purpose, are disproved by other Authors.

To pour Water into Wine without mixing.

You may pour Wine upon Water without mixing, if you put a tost of Bread upon the Water in a Glass, and while this tost swims above the VVater, pour in the Wine very softly; for then you'll see the VVater remain unmix'd at the bottom of the Glass without any alteration in its Colour.

To know if Milk is mix'd with Water.

Here by the bye I shall shew you a way of knowing when VVater is mix'd with Milk; put a little Stick into the Milk, then pull it out, and let a drop of the Milk fall from it upon the Nail of your Thumb; and if the Milk is pure, the drop being thick will stand for some time upon your Nail; but if 'tis diluted with VVater 'twill run off immediately.

To turn Water seemingly into Wine.

You may turn VVater into Wine in appearance, by setting a Vial full of Water in a Cask full of Wine, turning the Mouth of the Vial downwards; for then the Water will run out, and the Vial will be fill'd with Wine; which the Ignorant will take to be a turning of VVater into VVine.

PROBLEM XXIV.

Having two equal Bottles full of different Liquors, to make a mutual exchange of Liquor, without making use of any other Vessel.

I Suppose the two Bottles to be of equal Magnitude both in Neck and Belly, and the one to be full of VVine, and the other of Water. Clap the one that's full of VVater nimbly upon the other that's full of VVine, so that the two Necks shall fit one another exactly, as in the Figure, where the Bottle AB represents that which contains the VVater, and BC that which contains the Wine. In this case, the Water being heavier than the VVine will descend into the place of the VVine, and make the VVine ascend into its place; but in this case the Wine will be considerably alter'd, for 'twill have lost its Vapours and Fumes, and be uncapable to Intoxicate. (Plate 24. Fig. 74.)

Remark.

As the Wine can't Intoxicate, so it do's not drink Palatably, as having lost all its Strength. But if you

How to avoid being drunk with Wine.

want to prevent the intoxication of good Wine, *Wecker* and *Alexis* advise you, for this purpose, to take, before you begin to drink, an Ounce of the Syrup prepar'd of two Ounces of the Juice of Coleworts, two Ounces of the Juice of four Pomgranates, and an Ounce of Vinegar, all boil'd together for some time.

VVe are inform'd by the same *Alexis*, that, to prevent Drunkenness, you should break your fast with six or seven bitter Almonds, or with the Juice of Peach Leaves, or else with four or five Sprouts of the Leaves of raw Coleworts. We are told that when the *Egyptians* prepar'd for a Drinking Match, they eat Coleworts boil'd in VVater, before any thing else.

PROBLEM XXV.

To make a Metallick Body swim above Water.

THO' the Specifick Gravity of VVater is inferior to that of Metals, and consequently VVater is uncapable, absolutely speaking, to bear up a Metallick Body, such as a Ball of Lead; yet this Ball may be flatted and beat out to a very thin Plate, which when very dry and put softly upon still Water, will swim upon it without sinking, by vertue of its dryness. Thus we see a Steel Needle will swim upon VVater, when 'tis dry and laid softly lengthways upon the surface of still VVater.

But if you would have a Metallick Body to swim necessarily upon VVater, you must reduce it to a very thin Plate, and that Concave like a Kettle, in which case the Air it contains weighs less than the VVater whose room it possesses. 'Tis by this Contrivance that Copper Boats or Pontons are made for passing whole Armies over Rivers without any Danger.

Remark.

If you put this Concave Metallick Vessel upon the VVater with its Mouth perpendicularly down, 'twill still swim, by reason that the Air contain'd in its Cavity finds no exit; insomuch that if you push it under VVater and hold it there by force, the detain'd Air will keep the bottom from being wet on the inside. And by the same Reason, you may have a burning Coal in the bottom, and find it not extinguish'd when

you

you take it out of the VVater, provided you do not hold it long under Water, for Fire stands in need of Air to keep it in.

PROBLEM XXVI.

To make Aquafortis *put up close in a Bottle boil without Fire.*

PUT a small quantity of *Aquafortis*, and of the Filings of Brass in a Bottle, and you'll see so great an Ebullition, that the Bottle will appear quite full, and be so hot that you cannot touch it without burning your self.

Remark. In like manner if you mix Oil of Tartar and Oil of Vitriol together, you'll presently see a very great Ebullition with a sensible Heat, tho neither of these Liquors is compos'd of any Combustible Matter.

Of Aquafortis and Aqua Regia. *Aquafortis* is so call'd with respect to its Strength in dissolving almost all Metals and Minerals. 'Tis commonly a Distillation from Saltpetre and Vitriol or Green Copperas; and 'tis yet better, if it be a Distillation from Saltpetre and Roach Allum. It dissolves all Metals, but Gold; but is render'd capable of the dissolution of Gold by dissolving Sal Armoniack or Sea-Salt in it, after which it assumes the name of *Aqua Regia*.

Of Sal Armoniack. Roch Allum. To avoid all obscurity of Terms; I shall here acquaint you by the bye, that *Sal Armoniack* is a Composition of Bay-Salt, Chimney-Soot, and the Urine of Animals: That Roch Allum is a mineral earthy sharp Salt fill'd with an acid Spirit, which is oftentimes found condensated in the Veins of the Earth, or is taken from Aluminous Springs by Evaportion; or is found among Mineral Stones, and disengaged from them by dissolution in Water and Evaporation: And in fine, That Saltpetre is a Salt that's partly Sulphureous and Volatil, and partly Terrestrial; it is found in the dark Cavernous places of the Earth, and likewise in Stables, by reason of the great quantity of Volatil Salt in the Urine and Excrements of Animals, which joyns in with the Salt of the Earth by the continual action of the Air. (Saltpetre.)

The

Vitriol. The Oil of Vitriol (mention'd above) is a Caustick Oil distill'd by a strong Reverberating Fire from Vitriol. Now, Vitriol is a Mineral Salt, approaching to the nature of Roch-Allum, which is found crystallis'd in the Earth of such Mines as abound in Metals, which gives us to know that it contains in it some Metallick Substance, and particularly Iron or Copper. When 'tis loaded with Copper, if you rub it against Iron, 'twill stain it with a Copper colour. But 'tis best for all manner of Preparations when it partakes most of Iron.

Tartar. The Oil of Tartar (mention'd above) is distill'd from Tartar along with the Spirit, from which 'tis separated by a Funnel lin'd with brown Paper. Tartar it self is an Earthy incorruptible Substance, form'd like a reddish Crust round the inside of Wine-Casks, which thickens and congeals to the hardness of a Stone, and is separated from the pure parts of the Wine, by the action of the Fermentative Spirit.

PROBLEM XXVII.

To make the Fulminating or Thundring Powder.

TAke three parts of Saltpetre, two parts of Salt of Tartar, and one part of Sulphur, pounded and mix'd together; heat in a Spoon 60 Grains of this Composition, and 'twill fly away with a fearful noise like Thunder, and as loud as a Cannon, breaking thro' the Spoon and every thing underneath it, for it exerts it self downwards, contrary to the nature of Gunpowder which exerts it self upwards.

Salt of Tartar. The Salt of Tartar here used, is only a Solution in Water of the black Substance that remains after the Distillation of the Oil of Tartar, and an Evaporation of that Solution to a dry Salt, which must be kept very close, least the moisture of the Air should melt it.

PROBLEM XXVIII.

To make the Aurum Fulminans *or Thundering Gold.*

PUT into a Matrass upon hot Sand the filings of fine Gold, with a triple quantity of *Aqua Regia*, which will dissolve the Gold: Mix this Solution with a sextuple quantity of Spring Water, and then pour upon it drop by drop the Oil of Tartar or Volatil Spirit of Sal Armoniack, till the Ebullition ceases, and the Corrosion of the *Aqua Regia* is over; for then the Powder will percipitate to the bottom, which may be dulcified with warm Water, and dried with a very slow Fire.

This Powder is much stronger than that last described; for if you set fire to 20 Grains of it, 'twill act with more Violence and have a louder Crack, than half a pound of Gunpowder, and two Grains of it kindled at a Candle have a stronger report than a Musket Shot.

PROBLEM XXIX.

To make the Sympathetick Powder.

THE Sympathetick Powder is nothing else but the *Roman* Vitriol calcin'd and reduced to a white light Powder, which is said to cure Wounds at a Distance, by being put upon a Linnen Cloth dip'd in the wounded Person's Blood, or upon a Sword, whereon is the Blood or *Pus* that comes out of the Wound. This Cloth or Sword is wrap'd up in a white Linnen Cloth, which is open'd every Day, in order to strew some fresh Powder upon the Blood or *Pus* of the Wound. This course they continue till the Wound is perfectly Cured, which happens the sooner, if the Cloth upon which is the Blood and the Powder, is kept in a place that's neither too hot, nor too cold, nor too moist. Nay, 'tis necessary sometimes to shift the Cloth from place to place, according to the different dispositions of the Wound, by putting it for example,

ample, in a cold place, when the Patient finds an excessive heat in the Wound.

To calcine the Vitriol for the Sympathetick Powder, take some Roman Vitriol, when the Sun is in the Sigh of *Leo*, or in the Month of *July*, dissolve it in Rain-Water, and filtrate the Water thro' sinking Paper. Then let the Water evaporate upon a gentle Fire, and you'll find at the bottom the Vitriol in little hard Stones of a fire green Colour. Spread these Stones carefully, and expose 'em to the Rays of the Sun, stirring them often (with a Wooden Spatula; not an Iron Spatula, because the Spirits of the Vitriol are ready to joyn in with Iron, which would rob the Sympathetick Powder of its Volatil Spirits, in which all its Vertue consists) that the Stones may be the better penetrated by the Sun, and calcined and reduc'd to a Powder, which will be as white as Snow. And to render the Substance of the Vitriol more pure and homogeneous, the Dissolution, Filtration, Coagulation and Calcination ought to be repeated three times.

This wonderful Powder must be carefully kept in a Vial close stopt, and in a dry place, for the least moisture of the Air may turn it to Vitriol again, and so make it lose its Sympathetick Vertue.

We are told that this Powder stops all Bleedings, and mitigates very much all sorts of Pains in any part of the Body, particularly the Toothach; and that, by Application, not to the part affected, but to the Blood taken from it, and cover'd up in a Linen Cloth, as above.

Remark. Colcothar of Vitriol.

The Chymists have another Calcination of Vitriol call'd Colcothar, which being put into the Nose stops a Bleeding at Nose, and provokes to Sneeze; being of soveraign use for rouzing the Senses, wherefore 'tis given in Lethargies. 'Tis also successfully us'd for drying up Wounds and Ulcers. This Colcothar is only the Vitriol kept melted upon a Fire till all its Humidity is evaporated, and 'tis reduc'd to a hard reddish brown Mass, whereby 'tis render'd fit for the cure of the foresaid Maladies, and many others not here to be mention'd.

PRO-

PROBLEM XXX.

Of the Magnetical Cure of Diseases by Transplantation.

THE Magnetick Cure by Transplantation, is, that which is performed by communicating the Disease to some Beast, Tree, or Herb; and, as some will have it, is founded upon the efflux of the Morbifick Particles, which pass by insensible Transpiration out of the Body of the Patient into another Animal or Plant.

Froman informs us, that a young Student got rid of a Malignant Fever by giving it to a Dog that lay in the Bed with him, and died of it; which if true, must needs proceed from the insensible Transpiration of the subtile Matter, that thereupon entred the pores of the Dog.

Thomas Bartholin says, his Uncle was cur'd of a violent Cholick by applying a Dog to his Belly, which was thereupon seiz'd with it; and that his Maid-Servant was cur'd of the Toothach by clapping the same Dog to her Cheek, and when the Dog was gone from her, he howl'd and made such Motions, as gave 'em to know he had got the Maid's Toothach.

Hoffman speaks of a Man cur'd of the Gout by a Dog lying in the Bed with him, who thereupon was seiz'd with it. And frequently after the Dog had fits of the Gout, as his Master had used to have before. However this be, certain 'tis, that Dogs are often subject to the Gout, without any infection from Men; and this and the other Stories of Transplantation are not here offer'd for Conclusive Proofs, but by way of Recreation.

Mousieur *de Vallemont*, who seems inclinable to believe Transplantation of Diseases, says, 'tis done not only by insensible Transpiration, but likewise by Sweat, by Urine, by the Blood, by the Hair, or by taking up what falls from the Skin, upon a strong Friction. For this he brings several Instances, and particularly that which follows.

A

A Person of Quality in *England* used to cure the Jaundice at a great distance from the Patient, by mixing the Ashes of Ash-wood with the Patient's Urine; and making of that Composition three, or seven, or nine little Balls, with a hole in each of 'em, in which he put a Leaf of Saffron, and then fill'd it up with the same Urine. This done he hanged these Balls in a private place where no Body could touch them; and from that time the Disease began to abate.

Remark. The great Vertues of the Ash Tree.

The Ash, which is a common Tree all over *Europe*, has merited the Appellation of the *Vulnerary Wood*, by reason of its peculiar Property in curing several Diseases, and above all Wounds and Ulcers. Not to mention the almost incredible Vertues ascrib'd to it, 'tis said to stop Bleeding at Nose, if the Face be but rubb'd with the Wood, and then wash'd with fair Water, and if the Patient holds in the hand of that side where the Bleeding is, a piece of the Wood till it heats his Hand.

PROBLEM XXXI.

To stop a Bleeding at Nose, or at any other part of the Body.

FAther *Schott* the Jesuit says, that to stop a Bleeding at the Nose, you need only to hold to the Nose the Dung of an Ass very hot, wrap'd up in an Handkerchief, upon the plea that the Smell will presently stop it. *Wecher* did the same with Hogs Dung very hot done up in fine Taffeta, and put into the Nose.

I have several times experienced, that a piece of red Coral held in the Mouth, will stop a Bleeding at the Nose. Some tell you that the Constriction of the Thumb of the side of the Nostril that bleeds, will do the business.

To stop the bleeding of a Wound, take a Linnen Cloth in the Spring when the Frogs lay their Eggs in the Water, and wash it in that Water till it is well impregnated with the Frogs Eggs; then dry it at the Sun; and after repeating this Impregnation and Deficcation three or four times, keep the Cloth to be applied to the Wound twice in the form of a Cataplasm,

plasm. We are told the second Application will do.

PROBLEM XXXII.

To prepare an Ointment that will cure a Wound at a Distance.

THE Ointment mention'd by *Paracelsus* is prepar'd thus, according to *Goclenius*. Take of the Usnea or Moss of the Scull of a Man that was hang'd, two Ounces; Mummy, Human Blood, of each half an Ounce; Earth-worms wash'd in Water or Wine, and dried, two Ounces and a half; Human Fat, two Ounces; the fat of a wild Boar, and the fat of a Bear, of each half an Ounce; Oil of Linseed and Oil of Turpentine, of each two Drams.

John Baptist Porta prescribes it a little otherwise by throwing in some Bole Armeniack, and leaving out the Earthworms, and the Bears and Boars fat. But let the Composition be which it will, it must be well mix'd and beat in a Mortar, and kept in a long narrow Vial. Some say, it should be made when the Sun is in *Libra*. The way of using it is this.

Put into the Ointment the Weapon or Instrument that gave the Wound, and leave it there; then let the Patient wash his Wound every Morning with his own Urine, and apply nothing else to it; after 'tis well wash'd and cleansed, let him tie it up tight with a clean white Linen Cloth, and he'll find 'twill heal without any Pain.

Monsieur *Vallemont* says, if you can't get the Instrument with which the Wound was given, you may take another, which if gently convey'd into the Wound, and impregnated with the Blood and Animal Spirits residing there, will have the same effect. He adds, that if you want a speedy Cure, you must anoint the Instrument often, otherwise you may let it lie a day or two without touching it.

The effect of this Unguent he imputes to the subtle Particles, which are these little Agents that disengage themselves from the most spirituous and transpirable Ingredients of which this Unguent is compos'd.

To

To add to the Credibility of its Operation, he quotes Father *Lana*, who observ'd that when the Vines in *France* were in Flower, the Wines in *Germany*, tho' at a great Distance, suffer an Effervescence; which he explain'd by the effluvium's of the Subtile Matter, making these to reach as far as the Stars, and alledging that if the Atoms, which transpire from the Terrestrial Globe, were not carried to the Stars, and sent back from the Stars to the Earth by a perpetual Flux and Reflux, there would be no Physical Commerce between the Heavens and the Earth.

PROBLEM XXXIII.

When an Object appears confusedly by being too near the Eye, to gain a distinct view of it, without changing the place either of the Eye or the Object.

TAke a Leaf of Paper, or a very thin Card, make a hole in it with a Pin, as we use to do in viewing an Eclipse of the Sun to hinder the too great numerousness of the Rays from offending the Eyes; and the Object tho' so near your Eye will appear very distinctly; for then the Eye receives a lesser quantity of Rays from each Point of the Object, and so each point of the Object depicts its Representation in the bottom of the Eye only in a narrow Compass, and thus it is that two Images coming from two adjacent Points are not confused.

PROBLEM XXXIV.

Of the Origin of Springs and Rivers.

'TIS a hard matter to do Justice to this Subject, in the way of Demonstration; however I shall give you the divers Sentiments of Authors about it.

Aristotle attributes the Origin of Springs to the Vapours of the Earth, which mounting upwards, are stop'd in the Caverns of Rocks and Mountains form'd as it were into a Vault, where sticking to the Top, as in the Head of an Alembick, they are increas'd by the access

access of others till they're reduc'd to little drops of Water, as upon the lid of a Pot in which Water is boiling, and falling thence run down forcing their Passage.

Those who reject this Opinion, say, 'tis not probable that the Earth could contain so many Vapours, as to furnish Water for so great a number of Springs and vast Rivers. But to this, one may reply, that the Springs and Rivers are kept up and increased by the Rain and melted Snow, which penetrating into the Pores of the Earth, and Clefts of Rocks, gather into a sort of Cisterns or Heads, from whence they afterwards repair by Subterraneous Passages to the surface of the Earth, and there spread themselves.

Some may object with Father *Kircher*, that some Mountains have Springs and yet no Rain; as Mount *Gilboa* according to the sacred Text, and others both in and without the Torrid Zone. But I answer, that when the Ground hath not Vapours enough to produce Springs, they may come from afar by Subterraneous Passages to the highest Places, such being the nature of Water, that 'twill rise a'most as high as it descends.

I can't joyn with those who ascribe the Origin of Springs to the Waters of the Sea, conveyed by hidden Veins to the bosom of the Mountains, and to all the parts where we find Sources: For as 'tis the nature of Water, and of all liquid Bodies to descend and repair to the lowest Stations, so the Sea in which most Rivers disembogue must be the lower Station, and consequently the reascension of Water upon the Earth and the Mountains, would be contrary to the nature of heavy Bodies.

I believe indeed, there are several accidental Causes, that may make it rise, such as the Flux and Reflux of the Sea; but I do not think that can do much, or force it to the top of the highest Mountains. Father *Casati* imagines a Central fire in the Earth, which boils the Sea-water in its Abysses, and so forms it into Vapours; but that I think is useless, it being highly probable, that the Sun has force enough without it to attract Vapours.

'Tis offer'd by some Philosophers in Vindication of the Opinion ascribing the Origin of Springs to the

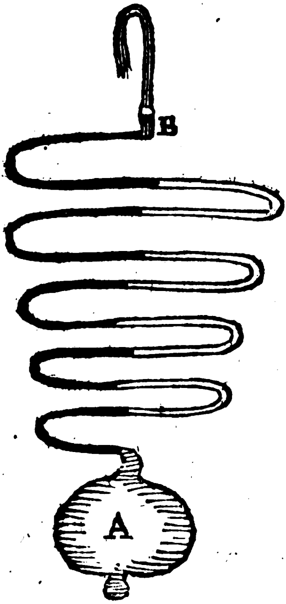
Sea, That if the Sea did not furnish Water to all the Springs, the greatest part of which are never dry, the Rivers which are a Collection of the Waters of Springs, would swell the Sea beyond its limits, which is contrary to Experience. But to this I answer, that the Water of all the Rivers is inconsiderable in respect of the wide Sea, that covers more than half the Surface of the Earth: Besides that the Water which runs upon the Earth, is in part imbib'd by the Earth, and continually reduced to Vapours; so that the Remainder of Water that flows into the Sea, supplies in a manner the place of the Vapours that ascend from it.

Thus you see that several Causes contribute to the Origin of Springs and Rivers; the Principal of which seems to be the quantity of Vapours so powerfully attracted by the Sun, not only from the Waters that run in open Channels upon the Surface of the Earth, but likewise from those that lie conceal'd in the Bosom of the Mountains, and the Bowels of the Earth.

Remark. Those who attribute the Origin of Springs on the tops of Mountains to Subterranean Fires, may alledge in Vindication of their Opinion, the following Experiment; by which we see, that, the Dilatation caus'd by the heat, makes a Liquor spout out of a Tube of Glass in such a manner, that it will produce an agreeable and curious Fountain.

Take

Take a Tube of Glaſs that's ſomewhat ſlender, and urn'd with windings as this in the Cut; at the lower nd of which there's a Glaſs Bottle A, into which you nay convey Water or any other Liquor by the other Extremity B, by heating the Air contain'd in the Tube, ſo as to make as much go out as is poſſible,



nd dipping the other Extremity B in the Liquor, which vill effectually enter the Tube as the Air within conenſates and takes up leſſer room. Then heat the ottle A, ſo that the Rarefaction may be greater than was before, and you'll ſee the Water aſcend and out like a Fountain out at the upper end B.

PROBLEM XXXV.

To know in what parts of the Earth, Sources of Water lie.

'TIS necessary for the conveniency of Life to have good Water, and consequently we can't be too diligent in learning to find out the places where the Sources of Waters are, in order to dig Wells or Pits for the Accommodation of Mankind, I shall therefore imploy this Chapter in laying before you the best Methods used by the Ancients and the Moderns, for discovering the Veins of Water that lie hidden in the Earth.

Pliny says, that to know if there be a Vein of Water under the Ground, you must have a particular Eye upon the places where you find moist Vapours and Exhalations; and in making this Observation, says *Palladius*, you must take care that the place where the Vapours rise be not moist in the Surface; for if 'tis not, you safely attribute the humid Vapours to Subterranean Sources of Water. This Experiment you had best make in *August*, when the Pores of the Earth are open, and give a freer passage to the Vapours.

But to make this Observation with all the certainty and facility that's possible, Father *Kircher* (in imitation of *Vitruvius*) advises to lie down with your Belly to the Ground, a little before Sunrise, and to bear upon your Chin with your Hand resting upon the Ground, that so your Sight may extend to the level of the Country, and the Eye being rais'd only to a just Height, may view the surface of the Ground by Visual Rays that graze upon the Horizon, and easily discern the places from whence moist, waving and trembling Vapours do arise; for in these places you'll infallibly find Veins of Water, there being no such Vapours observ'd upon the Grounds that are destitute of Water.

Vitruvius, and after him *Dechales*, acquaints us, that places which have Veins of Water conceal'd in the Bowels of the Earth, are distinguish'd by the sponta-

neous

eous growth of Rushes, Willows, Alder-trees, Rose-ushes, Ivy, and such other Aquatick Plants, that are ot planted there by Art, but come naturally. Ano-er sign, is the Frogs when they begin to brood, hich press down the Earth so much as to draw up the umidity; which doubtless proceeds from the Vapours at continually arise from the Veins of Water hid un-r those parts, and which reveals as it were what Na-re affected to keep secret.

Another Contrivance for the discovery of Water, commended by *Vitruvius*, and used by the Ancients, this. Dig a Ditch three Foot broad, and five Foot ep, where you suspect there may be Water; at Sun-place in the bottom of the Ditch a Brass or Lead essel or Basin, inverted or turn'd with its Cavity wnwards, and rub'd with Oil on the inside; cover is Vessel and the whole Ditch with Reeds and Leaves, d afterwards with Earth: And the next day if you d drops of Water hanging upon the inside of the essel, 'tis a sign of Water.

Instead of a Vessel or Basin of Metal, you may put the Ditch an Earthen Vessel not bak'd, without bbing it with Oil, or covering it with Reeds, Leaves Earth; and next Morning if you find it soft with oisture, you may conclude there's Water underneath: nd if instead of this Earthen Vessel, you put in ool, and next Morning you can express Water from e Wool, you may conclude there's a great deal of ater underneath.

Father *Kircher* shews us an admirable way of find-d out Water, having by his own Experience found e happy success of it. He orders it to be tried in e Morning when the Vapours are plentiful, and not t wasted by the heat of the Sun. He takes a small ck of two pieces of Wood joyn'd together, on the tremities being Alder or some such Wood that rea-y imbibes the Moisture; and having hung this Rod Needle (not unlike the Needle of a Compass) by Center of Gravity upon a Pivot, so as to make hang in *Æquilibrio*, he carries it thus hung, or else pended with a Thread, to the place where he sus-ts Water; and if there be any there, the Rod will put from its *Æquilibrium* by the Vapours penetra-g the Alder extremity, and making it incline to the Ground.

Ground. This he calls his *Baguette Divinatoire*, or *Divining Rod*.

Of the Divining or Conjuring Rod, call'd *Baguette Divinatoire*.

But now adays, we understand by a *Baguette Divinatoire*, a small forked Branch of light Wood, commonly of Hazelwood, which several have made use of to very good purpose in discovering not only the Sources of Water, but likewise the most noble Metals, which are now the bond of Society; and, as 'tis said, even Robbers and Murderers, of which we had a notable Instance in 1693, in one *James Aymar* of *Dauphiny*, who pursued a Murderer 45 Leagues, and found him out by this Rod; and when he came to *Paris*, he gave several Proofs of his Dexterity in making use of the Rod, by the discovery of Water, Metals, and hidden Treasures.

Plate 24. Fig. 77.

He takes a forked Branch of any sort of Wood, such as ABCD, and holds the two Prongs with his two Hands, but do's not grasp 'em hard. He holds them so, that the back of his Hands are turn'd to the Ground, the Point CD goes foremost, and the Rod or Stick is a'most parallel to the Horizon. In this fashion he walks softly along, and when he passes any place where there's Water, or Mines, or Silver hid, the Rod turns in his Hands and bends downwards; and the same thing happens in holding it over stolen Goods, and following the track of Robbers and Criminals, whom he easily distinguishes from the Innocent, for when he puts his foot upon one of theirs, the Rod turns towards the Criminal. Sometimes he makes use of a straight Stick, and holds it upon his fingers with his two Hands at some distance, as you see in Plate 24. Fig. 79.

Plate 24. Fig. 79.

As all Persons are not of the same Temperament, so this Divining Rod do's not succeed equally with all, for a great many have used it without Success, as being destitute of that gift of Nature. *Kircher*, and *Schott* and *Dechales*, do all speak of it as a thing frequently experienced; tho' every one is not capable of making the Experiment; and the last of the three says 'tis absolutely the easiest and most certain means yet tried for the discovery of Water.

Plate 24. Fig. 80.

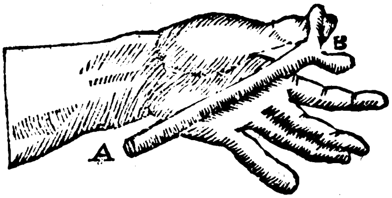
Some take a long straight and smooth shoot of Hazel, or any other Wood, such as AB, and hold it by the two ends bending a little Archwise, and keep it

parall-

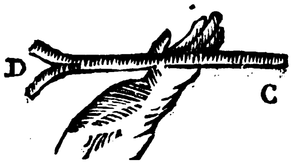
parallel to the Horizon that it may turn more readily to the Ground, when it passes over a Source of Water.

Father *Kircher* has seen the *Germans* practise this piece of Divination another way. He says, they cut a small Hazel Stick, such as AC, CB, into two almost equal Parts; making the end of the one hollow, and cutting the other to a Point, and so inchasing the one in the other. The Stick or Shoot thus used must be very straight, and without Knots. They carry it before them between the tops of the fore-fingers of each Hand, as you see in the Figure; and when they pass over Veins of Water or of Metal, the Shoot moves and bends. Plate 24. Fig. 81.

Some make use (as we are told) of a forked Rod a Foot long, holding it upon the extended palm of



their Hand, as AB. Others lay it in *Æquilibrio* upon the back of their Hand, as CD, that it may



move with more facility when they pass over a Spring of Water.

Tho' the Modern Authors abovemention'd take this Divining Rod to be a new thing, yet 'tis certain the Ancients spoke of it, and gave it different Names. *Neubusius* call'd it *Virga Divina*, and *Varro* seems to have meant some such thing, by entituling one of his Satyrs *Virgula Divina*. *Peter Belon* call'd it *Caduceus*; *Willenus*, *Virga Mercurii*, and Agricola *the inchanted* Remark.

 Rod;

Rod; some have call'd it *Aaron's Rod*, others *Jacob's Stick*, others *Moses's Rod*, with which he brought Water out of the Rock; and *Cicero* in his *Offices* speaks to his Son of a *Divine Rod.*

Some say this *Divining Rod* turns likewise to a Loadstone; others that it turns to the Bones of dead Corps, and has been used with Success, in distinguishing the Bones of Canoniz'd Saints from those of others.

Several other things are said of it, which I shall not here mention, because they seem incredible. I leave every one to their own Experience; for my own part I ne'er try'd it, and so can neither refute, nor vouch for the truth of what is said of it.

PROBLEM XXXVI.

To distinguish those parts of the Earth, in which are Mines or hidden Treasures.

Fumes and Exhalations one Sign.

Without insisting further upon the Vertues of the *Divining Rod* in turning upon Metals and Treasures; we shall observe in the first place, that the Mountains which contain Mines, do generally fill the Air with Fumes and Exhalations, such as the Workmen meet with in Mines, who find 'em almost always very Malignant. *Pliny* says, there rises a Vapour from the Silver Mines, that's unsufferable to all Animals, and especially to Dogs.

These Vapours and Exhalations, which contribute to the Generation of Metals and Minerals, are caus'd, without doubt, not by the heat of the Sun, which, in my Opinion, can't penetrate so far (there being some found 500 Cubits deep) but by the heat of the Subterranean Fires, of the existence of which we have no room to doubt, since we see Mountains and other places of the Earth vomit up Flames and Ashes. To convince us that these Vapours proceed from Subterranean Fires, rather than from the heat of the Sun, we need only to consult those who work in the Mines, who assure us, that the deeper they penetrate into the Earth, the more sensibly they feel the heat that issues from its Bowels, and to all appearance is the effect of Subterranean Fires; insomuch that they can't work but

but stark naked at the bottom of the Mine-Pits. They tell you, that sometimes there rise such Mineral Vapours as put out their Lamps, and would stifle themselves if they did not speedily retire. To remedy this Inconveniency, they have long Pipes which suck the Malignant Air from the bottom of the Mines, and so give place for that which is purer and wholsomer. *Agricola*, in his Book *de re Metallica*, describes several other Contrivances for the same purpose, which we leave the Curious to Consult.

Besides this Heat that's observ'd at all times in the Abysses of the Earth, we have intimation of the Subterranean Fires from the hot Springs, and the boiling Springs, such as that at *Grenoble*, which from time to time throws out Flames, especially when it Rains, or is about to Rain; as well as from the burning Mountains, such as Mount *Ætna* in *Sicily*, Mount *Vesuvius* in *Campania*, Mount *Hecla* in *Yslandia*, that in *Guatimala* in *America*, and others in *Perou*, in the *Molluca* Islands, and in the *Philippine* Islands. And these Subterranean Fires I take to be the cause of the thick Vapours or Smoak that I have oftentimes seen rise in the Winter time from the Caverns of the *Alps*; and which are sometimes seen by Mariners, as rising from the bottom of the Sea, and presaging the speedy rise of Winds and Storms.

Barrenness of the Earth another.

As Fumes and Exhalations are one sign by which the Mineral Philosophers distinguish the places that are stored with Mines; so another distinguishing sign is the Barrenness of some Places, which produce neither Trees nor Plants; for doubtless that proceeds from the dry and hot Vapours or Fumes, which scorch and dry up the Roots of the Plants and Trees, and so kill 'em. For the same end, we take notice of the places upon which Snow do's not lie long, or where we observe no Hoar-frost, for the heat of the Subterranean Vapours arising from the Mines melts the Snow in a little time, and keeps off the Frost.

Also no Snow nor Hoar-frost.

'Tis well known that *Hungary* abounds with Gold and Silver Mines, as well as those of Iron and Steel; and that the Gold Mines throw out very thick or gross Vapours, which are sometimes so Malignant, that in a little time they suffocate the Workmen. Now, those who have travel'd into *Hungary* on purpose

Several other Marks.

poſe to ſee the Golden Mines, inform us, that the Leaves of the Trees in thoſe parts are oftentimes cover'd with a Gold colour, owing to the Exhalations. *Alexander ab Alexandro* ſays, that in *Germany* they have found over the Gold Mines the Vine-leaves all over gilded, and ſome even pure Gold, which may ariſe from the inſinuation of the Metallick humour into the Root of the Vine, which being very Porous, may have drawn up in the Intervals of its Fibres ſuitable Nouriſhment. Thus we've known by Experience, that Metals Vegetate, and ſometimes have riſen up in Trees, with Trunks, Roots, and Branches.

'Tis ſaid, that if you carry a lighted Candle of Human Greaſe to a place where Treaſure is hid, 'twill diſcover the Treaſure by its continual noiſe, and by going out when it comes very near it: And Father *Tylkowski* a *Poliſh* Jeſuit, aſſures us, that, when Vapours are ſeen to riſe upon a Mountain at Sun-riſe, when the Air is clear and ſerene in *April* or *May*, 'tis a ſign that the Mountain contains a Quick-ſilver Mine.

With Reference to what I mention'd but now, the ſpeedy melting of the Snow, and there being no Hoar-froſt upon the places that cover Mines; I call to mind, what *Vallemont* ſays in his Occult Philoſophy, that the Soldiers when they go into Winter Quarters, are not ignorant of that Sign; for they obſerve narrowly in the Garden or Orchard, ſuch places as bear no Snow nor Hoar-froſt, in order to ſee if the Landlord has not hid ſome Treaſure there; for they conclude that the Earth of ſuch parts has been lately ſtir'd or dig'd up, and ſo being more Porous, gives a freer paſſage to the Exhalations, which crouding thither melt the Snow and the Hoar-froſt.

As pieces of Oar found. The ſaid Author Monſieur *Vallemont* has ſeveral other marks of Mines in the Bowels of the Earth. One is the finding of pieces of Oar or Metal upon the Ground; by which means the rich Mine of *Kuttemberg* in *Bohemia* was diſcover'd by a Monk, who obſerving by chance, as he walk'd in a Wood, a ſmall Stalk (as 'twere) of Silver ſhooting out of the Earth, very gravely left his Habit upon the Spot, that he might know it again, and ſo run back to acquaint the Convent.

Another

Another sign of Mines, which is reckon'd pretty sure, is, if towards the end of the Spring the Plants and Trees round a place have but little Vigour, and their Leaves are speckled with different Spots, their Green being not very bright. *Plants speckled, and not vigorous.*

Again, When the Foot of a Fountain points to the North, and its Head to the South, it oftentimes has Silver Mines. which usualy run from East to West.

A fourth sign given in by Mr. *de Vallemont*, is taken from the Colour of the Earth, and the Stones. If the Earth be Green, 'tis the sign of a Copper Mine; if Black, it promises Gold and Silver; if Gray, we expect from it Iron and Lead. *The Colour of the Earth.*

His fifth sign is the Barrenness of the Earth mention'd above; upon which Head he adds, that perhaps *Job* alluded to it, when he said, that no Fowl knoweth the Ground where Precious Stones are, and the Vultur's Eye hath not seen it, *Job* 28.

Again, if the Stones or Earth of any Place are heavier than usually, it gives us ground to suspect that Metals are there. *Weight of the Earth.*

In a seventh Place, we must mind the Springs that flow by the foot of Mountains; for not only their Colour and Smell serve to inform us, but even the Channel of such Water do's always bear some Grains, and other Vestiges of Metals. *Agricola* says, the Inhabitants of *Navarre* took out of the bottom of their Wells a sort of Earth loaded with Gold, which gave 'em to think, that there were Rich Gold Mines in that part of *France*. *Agricola de re Metallica*, lib. 2.

Mr. *De Vallemont*, informs us further, that some few Plants which bear a Sympathy and Analogy to Metals, grow commonly over Mines, and such are Juniper, Tree-Ivy, the Fig-tree, Wild Pine-trees, and most of the Plants that are pointed and prickly. *Sympathy between Plants and Metals.*

The last sign he mentions, is, the Exhalations of Vapours round the top of a Mountain.

'Tis a certain Truth that we do not always light on the secrets of Nature, when we hunt for 'em; Chance has the greatest hand in most Discoveries, particularly those of Mines; thus, Mines have been discovered by the Wind blowing up Trees by the Roots, by a Horse's foot striking against the Ground, by Hogs grubbing up the Earth; and *Diodorus Siculus* says, *Remark.*

the

the Mines in *Spain* were discover'd by a Forest taking fire Accidentally; and *Athenæus* says, very rich Silver Mines were discover'd in *Gaul*, by an accidental fire in the Woods, which melted the Silver so as to make it run in Brooks. Near *Fribourg* in upper *Saxony*, Silver Mines were discover'd by great Rains washing away the Earth that cover'd the Minerals; and the like Discovery has been made elsewhere by the fall of Snow, by Thunderbolts and Earthquakes, tearing Rocks from the tops of Mountains. 'Tis said, than in the Country call'd *la Brie* in *France*, a Gold Mine was lately discover'd in tilling the Ground, which the *French* King has order'd to be inclosed; and *Justin* says the like of *Gallicia*, but the Gold of that Country is not now much minded, by reason of its being blended with other Metals, that are hard to be separated and refined.

PROBLEM XXXVII.

To measure at all times the dryness and humidity of the Air.

As the *Thermometer* spoken of in *Probl. 6.* of the *Mechanicks*, measures the Degrees of Cold and Heat, and the *Barometer* those of the weight of the Air; so we make use of a Machine call'd an *Hygrometer* or *Hygroscope* to measure the dryness or humidity of the Air; for certain it is, that the Air is more or less moist, as 'tis more or less stock'd with Vapours. Now, since Fir-wood is extreamly susceptible of dryness or moisture, it seems to be the most proper for a *Hygrometer*, or for discovering the least change in the Air, as to dryness and moisture.

The first Hygrometer.

The first *Hygrometer* we shall here mention, was invented in *England*, and is compos'd of two very thin boards of Fir, in the middle of one of which is a Needle like the Hand of a Watch, made fast to the Centre of a Circle divided into several equal Parts, which represents the Degrees of the Moisture or Dryness of the Air, pointed to by the Needle as it moves round its Center by vertue of the two Fir-Planks, which move

move in two Grooves, according as they swell or shrink thro' the moisture or dryness of the Air.

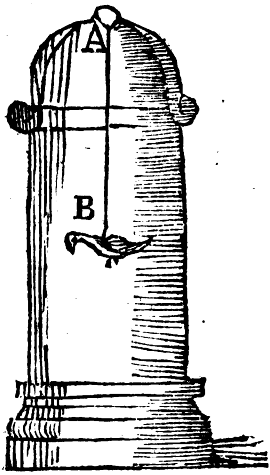
Another *Hygrometer* made in *England*, and more esteem'd than the former, is this. They take the Beard of a green Ear of Barley, and twist it round a Pin such as AB, rais'd perpendicularly upon the bottom of a round Box, like that of a Compass, as CD, the upper Circumference of which is divided into equal Parts, commonly 60. This Pivot or Pin AB, is as high as the Box CD, to the end that the light Needle EF which they clap upon the Point B, where the Beard terminates in a Hole made in the middle of the Needle, may appear above upon the lid of the Box, and mark upon its side how many Degrees the Air is dryer or moister than 'twas the day before, in moving round the Point B, as the Beard twists or untwists, in proportion to the greater or lesser dryness of the Air: Mr. *de Vallemont* says, the moisture turns the Beard from East to West by the way of the South, and the dryness from East to West by the North. The second Hygrometer. Plate 24. Fig. 76.

At the Emperor's Court, we meet with another *Hygrometer* made thus. Choose a Room that's not very large, to avoid the too great agitation of the Air, and with a String or Rope AB, hang up in it a round flat piece of Wood, CD, by its Center of Gravity B, so that it may hang Horizontally, and always in *Æquilibrio* round the Point B. This piece of Wood or Cylinder CD, must be about half a Foot broad, and almost as thick as one's Finger, and its Circumference must be divided into 60 equal Parts, mark'd all round upon the thickness, to denote the Degrees of the dryness and moisture of the Air, easily distinguish'd by the Finger of a Hand, as EF fix'd near to it, for then the Cylinder CD will turn round the Point B to the Right or to the Left, according as the Air is moister or dryer. A third Hygrometer. Plate 24. Fig. 78.

To avoid the inconveniency of the continual agitation of the Air in a large Room, the least Motion being capable to turn the Cylinder CD, while 'tis suspended by its Center of Gravity B; you may cover the Cylinder with a Glass Bell perforated above, so as to give passage to the String AB, and to suffer it to move without any hindrance; for then you may see the Alterations of the Air thro' the Glass.

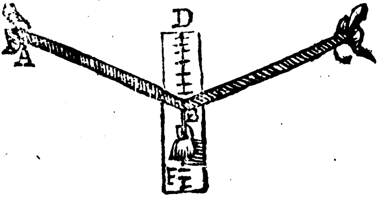
The

The Ingenious Mr. *Richard* informs me, he has used this Hygrometer with great Satisfaction; only instead of a common String, he takes a String of Catgut, as AB, and hangs it in a hollow Glass Cylinder with a Foot to it, and a little perforated Cupola; to the lower end of the String B, he ties an Artificial Bird



which by turning to the Right or Left, as the String untwists by the moisture, or retwists by the dryness of the Air, shews the Degrees of that moisture or dryness, upon equal divisions made upon the Circumference of the Cylinder.

The fourth Hygrometer. Another Hygrometer as easie as the former is made in *Germany*, of a String of a Catgut ABC, made fast



at its two Extremities A and C, and loaded in the the middle with a small Weight F, tied with a Thread to B, which lowers the String ABC more or less according,

according to the degrees of the dryneſs or moiſture of the Air, reckon'd upon the perpendicular Plain DE divided into equal Parts, which Diviſions the Point B touches in riſing or falling according to the moiſture or dryneſs of the Air; for we know by daily Experience, that when the Air is moiſt, the watry Vapours inſinuate themſelves eaſily into a String, and ſwell and ſhorten it, which makes the String ABC draw in and raiſe the weight F, as the Air grows moiſter.

Inſtead of a Gutſtring you may take a piece of Packthread, which indeed ſeems to be more ſuſceptible of Moiſture; for Moiſture eaſily inſinuates into all Porous Bodies, and above all, into the Strings that ſhorten ſenſibly upon the acceſſion of the leaſt Moiſture. Thus, we find, that when *Sixtus* V. ſet up the great Obelisk of the Vatican, the Cables being made longer by that huge Weight, which weigh'd one Million ſix Thouſand forty eight Pound, he order'd the Cables to be ſoak'd, upon which they ſhrunk ſo, that they ſet that huge Maſs upon its Baſe, as it now ſtands.

Theſe moiſt Vapours do likewiſe inſinuate readily into Wood, eſpecially that which is light and dry, as being extream porous; inſomuch that they are ſometimes made uſe of for dilating and breaking the hardeſt Bodies, particularly, Mill-ſtones; for when a Rock is cut into a Cylinder, they divide that into ſeveral leſſer Cylinders, by making ſeveral Holes round the great Cylinder at diſtances proportional to the deſign'd thickneſs of the Millſtones; and filling them with as many pieces of Sallow Wood dried in an Oven; for when wet weather comes, theſe Wedges or pieces of Wood are ſo impregnated with the moiſt Corpuſculums in the Air, that they ſwell and break, or ſeparate the Cylindrical Rock into ſeveral Millſtones.

The Humidity of the Air inſinuates it ſelf not only into Wood, but likewiſe into the hardeſt Bodies which are not deſtitute of Pores, and eſpecially into the light Bodies, which take up a great Space; and hence 'tis, that Mr. *Paſcal* in his Treatiſe of the *Æquilibrium* of Liquors ſays, that, if a pair of Scales continues in *Æquilibrio*, when loaded with two equal Weights, Fifth Hygrometer.

Weights, one of which is of a more voluminous Substance than the other, as Cotton or any Body of a lesser Specifick Gravity, the Ballance will depart from its *Æquilibrium*, and incline to that more voluminous Weight, when the Air is stuff'd with Vapours; for the watry Particles, with which the Air is loaded, will insinuate themselves more readily into this, than into the other Weight, which being less Voluminous, must needs have lesser Pores.

But of all the Bodies that are apt to imbibe the moisture of the Air, I know none more such, than the Salt af any hot Plant, or Saltpetre well calcin'd, which upon the least moisture of the Air, melts readily into Water, so as to weigh three or four times as much as before. For this is the common quality of a'most all Salts, that they are easily impregnated with the Bodies contain'd in the Air; and accordingly when the Salt at a Table is moister than ordinary, we take it for a certain Sign of approaching Rain, as denoting that the Air is loaded with moist Vapours, which will quickly dissolve into Rain.

So, if you want a good Hygroscope, put a certain quantity of Saltpetre well calcin'd into one Scale of a just Ballance, and an equal weight of Lead drops into the other, so as to make the Scales hang perfectly in *Æquilibrio*; then add to the Center of the Motion of the Ballance, a small Circle divided into equal Parts, representing the Degrees of the dryness or moisture of the Air, which the tongue of the Ballance will point to as the Air grows moister or dryer, for the moister the Air grows, the more will the Lead rise.

Sixth Hygrometer.

Another way of using Gutstring for Hygrometers, is this; Tune a Lute or any other String-Instrument, to the tune of a Flute or a Flageolet, which are less liable to the alterations of Weather; and while the Air continues in the same Temperature, you'll find the Instruments keep in Tune; but when the Air grows drier, the String sounds sharper, and more upon the Bass when the Air is moister.

Remark.

The variety of Hygrometers is infinite; you may invent as many as you will; for the very hardest and solidest Wood will swell by the moisture of the Air, as appears by the difficulty of shutting our Doors and Windows in wet Weather.

Nay,

Nay, the very Body of all Animals and Vegetables, is, as 'twere, a Contexture of Hygrometers, Barometers, and Thermometers; for the Humours with which the Organiz'd Bodies are replenish'd, increase or decrease according to the different Dispositions of the Air; and Plants are compos'd of an infinite number of Fibres, which are like so many Canals or Pipes, thro' which the moisture of the Air, as well as the Juice of the Earth, is conveyed into all their Parts.

Mr. *Foucher* says, he has experienced by the means of an Hygrometer, that in Summer the Weather is moistest between seven and eight at Night, and in Winter between eight and nine in the Morning; and that the Air is moister at Full-Moon, than when the Moon is near the Change.

PROBLEM XXXVIII.

Of Phosphorus's.

We give the name of *Phosphorus* to a Body that's fraughted with such a quantity of the Corpusculum's of Light, that by its means one may easily see in the darkest Night the next adjacent Objects, and even read a Manuscript without much difficulty.

Some Phosphorus's are Natural, and some Artificial. The Natural are a sort of Worms with Wings, which shine at a distance in the Hedges in the Summer Nights, and are commonly call'd *Glow-Worms*, by the Latins *Cicindelæ*, *Nitedulæ*, *Nitelæ*, *Luculæ*, and *Luciolæ*, and by the Greeks *Lampyrides*; which give your Husbandmen to know the season for cutting down their Corn, and bringing in their last Harvest, as the Mantuan Poet has elegantly express'd it in the following Lines. Of Glow-worms.

His tandem studiis hyemem transegimus illam.
Ver rediit, jam sylva viret, jam vinea frondet.
Jam spicatæ Ceres, jam cogitat horrea messor.
Splendidulis jam nocte volant Lampyrides Alis.

Besides these Glow-worms, which cease to shine when they are dead, there's likewise an *Indian* Snail which ſhines while alive, and ceaſes ſo to do when dead, as indeed all Animals do. But there's a ſort of Shell-Oyſters that preſerve ſome fiery Spirits, and give ſome light after their Death. Rotten Herrings give ſome light in the Night; and rotten Wood a great deal. Some Diamonds when rubb'd have the ſame effect; and *Gonzalo Doviedo*, ſays, there is a Fowl in the *Indies* call'd *Coërno*, which has ſuch ſparkling Eyes, that it ſerves for a Candle at Table.

The Artificial Phoſphorus's are made of a ſort of Stone, like unto Plaiſter, heavy, clear and Tranſparent, found in Mount *Paterna* near *Bologna*, and from thence call'd the *Bolonian* Stone. This Stone being calcin'd and expos'd to the light of the Day, imbibes that light without burning, and keeps it for as long a time as it has been ſet to receive it, as we obſerve by conveying it into a dark place where it ſhines like a burning Coal.

Some Artificial Phoſphorus's are made of Chalk, Urine, Blood, and other Sulphureous Subſtances; and theſe burn with a Flame that's quite different from that of other burning Bodies; for it ſpares ſome Subſtances that other Fires conſume, and conſumes thoſe that another Fire ſpares; what extinguiſhes other Fires kindles it, and what kindles other Fires extinguiſhes it.

There are ſome things that this Phoſphorus do's not inflame when it touches 'em, and yet puts them in a flame when it do's not touch them. Its flame is more hot than that of Wood, more ſubtil than that of Spirit of Wine, and more penetrating than that of the Sun, the Rays of which collected by a Glaſs, burn black Subſtances ſooner than white, whereas the Phoſphorus attacks them equally.

The flame of ſuch a Phoſphorus is ſaid to paſs thro' Paper or Linnen without burning 'em, unleſs it be old Linnen, or old Paper without Gum. 'Tis alſo ſaid, that if this flame runs upon a little ball of Sulphur, 'twill not ſet it on fire, nor yet Gunpowder; but if you bruiſe 'em together 'twill put them into a flame, Camphyr always takes fire preſently.

The Phoſphorus has always been reckon'd one of the moſt curious and ſurpriſing productions of Chymiſtry, by reaſon of its uncommon and peculiar Properties;

perties; for besides those already mention'd, 'tis possess'd of many more, some of which we shall briefly hint at.

If you write in the dark with a Phosphorus, the Letters will appear light like a Flame; and if you rub your Face with it, which you may do without any danger, your Face will be luminous in the dark; and in fine, if you beat it up with some Pomatum, 'twill make it shine in the dark.

If you dip one end of a piece of Paper or Linen in Spirit of Wine, or good Brandy, and rub some Phosphorus upon the other end, the Spirit of Wine or the Brandy, will be put in a flame by the Phosphorus, tho' it do's not touch 'em immediately, and will set fire to the Paper or Cloth; which would not happen, if the end of the Paper or Cloth had been dip'd in Oil of Spike or of Turpentine: And if you rub the Phosphorus upon the end that's dip'd in the Spirit of Wine, the Phosphorus will not take fire; but if the Cloth be dip'd in common Water, 'twill then take fire notwithstanding that 'tis preserv'd by being kept in Water; and this Water stir'd about will give Light, tho' Spirit of Wine with Phosphorus dip'd into it will not; but if you pour some drops of this Spirit of Wine into the Common Water, each drop will produce a light that presently disappears like Lightning, &c.

The Composition of the Artificial Phosphorus.

I've already intimated that to preserve the Artificial Phosphorus, we must keep it in Water; and now I come to shew you the way of preparing it with Urine.

Evaporate upon a gentle fire what quantity you will of fresh Urine, till there remains a black Substance almost dry; let this Substance rot for three or four Months in a Cellar; then mix it with double the quantity of Sand or Bole-Armeniack; and clap the mixture upon a gentle Fire, in a stone Retort with a Recipient well luted and half full of Water. Raise the Fire by degrees for three Hours; and there will pass into the Recipient first a little Phlegm, then a little Volatil Salt, then a great deal of black stinking Oil, and at last the Substance of the Phosphorus will remain sticking to the Vessel, in a white Mass, which you must melt in Water to reduce it to a Roller. This you may keep ſeveral years in a Vial full of Water cloſe ſtop'd.

The Phoſphorus being the fat and volatil part of the Urine, it may likewiſe be drawn from other Excrements; alſo from Fleſh, Bones, Hair, Feathers, Nails, Horns, Tartar, Manna, and any thing that yields by Diſtillation a fetid Oil.

Another ſort of Artificial Phoſphorus is made of the *Bolonian* Stone, calcin'd after the following manner. Take five or ſix great Stones, pound two of them in a Mortar to a very fine Powder, and with that make a Cruſt round the other four. Then put all in a little Furnace upon a Grate, cover them with Coal, and continue the Fire for three or four Hours, or till the Coal is conſum'd to Aſhes. This done take out the Stones, and clear 'em, and ſo your Work's done.

Remark. Writing that ſhines in the dark.

I intimated above, that with the Artificial Phoſphorus one may Write, ſo as that the Letters ſhall ſhine as a flame in the Dark; and *Wecker* ſays, after *Porta*, that this may be done by the Natural Phoſphorus, that is, by Writing with the Liquor of Glow-Worms. But this wants to be confirm'd by Experience; for, as I ſaid before, Glow-Worms give no light after Death.

Wecker, in imitation of the ſame Author, makes an Artificial Phoſphorus of Glow-Worms, after the following manner. Beat ſeveral Glow-worms together, put them in a Matraſs well ſtop'd for fifteen Days in Horſe-dung, then draw off with an Alembick a Water, which put into a Vial, will caſt ſuch a light in the Dark, that you may read and write by it.

How to make good red Ink.

But now that we are got upon the Subject of Writing, I ſhall here ſhew by the bye, the way of making good red Ink. Soak the White of an Egg thirty Hours in a Spoonful of good Roſe Vinegar; then throw away the White, which you'll find half boil'd, and ſtrain what remains thro' a clean Cloth, and ſo you have a Gummed Water, which you're carefully to keep in a little Vial, to be made uſe of on occaſion in the following manner.

Put a little of your Gummed Water into a Gally Pot, ſuch as your Apothecaries uſe for their Ointments, and mix it with a little Powder of Vermillion or Cinnabar, till 'tis red enough to Write without

being

being too thick; and so you will have a very good sort of Ink that will stick close to the Paper, and not set off to the opposite side, when the Paper is beat by the Book-binders or others, as it do's when made of bare Water or Common Gum. This red Ink must be stir'd with a Pencil from time to time, when you go to Write, because the Vermillion or Cinnabar sinks by its weight to the bottom of the Pot.

Another sort of red Ink which do's not want to be so often stir'd, and may be used as Common Ink, is this. Take four Ounces of Brasil Wood cut small, one Ounce of Cerus, one Ounce of Roch-Allum; pound all in a Mortar, and pour on Wine till all's cover'd. After three days standing, strain the Liquor three or four times thro' a very clean Cloth. then put it in a white earthen Mortar, and let it dry in a dark place, where Sun nor Day-light can't reach it. At last scrape off the Flower of this dry Substance, and keep it to be diluted in Gummed Water for use upon occasion.

Writing upon Paper that will not be seen without it be wet.

I shall here subjoyn *Alexis*'s Directions for Writing upon Paper, so as that the Writing shall be invisible till the Paper is dipt in Water, Put the Powder of Roch-Allum into a little Water, and with that write upon the Paper when you please. When the Letters are dry they will disappear; but clap the Paper in fair Water, and the Letters will look white and shining, the Paper being a little black'd with the Allum.

Writing that can't be read without fire,

The same Author directs to Write so as that the Writing shall not be read but before the Fire, by Writing with the Water in which Sal Armoniack well pulveris'd is dissolv'd. For when the Letters thus Written are dry, they will disappear, but hold them near the Fire, and then they become visible again. The same is the case if you Write with the juice of a Lemon, or of an Onion.

PROBLEM XXXIX.

To make the Sympathetick Ink.

THE Sympathetick Ink is made of two different Waters, the first of which discovers the Letters written with the second, which do not appear of themselves when they are dry; but when a Spunge moisten'd never so little with the first is drawn over them or near them, they appear of a red colour inclining to the Black. When these two Waters are filtrated, they are very clear and Transparent, but when mix'd together they become Opaque, and assume a very brown Colour. Their Composition is as follows.

The Water which discovers the Letters, and which we call the *First*, is thus made. Put into a new and very clean earthen Pot some fair Water, in which infuse a little Orpiment, with a piece of quick Lime for 24 Hours, and so you have your first Water. As for the *Second* Water with which you write the invisible Letters, 'tis a Gallon of distill'd Vinegar boil'd for half a quarter of an Hour with an Ounce of Litharge of Silver.

When these two Waters are fresh made, and care is taken to stop the Pot well which contains the *First*, the first Water has such a Vertue by the force of the Lime infused in it, that if you cover a Letter written with the second Water with a Quire of Paper, 'twill blacken the Letters and make 'em appear, tho' it be only pour'd upon the upper sheet of the Paper that covers the Letter. Take notice that these two Waters must be strain'd apart, for 'tis that which renders them clear and transparent.

A Sympathetick Ink that penetrates a Wall.

But there's another sort of Sympathetick Ink, that penetrates not only thro' a quire of Paper, but thro' a thick Book, and even thro' a Wall, provided there be Planks on the two sides to hinder the Evaporation of the Spirits. In this case the first Water is the same as above; but the second is an Impregnation of Saturn or Lead, as clear as Rock-Water, made thus. Take an unglazed earthen Pan, melt Lead in it, and stir it continually upon the Fire with a Spatula, till 'tis all redu-

reduced to Powder; dissolve this Powder in distill'd Vinegar, and so you have a clear transparent Liquor, with which you may write what you will upon a piece of Paper, and then put the Paper between the Leaves of a very thick Book; which being turn'd, observe as near as you can the part of the last Leaf that corresponds to that in which your Paper lies, and rub that last Leaf with Cotton impregnated with the first Water (made with quick Lime and Orpiment;) then leave the Cotton upon the place, with a double piece of Paper over it, and quickly shut the Book, giving it four or five knocks with your Hand. This done turn the Book, and put it in a Press for half a quarter of an Hour, after which you'll have a distinct appearance of the Letters that were formerly Invisible.

PROBLEM XL.

Of the Sympathy and Antipathy observ'd between Animate and Inanimate Bodies.

BY *Sympathy* we understand a Conformity of the natural qualities of Humours or Temperament, or a suitableness of occult Vertues, so distributed to two things, that they easily agree and bear with one another, nay love, so to speak, and court one another.

We find in our selves the effects of *Sympathy*, when we have a particular Affection or Esteem for an unknown Person, as soon as we see him; and of *Antipathy* when we avoid a Person that has never disobliged us, and in whom we have discovered no considerable Fault. A'most all of us hate to hear the grating of a Knife against any other thing. I know some would die rather than tarry for any time in a close Room with a Cat; some can't see Cheese without fainting; and it must be by the like Antipathy, if it be true, what is said, that the Blood of a Murdered Person will flow from the Wound in the presence of the Murderer; some have an Antipathy against the agreeable smell of Roses; Women in Childbed hate Perfumes, particularly Musk; some will Swoon away at the smell of an Apple. The Cock seems to have a Sympathy with the Morning, which it welcomes with Crowing and Clapping its Wings; Turn-Sol with the Sun, to which it turns; the Hazel Rod with the Metals which it discovers by its turning: there seems to be an Antipathy between a Horse and a Wolf, since, as 'tis said, the former will not eat if there be a Wolf's Tail hung upon the Rack; between the Vine and Coleworts; between Hemlock and Rue; between a Man and a Serpent; between a Hart and a Serpent; between a Weazle and a Serpent, and an Infinity of other things, which for Brevity's sake we here omit.

We are told there's such a Sympathy between Elephants and Sheep, that the *Romans* by that means defeated King *Pyrrhus* with his Elephants. *Ireland* produces no venomous thing, nor indeed any thing that do's Harm, except Wolves and Foxes; and near *Grenoble* in *France*, there's an old Town standing on a Mountain, where neither Serpents nor Spiders, nor any other poisonous Animal will live.

Mr. *Boyle* speaks of a venomous Tree in *America*, call'd *Manchinelle*, which the Fowls will not so much as pearch upon. The *Agnus Castus* is said to banish all venomous Plants; and every one knows that the Sensitive Plant shrinks up it self if it be but touch'd.

An Artificial Stone is said to be imported from *Goa*, which the *Portuguese* call *Capellos de Colubras*, the Snake-Stone, as being made of the bones of certain Snakes, which being made up with another Drug that few People know, composes that marvellous Stone that draws all poyson out of Wounds made by the biting of Venomous Creatures. But Mr. *Charras* tried this upon Pigeons bit with Vipers, to no effect.

Quicksilver which penetrates the Pores of all other Metals, and reduces 'em to a Past, has such a Sympathy with Gold, that if you put one end of a Rod of Massy Gold into it, 'twill insinuate it self all over the Rod to the other end, both on the outside and inside. This dry Liquid is such, that if you stir it with your Hand, a Gold Ring upon the other Hand will be white and cover'd with Quicksilver; and in like manner a piece of Gold held in the Mouth attracts the Spirits of Mercury. 'Tis needless to mention the

force

force of Quickſilver in paſſing thro' Leather when 'tis heated but never ſo little; and the re-union of its Particles in the primitive form, after being diſpers'd into Vapours by Diſtillation.

Few People are ignorant of the force of *Electrical* Bodies, which are ſo call'd, becauſe, like Amber, they attract Straw, *&c.* without touching them. Every one knows the Power of the Loadſtone, of which more at large in the next Problem.

PROBLEM XLI.

Of the Loadſtone.

THE Loadſtone is a very hard and very heavy Stone, the colour of which approaches commonly to that of *Iron*, which it attracts by a peculiar vertue at a reaſonable Diſtance, and that with a force that makes a ſenſible Reſiſtance when you go to part 'em. This admirable Stone has many fine Properties, which I am now briefly to hint at.

The Loadſtone has not only the vertue of attracting Iron even by penetrating the intervening Bodies; but likewiſe that of communicating to the Iron that it touches, the vertue of attracting other Iron, which in like manner acquires the power of attracting another: For we ſee with our Eyes, that an Iron Ring touch'd by a good Loadſtone lifts another Ring, and that ſecond Ring lifts a third, and ſo on. We ſee likewiſe, that a blade of a Knife touch'd by a Loadſtone, raiſes Needles and Iron or Steel Nails.

The Loadſtone not only attracts, but communicates its attractive Vertue to Iron.

If you lay ſeveral ſewing Needles cloſe to one another upon a Table, and bring a Loadſtone near the firſt, 'twill attract the firſt, which acquiring a Magnetick Vertue, will draw the Second, and that the next, and ſo on, till all the Needles hang to one another, as if they were link'd together, unleſs you part 'em by Violence.

Iron reciprocally attracts the Loadſtone at a reaſonable Diſtance, when that Stone can move freely, as when 'tis hung up, or floats in Water; notwithſtanding the intervention of another Body. For example, put a piece of Loadſtone in a light Boat made like a Gon-

Gondola, ſo as to make the Loadſtone float upon the Water, and preſent to it a piece of Iron at a reaſonable Diſtance, you'll ſee the Gondola cut the Water to go and joyn the Iron.

This puts me in mind of a Clock I once ſaw at *Lyons* in Mr. *Servieres*'s Cloſet, which ſhew'd the Hours by throwing an Artificial Frog into a Baſin of Water, round which the Hours were mark'd, as upon a Dial; for the Frog ſwimming upon the Water, ſtop'd and pointed to the reſpective Hour, and inſenſibly follow'd the Hour of the Day, like the Hand of a Clock. I judge this was done by a Loadſtone hid under the Baſin, which followed the hour of the Day by the vertue of Clock Wheels, and drew to the ſame Hour the Frog, in which no queſtion was hid a piece of Iron.

It affects the ſame aſpect in the Univerſe.

When a Loadſtone floats upon the Water, without any thing about it to cramp its free Motion, or hinder it to take what Situation it finds moſt convenient, it turns always the ſame way with reſpect to South and North; ſo that one particular part of the Stone always looks to the North, and its oppoſite to the South; whence theſe two places pointing to the two Poles of the World, are call'd the *Poles* of the Loadſtone; and the ſtraight Line paſſing from one Pole to the other, is call'd the *Axis* of the Loadſtone. Now, all the force and efficacy of the Loadſtone is in this *Axis*, for the other parts off of the *Axis* have very little Vertue; and 'tis chiefly from the two Extremities or Poles, as from two Centers, that the Vertue is diſtributed.

That part which is equally remote from its two Poles, we call the *Æquator* of the Loadſtone; and this has ſuch a quality, that if you lay a ſewing Needle upon it, 'twill lie all along it parallel to its *Axis*; but if you take it off of that Line, it riſes more and more as it approaches to one of the two Poles, where it ſtands upright. This is diſtinctly obſerv'd in the Spherical Loadſtones, which I here ſuppoſe Homogeneal, as they commonly are, for otherwiſe they may have more than two Poles. I know a Gentleman at *Lions* who has a Loadſtone that has four Poles, two on the South ſide lying oppoſite one to another, and two after the ſame manner pointing to the North.

The

The Loadstone communicates its Vertue not only to the Iron that it touches, but even to that which passes near it; it attracts likewise another Loadstone, and sometimes repulses it, according to the different Aspects of their Poles, which are call'd *Friendly Poles* when they're of a different Denomination, that is, the one Meridional, the other Septentrional; and *Hostile Poles*, when they're of the same Denomination, that is, both Meridional or both Septentrional: For the North Pole of one Loadstone attracts the South, and repells the North Pole of another, and *è contra*; provided the other can move freely, as when it floats in Water, Mr. *Puget* has a Loadstone, that in stead of attracting another that floats upon Water, when the Poles are friendly, draws it indeed to a certain distance, but repells it if it comes nearer.

We observe in all Loadstones, that when the North Pole of one has attracted the South Pole of another, the Aspect of the North Pole of a third parts 'em. Here I purposely wave the Reasons of these Phænomena, because they are Abstruse, and improper for Recreation.

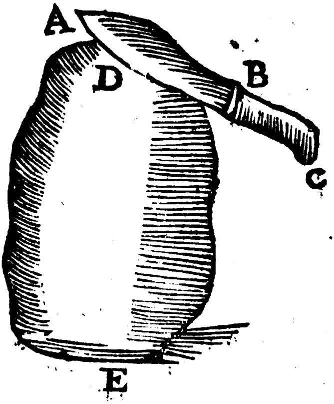
When we say, a Loadstone in attracting Iron penetrates all sorts of intervening Bodies, as freely as if there were none between; we must except the intervention of Iron it self; for we find by Experience, that the intervention of a plate of Iron impairs the activity of the Magnetick force; doubtless, because the Vertue taking hold of the Plate, is partly spent upon it. Several Experiments of Loadstones.

When we say, that the Loadstone draws Iron to it, we must suppose that it can draw it; for if it can't, and if 'tis at liberty to move, the Iron reciprocally attracts it, and when joyn'd together they sensibly resist the efforts of Separation.

Tho' the Magnetick Vertue penetrates all intervening Bodies, Iron excepted, with as much Facility, as if nothing interven'd, yet 'tis observable that this Vertue is communicated with more difficulty thro' Flesh, than thro' any Metal whatsoever.

When we say, that the blade of a Knife acquires the Magnetick Vertue by being touch'd with a Loadstone, we must add, that this Vertue is communicated to the part of the Knife that's last touch'd; so that if you rub the blade of a Knife from the Haft to the Point along a Loadstone, all the Magnetick Vertue will remain in the Point, and the other end towards the Haft will have no attractive Force; and if you rub it the contrary way, the Virtue will be transplanted to the other end. Farther, the Vertue thus imparted will be greater or lesser, according to the place of the Loadstone that the Blade is rubb'd upon; so that, if you rub it upon one of the Poles where the Vertue is most Efficacious, 'twill receive the greatest attractive force that 'tis capable of.

This Rubbing is done by drawing the Blade AB of the Knife ABC, lengthwise, from the Haft BC to the Point A, or from the Point to the Haft, along the Pole D of the Loadstone DE, the other Pole of which is E; and then the Blade AB acquires the Vertue of raising as much Iron as is possible; and if the Blade is drawn over the Pole from B to A, so that



B touches the Pole first, and A last, all the Magnetick Vertue lies in the Point A. But if after thus touching, you rub it again the contrary way, drawing it over the Pole D from A to B, in that very instant it loses that attractive Vertue it had acquir'd.

All Loadstones are not equally good; and we must n t always judge of the goodness of 'em by their W eight; for sometimes an Ounce of Loadstone is al le to lift a pound of Iron; tho' indeed of two Load-

stones

ſtones of equal Vigour, the greater, has always more force than the leſſer. The more ſolid and leſs porous that the Stone is, the greater is the force; and it has more vigour when poliſh'd than when rough, and more ſtill when arm'd with a plate of Steel or poliſh'd Iron. But here you muſt obſerve, that if a Loadſtone thus arm'd holds Iron by one of its Poles, and the friendly Pole of another naked or unarm'd Loadſtone is preſented to it, it holds it the more forcibly; but upon the preſenting of the Hoſtile Pole it loſes the force and lets it drop. In breaking a Loadſtone, you may find one part of it to have more force than the whole Stone.

The Loadſtone attracts twice as much Steel as Iron, and at a greater Diſtance; for the former being ſolider and leſs porous than the latter, it joyns more intimately with the former; and when thus joyn'd with fine well poliſh't Steel, it attracts a greater Weight, than when faſten'd to groſs unpoliſh'd Iron. A ſtronger Loadſtone draws a great weight with more Expedition, and at a greater Diſtance, than a weaker Stone. We ſeldom ſee a large Loadſtone raiſe more than its own Weight, unleſs it be arm'd; but oftentimes we meet with little ones, that raiſe ten, twelve, and ſometimes eighteen times their own Weight; thus an Ounce Stone will raiſe a pound of Iron, as above.

We ſometimes obſerve with Aſtoniſhment, that a large fine Loadſtone ſtrips a little one that comes too near it of its Vertue; but the little one recovers it again in two or three Days, We obſerve likewiſe in breaking off a part of a Stone, the *Axis* and the two Poles ſhift their places. Father *Schott* the Jeſuit, tells you, that if you cut a Loadſtone by its Æquator into two parts, each part will have two Poles, a new Pole at the Section, and the old one at the old place bearing the ſame Name; and if you cut it in two by its *Axis*, each part acquires new Poles, of a ſimilar Situation to that of the Poles of the firſt Stone, and likewiſe with the ſame Properties.

This Stone is ſo hard, that ſcarce any Iron Inſtrument will touch it, and it can't be cut but with a braſs Saw without Teeth, made as ſharp as a Knife, and with the Powder of Emmery diluted in Water; it it being impossible to cut it with any other Saw tho' of the finest Steel.

I forgot to acquaint you, that by the *North Pole* of a Loadstone, we understand that Pole which turns or points to the North, when the Stone hangs free by its *Æquator*; and by the *South* Pole, the opposite Pole that points to the South. I said, *when it hangs free by its Æquator*, for if 'twere suspended by one of its two Poles, 'twould continue unmovable, because the North Pole could not then turn to the North, nor the South Pole to the South.

Remark. Whence we have the Loadstones.

Some will have the Loadstone to be call'd in Latin *Magnes*, from *Magnesia*, a County in *Macedonia*, where 'tis frequently found. Now, the *Magnesia* Loadstone is sometimes black, sometimes red; the *Natolia* Loadstone is white; but, as Historians tell us, neither of these has much Vertue. The *Ethiopian* Loadstone, which is very heavy and very vigorous, is sometimes yellow. The best Loadstones we have in *Europe*, are for the most part found in *Norway*. There is a sort of red and of blew Loadstone, which *Dioscorides* prefers to that of the rusty Colour. In *Italy* they have a sort of Loadstone, that's red on the out-side, and blew within, which when beaten gives a sort of Flower that Iron attracts at a certain Distance.

If the name of Loadstone he allowed to the Stones that attracts other Metals, we may reckon in this List a Stone call'd *Pantarbe*, which attracts Gold, and another call'd *Andromantie*, which attracts Silver. *Cardan* says, there's a Stone call'd *Calamites* that attracts Flesh. In *Æthiopia* there's a Stone call'd *Theamedes*, that instead of loving Iron can't indure it, and repells it; which has given some occasion to say, that as those who carry Iron about 'em to the Loadstone Mountains can't stir, so on the other hand if these Mountains produced the *Theamedes* they could not keep to a fixed Station.

The best Loadstones.

To conclude this Problem, the best Loadstones are commonly those of a watry or of a shining black Colour, and very little Red; and of a solid Homogeneous Substance, that is, they have but few Pores, and are free from the mixture of a foreign Matter. The figure of a Loadstone contributes very much to its Force, for 'tis a standing Truth, that of all Load-

stones

ſtones of equal Goodneſs, that which is the longeſt, the beſt poliſhed, and ſo cut that its two Poles are at the two Extremities, is the moſt vigorous. A Spherical Figure is likewiſe very advantageous to a Loadſtone.

The Loadſtone preſerves its Vertue in Filings of Steel, tho' the filings may ruſt with it, and likewiſe impair its Vertue; but the violence of Fire impairs it more in one Hour, than the Ruſt does in ſeveral Days. Father *Dechales* ſays, the Loadſtone do's not attract red hot Iron, the occaſion of which is undoubtedly this, that the Heat diſſipates the Magnetick Spirits by putting them in Motion.

In fine; a Loadſtone alſo loſes its Vertue of attracting Iron, when 'tis beat too violently upon the Anvil; for that changes the Diſpoſition of its Parts, and the Figure of its Pores. This Reaſon is confirm'd by the Experience of Mr. *Puget*, who having put filings of Steel into a Glaſs Tube, and placed a good Loadſtone near the filings in order to communicate its Vertue, obſerv'd that theſe filings loſt their Magnetick Vertue by being ſtir'd and mov'd, ſo that they could not attract Needles as they had done before. To this purpoſe, 'tis ſaid that if a Magneted Steel Needle changes its Figure, *i. e.* is turn'd from a ſtraight to a bended, or from a bended to a ſtraight Figure, it loſes its Vertue quite.

PROBLEM XLII.

Of the Declination and the Inclination of the Loadſtone.

THE foregoing Problem diſcover'd three conſiderable Vertues in the Loadſtone, *viz.* its affecting a certain Aſpect in the World, its drawing Iron, and its communicating the ſame attractive Vertue to Iron. And in the Problem we are now upon, we are about to ſhew that nothing in the World is more variable than the direction of the Loadſtone, and hence ariſes what we call the Declination of the Loadſtone: For under the ſame Meridian the Loadſtone declines ſometimes to the Eaſt, ſometimes to the Weſt, as appears by the Angle which the Compaſs Needle makes with the Meridian Line, which is call'd *the variation*

of

of the Needle, reckon'd from North to East, in which case 'tis an *Oriental Variation*, or from North to West, in which case 'tis call'd *Occidental*.

The Irregularity of the Declination of the Needle.

This Variation or Declination is very irregular, for under the same Parallel it sometimes vary's very much in a little space, and oftentimes but little in a great many Leagues. Neither is it always the same at all times, for we find a Declination now where there was none before. In former times, the Declination at *Paris* was very small, and now 'tis almost six Degrees from North to West; which evidently shews, that Mr. *Riccioli's* large Table of Variations of the Needle, inserted in his Geography, is altogether useless.

All Loadstones and all Magneted Needles, of what length soever, decline after the same manner in the same place at one and the same time; which shews that the different sorts of Loadstones, or the different length of Needles, have no hand in the Declination. Since the Eruption of Mount *Vesuvius*, we find a considerable change in the Declination at *Naples*; and in several other places, we find no such Declination as our Ancestors observ'd.

What we call its Inclination.

As the Philosophers are puzzled in accounting for the variable Declination of the Loadstone, so they are equally gravell'd upon the score of its *Inclination*, by which we see a rod of Iron or Steel, suspended by its Center of Gravity in *Æquilibrio*, before 'tis touch'd by the Magnet; we see it, I say, lose its *Æquilibrium* after 'tis touch'd; for that End which points to the Pole that's elevated in the Horizon, where 'tis suspended, becomes heavier, and consequently inclines towards the nearest Pole of the Earth, when the Rod is in the Plan of the Meridian. And this is evidence, that the Magnetick Matter comes from North and South, and that the Earth may be considered as a great Loadstone, and a Loadstone as a little Earth, as you shall see in the Sequel.

'Tis for this Reason, that the Workmen, who make Needles for the Portable Dials, make the South Point of the Needle a little heavier than the North Point; that so when 'tis touch'd with the Magnet in the North Point, the Needle may rest in *Æquilibrio* upon its Pavit, that is, be parallel to the Horizon.

To

To make the end of a Needle point to the North, you must make it to touch the South Pole of the Loadstone, gliding it along from the middle to the end; and if after that you touch it again, gliding it contrariwise from the same end to the middle, the touch'd Point that formerly turn'd to the North, will then point to the South, and instead of inclining to the North Horizon, will rise towards the South.

As an Iron Needle applied to a Loadstone do's not incline equally upon every part of the Stone, insomuch that upon its Æquator it do's not incline at all, and the further 'tis from the Æquator, it still inclines the more, till it arrives at the Pole of the Loadstone where it rises perpendicularly, as if it sprung out of its Pole, and meant to continue the *Axis*, as we shew'd in the foregoing Problem; So the *Inclination* of the Loadstone is not the same in all Climates: for under the Equinoctial Line the Needle is certainly in a perfect *Æquilibrium*, and the nearer it approaches to a Pole it inclines the more, but not in the same Proportion; for if it did, we might thereby find out the Latitude of a place, as some have thought without ground.

'Twas likewise a groundless thought of some, that the end of a Magneted Needle that turns to the North, rises towards the Pole or the Polar Star, for on the contrary, it inclines to the Earth, and at *Paris* where the Elevation of the Pole is about 49 Degrees, the Needle inclines to the Horizon, almost 70 Degrees according to Mr. *Rohault's* Observations. In *England* in the Latitude of 50, it inclin'd 71 Degrees and 40 Minutes; and in *Italy* in the Latitude of 42, which is near to that of *Rome*, it inclines to the Horizon about 62 Degrees.

When a Magneted Needle sets one of its Points to the North, and the other to the South, we conclude it has been touch'd by one of the Poles of the Loadstone; for if you rub it against the Æquator of the Loadstone, or only cross its middle, 'twill have no Direction. When your Compass-makers magnet their Needles, they touch 'em only at one end, (namely, that which is commonly mark'd with the Flower de Luce) drawing the Needle over the meridional Pole from the middle to the end, that so it may turn to the North.

You may likewise touch the Needle if you will, beginning to glide it from the Flower de Luce end to the middle; and then the touch'd part of the Needle will turn to the same part of the World with that part of the Loadstone that touch'd it. And therefore if you would have the Flower de Luce turn to the North, as it commonly do's, run the Needle softly over the North Pole from the Flower de Luce to the middle; and if you want to change the touch of your Magneted Needle, rub the opposite end against the same Pole of the Loadstone, after the same manner as you did before, or else touch with the opposite Pole the same part that was touch'd before.

A generous Loadstone communicates its Vertue, to an Iron Needle, at a reasonable distance without touching it; and nothing can rob the Needle of this its derived Vertue, unless you bend it when 'tis straight, or turn it from a bent to a straight form: For if you heat it in a fire red hot without melting, if you rub it, if you file it, it still retains the Direction. It always follows the Pole of the Loadstone that has touch'd it, tho' when 'tis at liberty it points to the Pole of the World that's opposite to that of the Loadstone.

Of all the forms that can be given to Iron, a long straight Figure is the most proper for receiving the Direction, which is always according to the greatest length of the Iron. In an Iron Ring, the Direction lies in the touch'd part and its opposite Point. Hold a Knife over a Compass, and the Needle will turn the South end to it; hold it under, and the Needle will present that of the North to it.

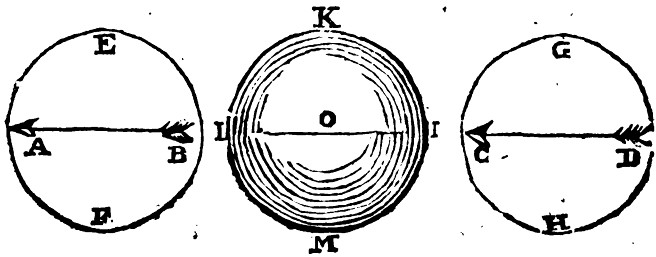
In the Needle of a Compass, we call that Point which turns to the South, the *Meridional Pole*, and that which turns to the North, the *North Pole*; and the South Pole of the Loadstone attracts the North Pole of the Needle, and *è contra*, when it can move freely, and is in the sphere of the activity of the Loadstone: The same is the case with two Loadstones placed by one another.

In two Magneted Needles, we call those the *Friendly Poles*, which have different Denominations, as the North

North and the South; for the one attracts the other, when the two Needles can move freely upon their Centers: And those are the *Hostile Poles*, which are of the same Denomination, *viz*, the two Meridional or two Septentrional Poles; for when two Compasses are put directly one upon another at a reasonable distance, the Similar Poles avoid one another, in the Plain of the Meridian, and so the two Needles take a contrary Situation, one to another, the stronger forcing the weaker to change.

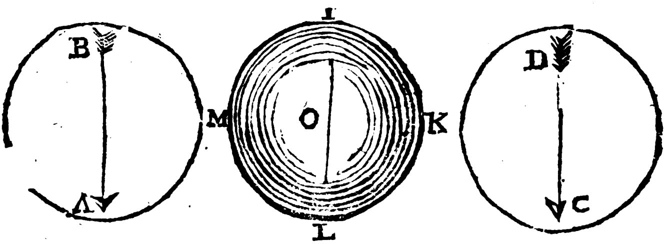
But if two touch'd Needles suspended freely upon their Centers or Pavets, be placed upon the same Horizontal Plain, at a reasonable distance, as AB, CD, so as to be parallel one to another, and to the true Meridian Line, and to have each Pole of the same Denomination turn'd to the same side: In this case, the Poles will continue in the same Situation; for in order to turn to the contrary Directions (as they would do were there no Impediment, and were one hung over the other, as CD is over AB, Fig. 82.) the two Hostile Poles which we have supposed to be on the same side, must of necessity approach one to another, which is contrary to their Nature, And therefore they are kept by force near one another, as if they were Friends. Plate 25. Fig. 83. Plate 25. Fig. 82.

If between two such Needles, as AB, CD, suspended in their Compasses AEBF, CGDH, you put a Spherical Loadstone at a reasonable distance, upon the Plate 19. Fig. 45.



same Horizontal Plain, as IKLM, the North Pole of which is I, and the South Pole L, so that the Axis IL is parallel to the Horizon, and in the Plain of the Meridian: In this case, each of the two Needles, AB, AB, CD, will place it self in the Plain of the same Meridian; that is to say, they'll put themselves in a Right Line with the Axis IL, the South Pole B of the Needle AB pointing to the North Pole I of the Loadstone, and the North Pole C of the Needle CD pointing to the South Pole L of the Loadstone.

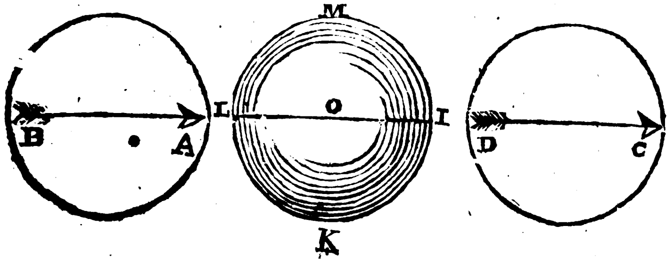
But if you turn the Loadstone IKLM round its Center O, so as to keep the Axis IL always parallel to the Horizon, and to make the North Pole I move to the right to K, and the South Pole L to the left to M, each Pole moving thro' a Quadrant of a Circle: In this case, the South Pole B of the Needle AB, attracted by the North Pole I of the Loadstone, will likewise run a quarter of a Circle from the right



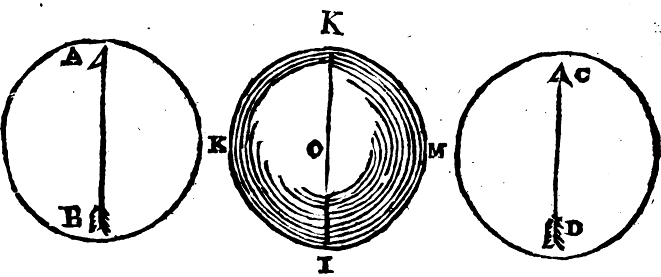
to the left to E, and in like manner the North Pole C of the Needle CD, attracted by the South Pole L of the Loadstone will move a quarter of a Circle, from the left to the right towards H; that is to say, the Poles, I, L, of the Loadstone having acquir'd their Situation as in the annex'd Cut, the Needles, AB, CD, will turn themselves Parallel to the Axis IL, and take the Situation here represented.

But

But if instead of making the Poles, I, L, of the Loadstone turn a Quadrant of a Circle, they be made to move a Semicircle, so as to assume the Situation represented in this Cut. The Needles, AB, CD, will



likewise move to the extent of a Semicircle, and turn as you see in the same Cut. Again if you make the Poles I, L, turn to the extent of three quarters of a Circle, so as to assume this Situation, the Poles of the



Needles, AB, CD, will move to the same extent, and range themselves as 'tis here represented.

Remark. How to have a Needle touch'd.

The Needles commonly made use of in the Boxes or Compasses for Dials, have one end pointed like an Arrow, and the other Plain; or else that end which turns to the North is cut like a Cross or a Flower de Luce, being touch'd with the South Pole of a good Loadstone as above.

Such a Needle ought to be straight, and made of fine polish'd Steel, with a little stud of Copper or Silver in the middle, perforated in the form of a Cone, that

that so the Needle may easily counterballance upon its Pin, which is rais'd at Right Angles from the Center of the Box. Father *Kircher* says, that if you would have a Needle well impregnated with the Magnetick Vertue, it ought not to be too small; because then it do's not so readily shew the Declination of the Loadstone; nor yet too big, by reason that if its length surpasses the Semidiameter of the Sphere of the activity of the Loadstone, 'twill receive a'most nothing of the Direction, and so be of no use. Upon this Consideration, when you are about to touch a Needle, you ought to examine before-hand, the Sphere of the activity of the Loadstone; and that Pole of the Loadstone which touches the Needle ought to be polish'd (if 'tis not arm'd) and that ought to be done not by beating it with an Iron Hammer, for that impairs its force, but rather with a gentle soft File.

PROBLEM XLIII.

To find the two Poles of a Spherical Loadstone, with its Declination and Inclination

To find the two Poles.

TO find the two Poles of a Spherical Loadstone; raise at Right Angles upon any Point of its Surface a small Pivot or Pin, upon which place a Compass-Needle, somewhat shorter than the Diameter of the Loadstone. This Magneted Needle will turn one of its Points to the North, and the other to the South, but 'twill not keep an Horizontal Position, unless it answer to the Axis of the Loadstone. If it don't, you must turn the Loadstone to the Pivot of the Needle, till the Needle is exactly parallel to the Horizon, and then the Pin which I suppose placed on the highest part of the Magnet, will be upon its Æquator, and the two Points of the Loadstone corresponding to the two Extremities of the Needle, will be the two Poles you look for.

Or else hold the Loadstone near to the Needle placed in the Compass, and turn it from one side to the other, till the Needle is perpendicular to the surface of the Loadstone, and then the Point of the Load-

stone

ſtone that anſwers perpendicularly to the Point of the Needle, will be one of the two Poles of the Loadſtone. But in ſtead of a Compaſs-Needle, you may make uſe of a good Steel Sewing Needle, ſuſpended by one end with a Thread, and turn the Needle thus ſuſpended round the Loadſtone, till it touches it at Right Angles, for then the point of Contact is one of the Poles ſought for.

Or again, clap the end of a fine Steel Needle upon the ſurface of the Loadſtone, and the Needle will incline to the Loadſtone divers ways, according as 'tis more or leſs remote from one of the two Poles, but when it comes to one of the Poles 'twill ſtand perpendicular, as intimated above. So that, to find the Pole, you need only to place the Needle in different parts of the Surface, and mark the Point where it comes perpendicular.

One Pole of a Loadſtone ſtronger than t'other.

We rarely meet with a Loadſtone, the two Poles of which are equal, that is, of equal force, for one is a'moſt always ſtronger than t'other. Moſt frequently they are Diametrically oppoſite, that is, they lie in the Line call'd the Axis, which paſſes thro' the middle of the Loadſtone; but ſometimes they are not directly oppoſite; and ſome Loadſtones are ſo vigorous and lively, that they have equal vigour every where, being, as 'twhere, all Poles, for every Point unites to Iron.

To find the Declination.

In the next place, to find at all times and in all places the Declination of the Loadſtone, mark exactly upon an Horizontal Plain the true Meridian Line, by the means of two Points of a ſhadow mark'd upon the Plain before and after Noon, as we ſhew'd you *Probl.* 31. *Coſm.* and after applying to that Meridian Line the ſide of a Square Compaſs, which has a Circle within nicely divided into 360 Degrees, and a Needle well magneted, the end of the Needle will ſhew upon the divided Circle the Degrees of Declination ſought for, counting them from the ſtraight Line that paſſes thro' the middle of the Compaſs, which is the ſide of the ſame Compaſs that was applied to the Meridian Line.

After this manner, we find, that, at *Paris*, the Magnet declines at preſent, from North to Weſt almoſt ſix Degrees; and by the ſame way we know the De-

The Declination of a Vertical Plain.

clination of a Vertical Plain, *viz.* by applying to that Plain the side of a Square Compass, or at least such a side as is perpendicular to the Meridian Line, drawn in the bottom of the Compass; and here you must take care that there be no Iron hid in the Wall, to hinder the Direction of the Magneted Needle, one of whose Extremities will shew upon the divided Circle the Declination sought for, reckoning from the Meridian Line of the Compass, where the Declination of the Loadstone ought to be mark'd, in order to take the Declination of the Vertical Plain more exactly.

Monsieur *Rohault* says in his Physicks, that the Compass Needles are scarce proper for shewing, in this and the other Northern Climates, how much the end of a Needle pointing to the North inclines towards the Earth, because their Center of Gravity is a great deal under the fix'd Point round which they move. For this reason we shall now propose a way of finding (as near as may be) the Inclination of a Magneted Needle.

To find the Inclination which vary's, as well as the the Declination.

Take a very straight piece of Steel Wire, equally thick all over, and of a proper length as four or five Inches. Run a piece of Brass Wire cross its Center of Gravity or middle at Right Angles, and that will hold it in *Æquilibrio*, just as a Beam of a pair of Scales is held by the Hook. Now, as soon as this Steel Wire or Needle is touch'd with a good Magnet, and placed in the Plain of the Meridian, 'twill lose its *Æquilibrium*, and the end that points to the North will incline to the Ground; and so the Needle will shew the Inclination of the Loadstone, which *Rohault* found to be at *Paris* in his time 70 Degrees, and others since only 65 Degrees; and from thence I conjecture, that the Inclination changes as the Declination; but a great many Experiments are wanting to fortifie the Conjecture.

But however that be, the Inclination do's not vary under the Æquator, for there there's none at all, and as it do's not begin till the Needle is moved to some Distance from the Æquator towards one of the Poles, so it still increases as it approaches to a Pole; and hence 'tis that the Navigators sailing Northwards, have been obliged in Sailing North, to clap a little

little Wax upon the South end of the Needle, because the other end bended down to the North Pole of the Earth; and to take it off under the Æquator; and in Sailing on the other side of the Æquator to put the Wax upon the North end of the Needle, the South end of which inclin'd there to the South Pole of the Earth.

Monsieur *Vallemont* very ingeniously explains the Inclination of the Divining Rod by that of the Magneted Needle, in the following Words. 'As the Magnetick Particles that circulate round the Earth, meeting with a Rod of Magneted Iron, range it in the direction of their Course, and render it parallel to the Lines that they describe round the Terrestrial Globe: So the Corpusculum's flowing from Veins of Water, from Mines, from hidden Treasures, and from the tract of fugitive Criminals, rising vertically in the Air, and impregnating the Hazel Rod, make it turn or bend downwards in order to be parallel to the Vertical Lines that they describe as they rise. The same thing happens in this case, that would happen to a Rod of Magneted Iron at the Pole of the Earth, where 'twould incline perpendicularly, by reason of the Magnetical Particles their rising Vertically. Just as when you make fast the branch of a Tree to the stern of a Boat, you see it quickly disposes it self lengthways according to the stream of the River, to which the branch always affects to be Parallel. Remark.

PROBLEM XLIV.

To represent the four Elements in a Vial.

THE four Elements of which the Author of Nature has composed the Elementary World, are the *Earth*, *Water*, *Air*, and *Fire*; of which, the Earth being the heaviest, is said to have the lowermost Station in the Center of the World; Water being lighter covers the Earth; Air being lighter than Water covers it; and at last Fire the lightest of all surrounds the Air. So that in this sense these four make four Concentrical Orbs, the common Center of which is the Center of the World.

We

We may represent the four Elements in this Order, in a long Vial of Glass or Crystal, as AB, by the help of four Heterogeneous Liquors, that is, Liquors of a different Specifick Gravity, which are of such Qualities, that, tho' shak'd together by a violent Agitation, they soon after return to their natural Stations, and all the Particles of one and the same Liquor unite in a

separate Body from the rest, the lighter giving way to the heavier.

To represent the Earth, make use of Crude Antimony, or blue Smalt well refin'd, or black Smalt coarsly pounded, which by its Weight will sink to the bottom of the Vial AB.

To represent Water, pour upon the last the Terrestrious Substance of the Spirit of Tartar, ar Calcin'd Tartar, or the clear Solution of Pot-Ashes with a little Roch-Azur, which will give a Sea Colour.

To represent the Air, pour upon this Composition Spirit of Wine rectified three times, till it has a colour of Air, or else the most Spirituous Brandy with a little Turnsol, which will give it a Celestial Blew or Air Colour.

To

To represent Fire, pour upon all three the Oil of *Behn*, which by its Colour, Lightness and Subtilty, will make a pretty near Resemblance.

PROBLEM XLV.

Several ways of Prognosticating the changes of Weather.

THE Winds are the cause of the most sudden and extraordinary alterations of the Gravity of the Air; and the nature of the Winds is such, that by the Experience we have of them, we may from thence predict (very near) the Weather that will insue for two or three days after; for the Wind that blows is readily known by the *Anemoscope*, of which *Probl.* 34. of *Mechanicks*. We know, for example, in this Climate, that a South Wind generally brings Rain, and a West Wind yet more (which is the Predominant Wind here, doubtless, because the Ocean lies on that side;) that the North Wind brings fair Weather, as well as the East Wind, which do's not last so long as the former.

The Inhabitants of the *Antilla* Islands have an admirable faculty of Prognosticating by Experience the Hurricanes that usually happen in those Islands, and are sometimes so Violent as to toss Men in the Air, raise up big Trees, &c.

We may foretel the alteration of Weather by a Barometer (of which *Probl.* 6. *Mechan.*) for when 'tis calm Weather, and about to Rain in a little time, the Quicksilver usually descends.

Mr. *Guerick* Bourgomaster of *Magdebourg* invented a Barometer, which he call'd an *Anemoscope*, because by it he pretended he could not only tell how the Wind stood in the Air, but likewise predict Rain, Drought, Storms, and Tempests two hundred Leagues off; and even the formation of Comets in the Heavens.

This Barometer is made like a Glass Tube, in which is a little Artificial Man of Wood, that ascends or descends according to the weight of the Air.

We

We are told, that in the year 1680 this little Man mounted so very high at *Magdebourg*, that all on a sudden he sunk quite down in the Tube for two or three Hours; upon which Mr. *Guerick* Prognosticated a great Storm, which accordingly happen'd soon after, and did great Mischief all over the Sea-Coast of *Europe*.

This Gentleman's Secret is said to be known to none but the Elector of *Brandenbourg*, who has one of his Barometers in his Library. But what he knew by his Barometer, the *Savages* know by a long habitual Consideration of the Temperament of the Air, when Hurricanes happen, or of the course of the Clouds, or of the Winds that oftentimes are the fore-runners of Hurricanes; sometimes they predict Hurricanes from the flight of certain Fowls.

The Labouring Men and Ancient Inhabitants of Rural Places, are not less expert in foretelling the alterations of the Weather; above all, the experienc'd Pilots never fail almost in predicting Storms from the precedent Signs formerly observ'd.

Some tell you, there's a hole in a Mountain in the *Alps*, the stopping of which brings a Storm in that part an hour after. We are likewise told that there are some natural Tubs or Caverns in the Rocks near *Grenoble*, which, when full of Water in the Spring, presage a good and fertile Year, and when dry a barren Year.

Those who apply themselves to the observation of the fore-running Signs of good or bad Weather, lay down the following Rules. When a thick white Dew lies upon the ground in a Winter Morning, you'll have Rain the second or third day after. When the Sun rises red or pale, it generally rains that day: When the Sun sets under a thick Cloud, you'll have Rain next Day; or, if it rains immediately, you'll have a great deal of Wind next day; which is almost always the Consequence of a pale setting Sun. A red Sky at Sun-rise is a sign of Rain; but a red Sky where the Sun sets, is a sign of fair Weather; indeed if the Sky be red at a great distance from the part where the Sun sets, as in the East, you'll have either Rain or Wind the next day. If just after Sun-set, or before Sun-rise, you observe a white Va-

pour

pour rising upon Waters, or Marshes, or Meads, you'll have fair warm Weather next day.

If a full Moon rises fair and clear, it portends a set of good Weather; a pale Moon is the fore-runner of Rain, a red Moon of Wind, a clear Silver-colour'd Moon of fine Weather; according to the Latin Verse.

Pallida Luna pluit, rubicanda flat, alba serenat.

When the Fowls pick their Feathers with their Bill, 'tis a sign of Rain. Other signs of Rain, are; When the Birds that usually pearch upon Trees fly to their Nests; When Coots and other Water-Fowls, especially Geese, keckle and cry more than usually; When the Land-Fowls repair to Water, and the Water-Fowls to Land; When the Bees do not stir (or at least not far) from their Hives; When the Sheep leap mightily, and push at one another with their Heads; When Asses shake their Ears, or are much annoyed with Flies; When Flies are very troublesom, dashing often against a Man's Face; When Flies and Fleas bite wickedly; When many Worms come out of the Ground; When Frogs croak more thau usually; When Cats rub their Head with their Fore-paws, and lick the rest of their Body with their Tongue; When Foxes and Wolves howl mightily; When Ants quit their Labour and hide themselves in the Ground; When Oxen tied together raise their Heads, and lick their Snouts; When Hogs at Play break and scatter their bottles of Hay; When Pigeons return to their Pigeon-House; When the Cock crows before his usual Hour; When Hens creep in Clusters into the Dust; When Toads are heard to croak upon Eminences; When Dolphins are often seen at Sea; When Deers fight, *&c.*

A Rainbow in the East is a sign of great Rain, especially if it be of a bright lively Colour; A Rainbow in the West presages an indifferent quantity of Rain, and Thunder; but a Rainbow in the East in an Evening, predicts fair Weather, and if its colour is lively and red, it foretells Wind.

An Iris round the Moon, is a sign of Rain with a South-Wind; an Iris round the Sun with a fair clear Air,

Air, is a sign of Rain, but in the time of Rain 'tis a sign of fair Weather.

We apprehend changes of VVeather, when the leaves of Trees move without VVind; when the Water dries more than usually, or where it did not use to dry; VVhen Spring or River VVater increases without Rain; VVhen we see an *Iris* round a Torch, a Candle or a Lamp; VVhen Fire kindles with Difficulty; VVhen the Flame instead of mounting upwards bends sideways, and the Rays reflect; VVhen salt Meat or Salt becomes moist, and when Stones sweat, that Humidity being a sign that the Air is overloaded with moist Vapours.

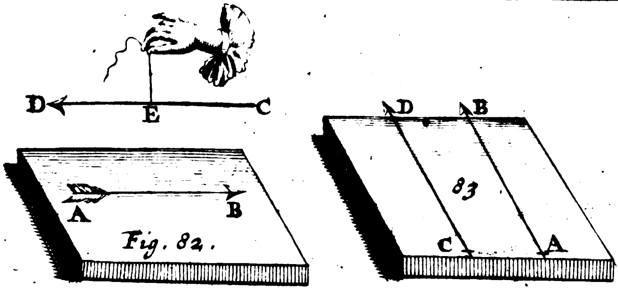
In Summer we apprehend a future Storm, when we see little black loose Clouds lower than the rest, wandring to and fro; VVhen at Sun-rise we see several Clouds gather in the VVest; and on the other hand, if these Clouds disperse, it speaks fair VVeather. VVhen the Sun looks double or triple through the Clouds, it Prognosticates a Storm of long Duration. Two or three discontinued and speckled Circles or Rings round the Moon, presage a great Storm.

PROBLEM XLVI.

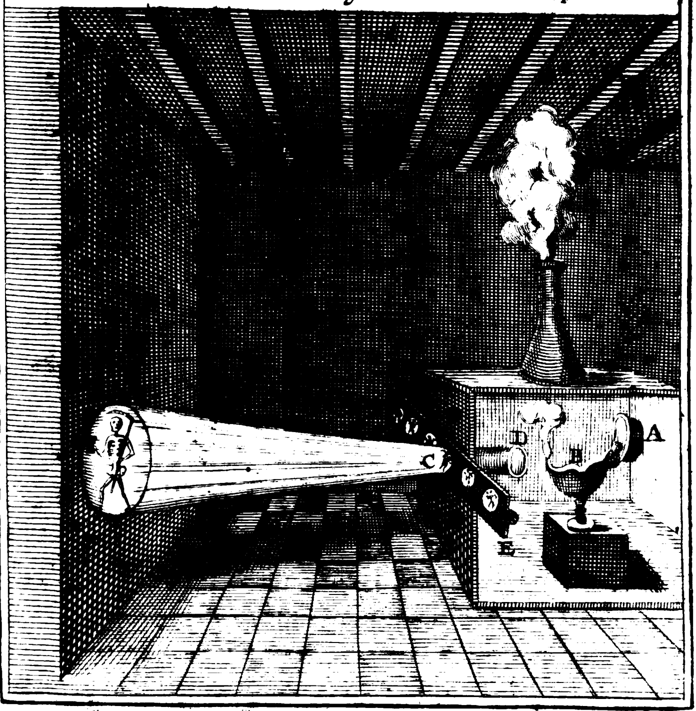
Of the Magical Lantern.

THO' I took notice already *Probl.* 27. *Opt.* of the Magical Lantern, the Invention of which is attributed to Frier *Bacon* of *England*, yet having there spoke but transiently of it, I think my self obliged to describe it a little more particularly in this place, since it has made so much noise in the VVorld of late, insomuch that some think 'twas known to *Solomon*.

This Lantern is call'd Magical, with respect to the formidable Apparitions that by vertue of Light it shews upon the white VVall of a dark Room. The Body of it is generally of white Iron, and of the Figure of a square Tower, within which towards the back part is a Concave Looking-Glass of Metal A, which



The Magical Lantern Prob. 46.



which may either be Spherical or Parabolical, and which by a Groove made in the bottom of the Lanthern, may be either advanced nearer, or put further back from the Lamp B, in which is Oil of Olives or Spirit of VVine, and of which the Match ought to be a little thick, that when 'tis lighted it may cast a good Light, that may easily reflect from the Glass A to the forepart of the Lanthern, where there's an Aperture C, with a Prospective CD in it compos'd of two Glasses that make the Rays converge and magnifie the Objects.

VVhen you mean to make use of this Machine, light the Lamp B, the light of which will be much augmented by the Looking-Glass A at a reasonable Distance; between the forepart of the Lanthern and the Prospective-Glass CD, you have a Trough made on purpose, in which you're to run a long flat thin frame EF, with several little different Figures, painted with transparent Colours upon Glass or Talk: Then, all these little Figures passing successively before the Prospective CD, thro' which passes the Light of the Lamp B, will be painted and represented with the same Colours upon the white VVall of a dark Room, in a Gigantick monstrous Figure, which the fearful ignorant People take to be the effect of Magick.

PROBLEM XLVII.

To pierce the Head of a Pullet with a Needle without killing it.

THIS is a very easie Problem, for there's a place in the middle of a Pullet's Head, that may be pierced without hurting the *Cerebellum*. But the Needle must not be kept in above a quarter of an Hour.

PROBLEM XLVIII.

To make handsom Faces appear pale and hideous in a dark Room.

BURN some Brandy and common Salt in a Glass, then put out the fire and all the Lights in the Room; and the Particles of the Salt and Brandy evaporating into the Air shut up in the Room, will make the Faces of the People in the Room appear thro' that Air hideous and frightful.

I intimated above, That, if instead of Brandy, you take good Spirit of Wine mix'd with Camphyr in a glaz'd earthen Pan put upon hot burning Coals; he that enters the Room with a lighted Candle will be agreeably surpris'd; for the Candle setting fire to the Particles of the Spirit and the Camphyr, with which the Air is replenish'd, that Air will seem to be all in a fire, and the Person will see himself in the midst of Flames without being burnt.

PRO-

PROBLEMS OF PYROTECHNY.

Pyrotechny is an Art that teaches to make Fire-works of all sorts, whether for War or for Diversion. Of the first kind, are *Grenades*, *Bombs*, *Carcasses*, *Petards*, *Mines*, and such other Machines of War fitted for the Terror and Destruction of an Enemy: Of the Latter, are *Rockets*, *Fire-Lances*, *Serpents*, and other artificial Representations of various things in Fire, which are fit for Diversion, and for Entertainment upon solemn Occasions of Joy; such as of *Suns*, *Stars*, *Rain of Gold*, *flying Dragons*, *Rocks*, *Towers*, *Pyramids*, *Arches*, *Coaches*, *Triumphal Chariots*, *Colosses* or *Gigantick Statues*, *Swords*, *Scymitars*, *Cudgels*, *Bagonets*, *Shields*, *Scutcheons*, &c. as will appear in the following Problems.

PROBLEM I.

To make Gun-Powder.

Gun-Powder, which is said to have been invented about three hundred years ago by a *German* Monk, being required to the making up of all Fire-works, 'tis necessary we should begin by shewing the Manner of its Composition, the Effects of which, when in whole Grains or Corns, are so sudden and violent, tho' when beaten small, it loses most of its Force, as Experience teaches; of which we shall not here trouble our selves to find out the Reason.

The principal Things of which Gun-Powder is made are three, *viz.* *Nitre* or *Saltpeter* which gives it the Force, *Sulphur* or *Brimstone* which makes it quickly to take fire, and *Wood-coal* Dust which unites the Composition, and qualifies the force of the Powder.

The *Saltpeter* must be very white, being well skimm'd and clarified, which is done in this Manner; first it must be boiled, with a quantity of Water sufficient to dissolve it, in a Kettle, or in a glaz'd Earthen Pot, on a Fire, slow at first, and increas'd by degrees till the Nitre is all dissolv'd, and the Liquor begins to thicken: After which some yellow Sulphur well pouder'd must be thrown in, which will immediately take Fire; this Injection being many times repeated, will consume the gross and viscous Humour of the Saltpetre, which hereby will be purified.

The *Salt-Peter* thus dissolv'd and purify'd, must be pour'd out upon a well-polish'd Marble, or upon glazed Tiles, or upon Plates of Iron or Copper, where, when cold, it becomes hard, and white as Marble: After which it must be reduc'd to a Flower or Powder, by drying it on a Coal-fire, and stirring it continually with a large Stick, till all the Humidity is exhal'd, and its become perfectly white; then more clear Water, or rather White-wine, must be pour'd upon it, sufficient to cover the Salt-Peter, which will dissolve it; and when it has acquir'd a somewhat thick Consistence, it must be perpetually stirr'd, and as quick as possible, with the same Stick, till this Moisture is also evaporated, and all is reduc'd into a very dry and white Powder, which must be afterwards pass'd thro' a very fine Silk Searce.

The *Sulphur* must also be well clarified and skimmed with a Spoon, being dissolv'd by little and little on a Coal-fire without Smoke, in an Earthen or Copper Pot: Then being taken from the Fire, it must be strain'd thro' a Linnen Cloth, into another Vessel, where it remains pure and clean, separated by the Cloth from all the gross and oyly Humour, of which it, no less then the Salt-Peter, did partake.

Some there are, who to make the *Sulphur* more active and violent, add to it, when dissolv'd as is before

before order'd, a fourth part of its Weight of *Quick-silver*, ſtirring and mixing it inceſſantly, and as faſt as poſſible with a Stick, till it be cold, and the Mercury is well united and incorporated with the Sulphur, inſomuch that all is reduced into one ſolid Body.

Others, to render the Sulphur more forcible, pure, and clean, inſtead of *Mercury* mix it with Glaſs finely powder'd, and pour upon it Brandy with ſome Powder of Allum. This is a good way to make fine Gun Powder for Piſtols, Carbines, and other ſuch Fire-Arms; but for ordinary Gun-Powder the common yellow Brimſtone is ſufficient, which makes a Noiſe when 'tis held to the Ear.

The *Coal* required in making of Gun-Powder muſt be light; for the lighter 'tis, the more thereof goes to make up the Weight, and when reduc'd to Powder it takes up moſt room, and goes the further. The lighteſt of all others is that made of pilled Hemp-ſtalks; but in my Opinion the Coal of the Willow-tree is better; or if this can't be had, we may uſe the Wood of the Hazel-tree, or that of the Lime-tree, or even that of Juniper for the ſame End. And 'tis done thus.

The Branches of the Wood you deſign to uſe, muſt be cut in *May* or in *June*, when fulleſt of Sap, of two or three Foot in length, and half an Inch thick; then with a Knife you muſt clear them of the Bark and Twigs, and tie 'em up into little Faggots, and dry them in a hot Oven; you muſt burn them afterwards in a large Pot, till they are reduc'd into live Coals, which muſt then be extinguiſh'd by covering the Pot cloſe with Earth ſomewhat moiſt, which after 24 hours may be uncover'd, and the Coal taken thence to be us'd upon occaſion when ever you have a mind to make up your Gun-Powder, which you muſt do in this manner.

Preparation of *Gun-powder*.

Having ſhewed *already* that theſe three things, *Salt-peter*, *Sulphur* and *Wood-Coal*, which we have already taught how to prepare, are required in the Compoſition of *Gun-powder*, what remains is only to determine the Proportion and Quantity of each, together with the Order and Method to be obſerv'd in mixing 'em. Wherefore, To make fine *Powder* fit for *Rockets*, you must add, to eight Pounds of good *Salt-petre* well refined, one Pound of Flower of *Sulphur*, and two Pounds of the *Coal* of *Willow-tree*.

Or, to fourteen Pounds of *Salt-petre*, add two Pounds of *Sulphur* prepar'd with *Mercury*, or in Flowers, and one Pound of *Coal* made of *Hemp-stalks*.

Or again, add to six Pounds of *Salt-petre*, one Pound of *Brimstone*, and one Pound of *Coal*.

Or, finally, to four Pounds of *Salt-petre*, add one Pound of *Sulphur*, that has been made to pass thro' a very fine Searce, and two Pounds of *Coal* taken from a Baker's Oven; and this to me seems the best of all.

If 'tis requir'd that this *Powder* should burn in Water, you must add, to one of these four Compositions, a quantity of *Quick-lime* equal to that of the *Sulphur*.

To make *Powder* fit to be us'd in Fire-Arms, and in the first place for *Cannons*, add to four Pounds of *Salt-petre*. one Pound of *Sulphur*, and one Pound of *Coal*; or else to twenty five Pounds of *Salt-petre*, add five Pounds of *Sulphur*, and six Pounds of *Coal*.

For *Musquets*, to fifty Pounds of *Salt-petre* add nine Pounds of *Sulphur*, and ten of *Coal*: Or else, to an hundred Pounds of *Salt-petre*, add fifteen Pounds of *Sulphur*, and eighteen of *Coal*.

In fine, for *Pistols*, add to an hundred Pounds of *Salt-petre*, twelve Pounds of *Sulphur*, and fifteen of *Coal*: Or to fifty Pounds of *Salt-petre*, five of *Sulphur*, and four of *Coal*.

The Proportions of the Ingredients being thus adjusted, all together must be thrown into a brazen Mortar, and with a Pestle of the same Metal well beaten, for seven or eight Hours and more, without ceasing, gently sprinkling the Mixture with Water from time to time, or rather with Urine, or with strong Vinegar, or, which is yet better, with Brandy; and if you desire a fine light Powder, use, instead of these abovesaid Liquors, the distill'd Water of Orange or Citron-peel, taking care that you moisten it not too much; and to hinder the Coal from flying away, you may dissolve a little Ising-glass in the Liquor: If 'tis required that the Grains of the

Powder

Powder be very hard after they are dryed, the Composition, towards the End, must be sprinkled with Water wherein Quick-lime has been quench'd.

The Mixture being thus sufficiently beaten and sprinkled, must be pass'd thro' a Sieve with round Holes, more or less wide, according as the Size of the Grains is desir'd; after this it must be put into a hair Searce, and shaken till all pass through but the Grains, which must be kept for use. But that which is not reduc'd into Grains, that is the Dust which passes thro' the Searce, must not be lost; for it may be dry'd in the Sun, or some hot Place, as in a Stove, and then put into the Mortar, pounded, sprinkled, pass'd thro' the Sieve, and searced, as hath just now been said, and the same Operations may be reiterated till all the Mixture is brought into Corns or Grains.

Some there are that don't bestow so much Pains in making this Powder, especially upon that for Cannons: For they judge it sufficient to put into an Earthen Pan some Salt-petre, Sulphur, and Wood-coal, in a Proportion approaching some of those formerly set down, or such an one as Experience has taught 'em to be the best, which they boil in Water over a gentle fire two or three Hours, till, the Water being consumed, the Mixture acquires some Consistence; after which they dry it, as formerly, in the Sun, or in some warm Place, and then make it to pass through a Searce of Hair, thereby to reduce it into small Grains.

PROBLEM II.

To make Gun-Powder *of any required Colour*

THE *Powder*, of which we have given the Composition in the preceding Problem, must of necessity be of a black Colour, by reason of the Coal mixed therewith; which yet is not absolutely necessary to it: For we are at liberty instead of it to use any other Matter that is easily inflammable, which will communicate its Colour to the Powder, to be made as has been taught above: But the following Proportions must be observ'd. To make *Powder* of any Colour.

White Gun-Powder. If 'tis requir'd to make *White Powder*, to six Pounds of *Salt-petre*, must be added one Pound of *Sulphur*, and one Pound of the *Pith* or Heart of *Elder* well dry'd: Or else to ten Pounds of *Salt-petre*, add one Pound of *Sulphur*, with one Pound of pilled *Hemp-stalks*.

Yellow Gun-Powder. If *Yellow Powder* is desired, add, to eight Pounds of *Salt-petre*, one Pound of *Sulphur*, with one Pound of wild *Saffron* boil'd in Brandy, and afterwards dry'd and pulveriz'd.

Blue Powder. To make *Blue Powder*, take, to eight Pounds of *Salt-petre*, one Pound of *Sulphur*, with one Pound of the Saw-dust of the *Lime-tree*, boil'd in Brandy with some blew *Indigo*, and after dry'd, and Powder'd.

Green Powder. If you would have *Green Powder*; with ten Pounds of *Salt-petre*, you must mix one Pound of *Sulphur*, and two Pounds of *rotten Wood*, boil'd in Brandy with some *Verdigrease*, and then dry'd and reduc'd to Powder.

Red Gun-Powder. Finally, *Red Powder* may be made, by adding to twelve Pounds of *Salt-petre*, two Pounds of *Sulphur*, one Pound of *Amber*, and two Pounds of *Red Sanders*: Or, to eight Pounds of *Salt-petre*, and one Pound of *Sulphur*, you may take one Pound of *Paper* dry'd and pulveriz'd, and afterwards boil'd in Water of *Cinnabar*, or of *Vermilion*, or of *Brasil-wood*, and then dry'd.

PROBLEM III.

To make Silent Powder, *or such as may be discharged without a Noise.*

Making of Silent Powder. THIS *unsounding Powder*, if any such there is, goes commonly under the Name of *White Powder*, because, possibly, the first made was of that Colour. 'Tis not probable it can be of any great Force, for as much as the Noise of Gun-powder, proceeds from the violent Percussion of the Air, occasion'd by the strength of it. I have not indeed seen this Powder, my self, yet I have read in Authors several Ways of making the same, of which the following two only occur to my Memory.

The

The first is thus: To one Pound of Common *Gun-Powder*, take half as much *Venetian Borax*, which having pulveris'd, mix'd, and well incorporated together, reduce the Mixture into Grains, as above directed, and you have the Powder required. The first Way.

The other Way is: To four Pounds of Common *Gun-Powder*, add two Pounds of *Venetian Borax*, one Pound of *Lapis Calaminaris*, and one Pound of *Sal-Armoniack*; pulverize 'em all together, to make of 'em a Powder in Grains, as before. The second Way.

PROBLEM IV.

To know the Defects of Gun-Powder.

THE Defects of *Gun-Powder* may be known several Ways: as first, *by the Sight*, when 'tis too black; for then it has too much of the *Wood-coal*, as you may perceive if you put some of it upon white Paper, which it will blacken: Now too much of the Coal renders it moist, and the Moisture dissolves the Salt-petre, separates it from the other two Parts of the Mixture, and so lessens its Force. The Powder that is good, shou'd be of a dark Ash-colour, inclining somewhat towards a Red.

Secondly, *by the Touch*; if you rub some Grains of it with the end of your Finger upon a well-polish'd Table, and they are easily reduc'd into Dust, 'tis a sign that the Proportion of the Coal therein is more than enough: And if the Grains don't crumble with equal Facility, some of them being so hard that they prick the Finger, 'tis an evidence that the Sulphur is not well imbodied with the Salt-petre, and the Powder therefore not duly prepared.

Thirdly, the Faults of Gun-Powder may be perceived by means of *the Fire*: For if when 'tis fired upon a smooth Board, it blackens it much, 'tis a token there is too much Coal in it; and if upon that Board or Table there remains only some black Mark, it appears thereby that much of the Coal has not been well burnt: And, in fine, if the Board remains as it were greasy, this discovers that the Sulphur and the Salt-petre have not been sufficiently purified;

rified; that is freed of that oily, greasy and viscous Humour, which is ever hurtful and superfluous.

'Tis likewise a sign that the Salt-petre has not been sufficiently refined, that is, separated from that gross terrestrial Matter which is prejudicial in the Composition, and that the Sulphur has not been beaten enough, nor well incorporated with the other Parts, when there appear in the Powder small Grains, white, or of a Citron-colour.

The good or bad Qualtity of Gun-powder may also be thus discerned by means of Fire, if you lay several little Heaps thereof upon a clean and well-polish'd Board, at the distance of four or five Inches from one another: For when 'tis well prepar'd, if you put fire to one of these Parcels, the Powder will take fire of a Sudden, and it will burn by it self with a little Crack, the clear white Smoak arising all at once like a Circle in form of a Crown.

PROBLEM V.

To amend the Defects of Gun-Powder, *and to restore it when decay'd.*

IF *Gun-powder* has not been well prepared, or, if, being kept in a moist Place, or being too old, 'tis altered, weaken'd, or spoiled, degenerating thus from its first Vigour, it may be recovered in the following Manner.

Take a quantity of good *Gun-powder* equal in bulk to that which you would amend or restore; that will be much heavier than this: To this last therefore a quantity of well clarified *Salt-petre* must be added, sufficient to make it of the same Weight with the former, which being beaten together in the usual Manner, must be reduc'd into Grains, as was elsewhere taught, which will be a very good Powder, that must be kept in some Wooden Box or Vessel, untill there's occasion to use it.

When the Powder is but a little altered, it will be sufficient to mix some of it with an equal quantity of good Powder newly prepar'd, upon a Table or a Cloth,

Cloth, with the Hand or a Wooden Shovel, and then to dry it in the Sun.

PROBLEM VI.

To prepare an Oyl of Sulphur, *required in Fire-works.*

HAving melted what quantity of *Sulphur* you think fit, upon a moderate Fire, in an Earthen, or Copper Veſſel, throw into it ſome old, or in defect of this ſome new Brick, that is well burnt, and was never wetted, broken into many ſmall pieces about the bigneſs of a Bean; ſtir them continually with a ſtick, till they have drunk up and conſum'd all the Sulphur; this done ſet them upon a Furnace to diſtil in an Alembick; ſo you ſhall have a very inflammable Oyl, fit for your purpoſe.

You may make it otherways thus: Fill one third or fourth part of a Glaſs-bottle with a long Neck with *Sulphur* pulveris'd; then pouring upon it Spirit of *Turpentine*, or Oyl of Walnuts, or of Juniper, till the Bottle is half full, ſet it upon hot Cinders, leaving it there eight or nine Hours; and you ſhall find an Oyl therein of the above-ſaid Qualtity.

PROBLEM VII.

To prepare the Oyl of Salt-petre *uſeful in Fireworks.*

PUT, upon a Fir-board well plain'd, and dry, what quantity of purify'd *Salt-petre* you pleaſe, and cauſe it to melt by putting thereupon burning Coals; and you ſhall ſee the Liquor to paſs thro' the Board, and to fall down Drop by Drop, which muſt be received in an Earthen or Copper Pot, where you have an Oyl of Salt-petre, fit to be uſed in Fire-works, as we ſhall declare in its proper Place.

PROBLEM VIII.

To prepare the Oyl of Sulphur and Salt-petre *mix'd together.*

HAving mix'd and well incorporated equal Portions of *Sulphur* and *Salt-petre*, reduce all into a fine Powder, which must be pass'd thro' a fine Searce: Put this Powder thus searced into a new Earthen Pot, or one that hath not been used, and pour upon it good White-wine Venegar, or else Brandy, till 'tis covered. Then cover your Pot so that no Air may get into it, and set it to stand in some hot Place, till all the Vinegar is consumed or disappears. Last of all, draw from the remaining Matter the Oyl by means of an Alembick, which will serve to several Purposes of Pyrotechny.

PROBLEM IX.

To make Moulds, Rowlers, *and* Rammers *for* Rockets of all sorts.

A *Rocket*, which the *French* call *Fusee*; the *Latins Rocheta*; and the *Greeks*, *Pyrobolos*, consists of a *Cartouch* or Paper-tube call'd the *Coffin*, and a combustible *Composition*, with which 'tis loaded; which being fired, mounts into the Air, in a manner most agreeable to behold.

There are three sorts of 'em; the *Small*, the *Middling*, and the *Great*. All such are reckon'd *small*, whereof the Diameters don't exceed that of a Lead-bullet of one Pound, or whose Moulds admit not a Bullet above that Weight. The *Middling*, are those the Moulds of which will admit Bullets from one to three Pound-weight. The *Great* will carry from a three Pound to an hundred pound Ball.

To determine the Bigness of these *Coffins* to a required Measure, that is Length and Thickness, and to make any demanded Number of 'em, of the same Reach, and of equal Force, they must be fitted to a concave Cylinder, made of some hard Matter, and turn'd turn'd exactly in a Lath: This is called the *Mould* or *Form*, which is sometimes made of Metal, but most commonly of hard Wood, such as Box, Juniper, Ash, Cypress, wild Plum-tree, *Italian* Walnut-tree, and such like.

Besides this, there is another, but a convex and solid Cylinder of Wood required, call'd a *Rowler*, upon which the thick Paper, whereof the Coffin is made, must be rowled, till 'tis of a bigness exactly to fill the Concavity of the Mould. This Rowler is here represented by the Letter B, and its Diameter must contain five eight Parts of that of the Mould A, the Length of which must be six times the Diameter of its Bore, in small Rockets; but in the Middling and the large ones, it must be only five, or four times the length of the Diameter of their Bore. Plate 23. Fig. 66. Pag. 391.

Another Cylinder of Wood must also be had, which is to be a little smaller than the former, that it may go into the Coffin with the greater ease. And this is to serve for a *Rammer*, as C, to drive down the *Composition* into the *Coffin* when you charge it. But first your *Coffin* must be straitned or choaked; which is done by winding a Cord about the end of it, after you have a little withdrawn the *Rowler*, turning in the mean time the *Coffin*, and drawing the Cord, till there remains only a little Hole, which then must be ty'd with strong Pack-thread. This done you must draw out the *Rowler*, and introducing the *Rammer* into the *Coffin*, put all into the *Mould*; and when you have struck five or six blows with a Mallet upon the *Rammer*, to give a good form to the Neck of the *Rocket*, the *Coffin* is finished, and ready to be filled upon Occasion.

This *Rammer* C, must be bored lengthwise to some depth, that it may receive into its Concavity the *Needle* DE, which must be in the *Mould* A, together with the *Coffin* and *Rammer*. The use of this Needle, which must be one third Part of the length of the *Coffin* or *Mould* is to make a vent for the Priming in the bottom of the *Composition*, of which we speak in the ensuing Problem.

PRO-

PROBLEM X.

To prepare a Composition *for Rockets of any size.*

THE *Composition* wherewith the Coffins are to be fill'd is different, according to the different bigness of 'em; for 'tis found by Experience, that what is fit for small Rockets, burns too violently, and too quickly in those that are large, because the Fire is bigger, and the Matter also driven closer together: Hence it is that no Gun-powder is us'd in the larger sort. In making up this Composition, according to the differing sizes of Rockets, the following Proportions must be observed.

For *Rockets* from 60 to 100 Pounds, you must to three Pounds of *Salt-petre*, add one Pound of *Sulphur*, and two Pounds of good *Wood-coal.*

If they are from 30 to 50 Pounds, to thirty Pounds of *Salt-petre*, put seven Pounds of *Sulphur*, and sixteen Pounds of *Coal.*

Rockets from 18 to 20 Pounds, to twenty one Pounds of *Salt-Petre*, require six of *Sulphur*, and thirteen of *Coal.*

From 12 to 15 Pounds, require to four Pounds of *Salt-petre* one Pound of *Sulphur*, and two Pounds of *Coal.*

If they be from 9 to 12 Pounds; to sixty two Pounds of *Salt-petre*, add nine Pounds of *Sulphur*, and twenty of *Coal.*

From 6 to 9 Pounds; add to seven Pounds of Salt-petre, one of Sulphur, and two of Coal.

From 4 to 5 Pounds; to eight Pounds of Salt-petre, add one Pound of Sulphur, and two of Coal.

From 2 to 3 Pounds; to sixty Pounds of Salt-petre, add two of Sulphur, and fifteen of Coal.

For one Pound; to sixteen Pounds of Gun-powder, add one Pound of Sulphur, and three of Coal: Or to nine Pounds of Powder, four of Salt-petre, one of Sulphur, and two of Coal.

For twelve Ounces; put to nine Pounds of Powder, four of Salt-petre, one of Sulphur, and two of Coal.

For 8 Ounces; add to thirty Pounds of Powder, twenty four of Salt-petre, three of Sulphur, and eight of Coal.

For 5 and 6 Ounces; to thirty Pounds of Powder, add twenty four Pounds of Salt-petre, three Pounds of Sulphur, and eight Pounds of Coal.

For 4 Ounces; add to twenty four Pounds of Powder, four Pounds of Salt-petre, two Pounds of Sulphur, and three Pounds of Coal.

For 2 and 3 Ounces; to twenty four Pounds of Powder, put four Pounds of Salt-petre, one Pound of Sulphur, and three Pounds of Coal.

For an half Ounce, and an Ounce; take fifteen pounds of Powder, and two pounds of Coal.

For the smaller Rockets; to nine or ten pounds of Powder, add one pound, or one and a half of Coal.

Here follow also other Proportions, which Experience hath taught to succeed extremely well.

For Rockets that contain one or two Ounces of Matter. Add to one pound of Gun-powder, two Ounces of good Coal: Or, to one pound of Musquet-Powder, take one pound of course Cannon-powder: Or, to nine Ounces of Musquet-powder, put two Ounces of Coal: Or to one Ounce of Powder, an Ounce and a half of Salt-petre, with as much Coal.

For Rockets of two or three Ounces; add to four Ounces of Powder, one Ounce of Coal: Or to nine Ounces of Powder, two Ounces of Salt-petre.

For a Rocket of four Ounces; add to four pounds of Powder, one pound of Salt-petre, and four Ounces of Coal, and if you please half an Ounce of Sulphur: Or to one pound two Ounces and an half of Powder, four Ounces of Sulphur, and two Ounces of Coal: Or to one pound of Powder, four Ounces of Salt-petre, and one Ounce of Coal; or to seven Ounces of Powder, four Ounces of Salt-petre, and as much Coal: Or, add to three Ounces and an half of Powder, ten Ounces of Salt-petre, and three Ounces and an half of Coal. The Composition will be yet more strong, if it be made up of ten Ounces of Powder, three Ounces and an half of Salt-petre, and three Ounces of Coal.

For

For Rockets of five or six Ounces; take two pounds five Ounces of Powder, to half a pound of Salt-petre, two Ounces of Sulphur, six Ounces of Coal, and two Ounces of Filings of Iron.

For Rockets of seven or eight Ounces; add to seventeen Ounces of Powder, four Ounces of Salt-petre, and three Ounces of Sulphur.

For Rockets from eight to ten Ounces; to two pounds five Ounces of Powder, put half a pound of Salt-petre, two Ounces of Sulphur, seven Ounces of Coal, and three Ounces of Filings.

For Rockets from ten, to twelve Ounces; take to seventeen Ounces of Powder, four Ounces of Salt-petre, three Ounces and an half of Sulphur, and one Ounce of Coal.

For Rockets from fourteen to fifteen Ounces, to two pounds four Ounces of Powder must be added, nine Ounces of Salt-petre, three Ounces of Sulphur, five Ounces of Coal, and three Ounces of File-dust.

For Rockets of one Pound, to one pound of Powder, take one Ounce of Sulphur, and three Ounces of Coal.

For a Rocket of two Pounds, add to one pound four Ounces of Powder, twelve Ounces of Salt-petre, one Ounce of Sulphur, three Ounces of Coal, and two Ounces of File-dust of Iron.

For a Rocket of three Pounds, to thirty Ounces of Salt-petre, put seven Ounces and an half of Sulphur, and eleven Ounces of Coal.

For Rockets of four, five, six, or seven Pounds, add to thirty one pounds of Salt-petre, four pounds and an half of Sulphur, and ten pounds of Coal.

For Rockets of eight, nine, or ten Pounds, take to eight pounds of Salt-petre, one pound four Ounces of Sulphur, and two pounds twelve Ounces of Coal.

The Proportion of the different Materials being thus determined, each of 'em must be well beaten, and searc'd apart, and afterward weigh'd and mix'd. Thus is your *Composition* ready wherewithal to charge your *Coffins*, which must be made of strong Paper well pasted.

PRO-

PROBLEM XI.

To make a Rocket.

YOUR *Coffins* and different *Compositions* being in readineſs, You muſt chuſe a Compoſition ſuitable to the largeneſs of your deſign'd *Rocket*, which muſt neither be too wet nor too dry, but a little moiſtened with ſome oyly Liquor, or with Brandy; then take your Coffin, the length of which muſt be proportion'd to the bigneſs of its Concavity; put it, with the Rammer C, into the Mould A; then put into it ſome of your Compoſition, taking good care not to put in too much at a time, but only one Spoonful or two; then put in your Rammer, and with a Mallet ſuited to the bigneſs of the Coffin, ſtrike three or four ſmart Blows directly upon it; then withdraw the Rammer again, and pour in an equal quantity of your Compoſition, and drive it down in like manner with your Rammer and Mallet, giving the ſame number of Blows; continue thus doing till the Coffin is fill'd to the height of the Mould, or rather a little below it, that five or ſix Folds of the Paper may be doubled down upon the Compoſition thus driven into the Coffin, which ſometimes inſtead of Paper is made of Wood. Plate 23. Fig. 66.

The Coffin being filled with the Mixture, and the Paper doubled down upon it, you muſt beat it hard with the Rammer and Mallet to preſs down the Folds of the Paper, upon which you may put ſome Corn-powder, that it may give a Report. In this Paper folded down, you muſt make three or four Holes as you ſee in A, with a Bodkin FG, which muſt penetrate to the Compoſition, to ſet fire to the *Stars*, *Serpents*, and Ground *Rockets*, when ſuch there are; otherwiſe it will ſuffice to make one Hole only, with a Broach or Bodkin, which muſt be neither too ſmall nor too great, but about one fourth of the Diameter of the Bore, as ſtraight as poſſible, and in the very middle, in order to fire the Corn-powder. Fig. 67. Fig. 66.

PROBLEM XII.

To make Sky-Rockets, *that mount into the Air with Sticks.*

Plate 23. Fig. 67. 'TIS to be noted, that the *Head of a Rocket*, is the highest end A, by which 'tis loaded, and which rises first when 'tis fired: The *Neck of the Rocket*, or its *Tail*, is the lower end B, where it was choak'd or straitned, and the Priming is put, which must be of good Corn-powder.

Fig. 68. Your *Rocket* being charg'd, as was taught in the preceding Problem, you must have a long Rod or Stick, as AB, of some light Wood, such as Osier or Fir, which must be bigger and flat at one end growing slenderer towards the other. This Stick must be straight and smooth, without Knots, and plained if need be. Its Length and Weight must be proportioned to the Size of the Rocket, being six, seven, or eight times the Length of it; to the larger End of this where 'tis flatted, you must tie your Rocket, its Head reaching a little beyond the end of the Stick, as you see in Fig. 68. and being thus fix'd, lay it upon your Finger two or three Inches from the Neck of the Rocket, which should then be exactly ballanced by the Stick, if 'tis rightly fitted; after which you have nothing to do, but to hang it loosly, upon two Nails, perpendicular to the Horizon, with its Head up, and then 'tis ready for Firing. But if you would have it to rise very high, and in a straight Line, you must put a pointed Paper Cap, such as C, upon its Head, and it will pierce the Air with greater Facility.

To these Rockets, for the greater Diversion of the Spectators, several other things may be added: as *Petards* or *Crackers*, thus; get a Box of Iron solder'd, of a convenient bigness, fill it with fine Grain-powder; put it into the Coffin upon the Composition, with the Touch-hole down, double the rest of the Paper upon it to hold it fast till the Mixture is consum'd, and then firing it will give a Report in the Air.

You

You may add to them likewise, *Stars*, *Golden-rain*, *Serpents*, *Fire-links*, and other such agreeable Works, the making of which shall be taught afterwards. In order to this, you must have in readiness an empty Coffin, of a larger Diameter than your Rocket. This must be choaked at one end, so as only to admit the Head of the Rocket, to which it must be fastned. Into this large Coffin, having first strewed the bottom of it with Meal-powder, you must put your Serpents, or Golden-rain, or Fire-links, with the prim'd end downwards; and amongst, and over your Stars you must throw a little Powder. Then you may cover this additional Coffin with a piece of Paper, and fit to it a pointed Cap as before, to facilitate its Ascension.

PROBLEM XIII.

To make Sky-Rockets *which rise into the Air without a Stick.*

SKy-Rockets without Sticks must be small, because they are held in the Hand, from whence they rise, after you have put fire to the Priming. They are made as the foregoing; but that they may the better fly into the Air, you must fit to 'em four *Wings* disposed Cross-wise, like the Feathers of Darts or Arrows, as A, A; their Length must be one third part of that of the Rocket, their Breadth at the lower part half their Length, and their Thickness about a sixth or eighth part of the Diameter of the Orifice of the Rocket. Plate 23. Fig. 69.

Instead of four of these Wings, you may use three of the same Dimensions with equal Success; but with this Caution, that in placing them upon your Rocket, the lower ends of 'em must be let down below the Tail of it the length of one Diameter of its Orifice. There are many other ways of making these Rockets, according to the various Fancies of Artists, which would be too tedious for this Work.

If the *Composition* for your *Rockets* is defective, as is known when they rise, either not at all, or with difficulty, or fall down again before consumption of Remark.

 the

the Mixture; or when they mount not with an equal and upright Motion, but turning and winding, or whirling in the Air; to amend your Composition, you must diminish the Quantity of Coal when 'tis too weak, and add to it if too strong, as it is when it bursts the Rocket, the Coal serving to abate the force of the Powder, and to give a fine Train to your Rocket. Wherefore it wou'd be convenient, before you make up a Quantity of Rockets, to try your Mixture and correct its Faults.

To preserve your Rockets in good Condition, they must be kept in a Place, neither too dry, nor too moist, but temperate; and the Composition should not be made up, but upon occasion to use it. Your Rocket must not be pierc'd, till you design to play it; which must not be in a Season of Wind or Rain, or when the Nights are moist with Fogs and Mists, all which are prejudicial to the agreeable Effects of a Rocket.

If you would have your Rocket to burn with a pale white Flame, mix some *Camphire* with your Composition; instead of which if you take Raspings of *Ivory*, the Flame will be of a clear Silver-colour, but somewhat inclining to that of Lead; if *Colophony* or Grecian-pitch, 'twill be of a reddish Copper-colour; if black or common *Pitch*, the Flame will be dark and gloomy; if *Sulphur*, it will be blue; if *Sal-armoniack*, it will appear greenish; if crude *Antimony*, or the Raspings of yellow *Amber*, it will emit Flames of a like Colour.

PROBLEM XIV.

To make Ground-rockets, *which run upon the Earth.*

ROckets that run along the Ground, call'd therefore *Ground-rockets*, require not so strong a Composition, as those that mount into the Air; and therefore continue longer, burning as well as moving more slowly: Wherefore they vary from the others, as well in the Demensions of their Coffins, as in the Composition wherewith these are charg'd. The length of the Bore or Concavity, may be eleven times that of its Diameter;

Diameter; the Rowler on which the Coffin is made, may be five Lines in Diameter, and the Rammer a little leſs, that it may go eaſily into the Coffin without ſpoiling it.

The *Compoſition* may be of Cannon-Powder only, well beaten and ſearc'd till 'tis as fine as Flower, wherewith you muſt fill the Coffin, by little and little, as before, within a Finger's breadth of the Brim of the Mould; then doubling down one third part of the Paper, knock it down with the Rammer and Mallet, and after, with a Bodkin, make a ſmall Hole which may penetrate to the Compoſition; then put in a Piſtol-charge of fine Powder, doubling down ſome more of the Paper upon it, the reſt of which muſt be choak'd tying it hard with Pack-thread, as you ſee in A. Plate 23. Fig. 70.

These Rockets being ſmall are charg'd only with Powder finely pulveriz'd, without any Coal, herein differing from the large ones, that have no Powder at all, except in their Priming, which in both ſorts muſt be of well grained Powder: The Reaſon of which is, becauſe in a greater Concavity there is a greater Fire acting upon a greater Quantity of Matter, and conſequently with more Violence; there being alſo a greater Quantity of Air to be rarified in a great than in a ſmall Rocket. Remark.

When you choak or ſtraiten the End of your Rocket, whether ſmall or great, you muſt have a Hook or Staple driven into a Poſt or into a Wall, to this tie one end of your Cord, which muſt be of a ſize proportionable to your Rocket, or to the Bar of a Window, and the other to a ſtrong Stick, which you muſt put between your Legs: Thus the Cord being winded about your Rocket in the deſign'd place, you may draw, turning, and ſtraitning it by Degrees as you deſire.

PROBLEM XV.

To make Rockets *that fly on a Line, call'd* Air-Rockets.

THIS is done with ordinary Rockets, that must not be too big, by fastning to 'em two Iron Rings, or, which in my Opinion is better, a wooden Pipe or Cane, thro' which must pass a well-streched Line: Thus if you set Fire to your Rocket, 'twill run along the Line without ceasing till all the Matter is spent.

If you would have your Rocket to run back, as well as forward, after you have fill'd one half of the Coffin with the Composition, separate this from the empty half by a Wheel of Wood fitted exactly to the Cavity; in the middle of this Wheel must be a Hole, from which a small Pipe, fill'd with Meal-Powder, must pass along the middle of the empty half, which then must be fill'd with the Composition; and so after the first half of the Rocket is consum'd, the Fire being communicated by the little Pipe, will light it at the other Extremity, and so drive it back to the Place from whence it came.

The same thing may be effected by means of two Rockets ty'd together, the Tail of the one to the Head of the other, one of which being burnt to the End fires the other, making it to run back: But least the second should catch fire at the Head, it must be defended with a Cover of Paper or wax'd Cloth.

Remark. This sort of Rockets is commonly us'd to set fire to other Machines in Fire-works for Diversion, to which, for the greater Pleasure, they give the Figures of several Animals, such as Serpents or Dragons, which then are call'd *Flying Dragons*; and are extremely agreeable, chiefly when fill'd with several other Works, as Golden Rain, Hairs dipt in Wild-fire, Small-nut Shells fill'd with the Rocket Composition, and many other diverting things, of which afterwards.

PRO-

PROBLEM XVI.

To make Rockets that burn in the Water, call'd Water-Rockets.

'THO' the Fire and Water are oppoſite Elements, mutually deſtroying one another; yet the Rockets we have hitherto deſcrib'd, being once lighted will continue to burn even in the Water, and will have their full Effect; but for as much as 'tis done under Water, we are depriv'd of the Pleaſure of beholding it. In order, therefore, to make them to ſwim upon the Water, we muſt alter ſomewhat the Proportions of their Mould, as well as the Materials of their Compoſition.

The *Mould*, then, requir'd to ſuch Rockets, may be eight Inches in Length, and its Bore an Inch over. The *Rowler* muſt be of nine Lines Diameter, and the *Rammer* not quite ſo thick: No Needle is required to this Mould.

The *Compoſition*, if you would have your Rocket burn on the Water with a clear Flame like a Candle, muſt be made of three Ounces of Powder beaten and ſearc'd, one Pound of Salt-petre, and eight Ounces of Sulphur mix'd together: When you deſire your Rocket to appear on the Water with a fine Tail, you muſt, to eight Ounces of common Powder, add one Pound of Salt-petre, eight Ounces of Sulphur, and two Ounces of Coal.

The Compoſition being prepar'd, and the Coffin charg'd with it, as is taught above, put a *Fire-Link* at the end of it; and covering your Rocket with Wax, Pitch, or Roſin, to preſerve the Paper from the Water, faſten to it a ſtick of white Willow about two Foot long, which will cauſe it to ſwim upon the Water.

Many other different ways may ſuch Rockets be made without altering either the Mould or Compoſition, for which the curious may conſult the Authors that have writ particular Treatiſes of Pyrotechny.

A Rocket also may be made, which, after burning some time in the Water, will throw up into the Air Sparkles and Stars; which is done by dividing the Rocket into two parts with a wooden Wheel having a Hole in the Middle, one Partition being fill'd with the common Composition, the other with Stars, having some Powder strew'd amongst 'em.

Plate 23. Fig. 71. Moreover you may contrive a Rocket, which, having burnt one half of its time in the Water, will mount up into the Air with great Swiftness; thus: Having fill'd two equal Coffins with good Composition, paste 'em together slightly only at the Middle A, the Head of the one answering the Tail of the other; betwixt them must pass a little Pipe at the Extremity *B*, to light the other when one is consum'd. Then fasten the Rocket D, to which the other is joyn'd, to a stick of such Length and Bigness as is requir'd for ballancing it, and to the lower end of the Rocket C, tie a Pack-thread at F, to which you must fasten a large Musquet-Ball that must hang upon the stick at E by means of a bent Wire. This done set fire to C, your Rocket being in the Water; and its Composition being consum'd to B, will light, by means of the little Pipe, the other Rocket, which will mount into the Air, through the strength of the Fire, the first being kept down by the Weight it sustains.

PROBLEM XVII.

To make Fire-Links.

A *Fire-Link*, so call'd from its resemblance to the Links of a Saucidge, is a kind of Rocket, that is usually tied to the end of a bigger one, to render the Effect more agreeable. I said *usually*, because there are some of 'em made that fly into the Air as Sky-rockets, and are call'd *Flying Fire-Links*, to distinguish 'em from the others which are nam'd *fixed Fire-Links*. We shall here briefly teach the Making of both Sorts.

And first the *fixed* kind to be fastned to a Rocket is made thus: Take a Coffin of what Bigness you think fit, and having choak'd it at the End, fill it with

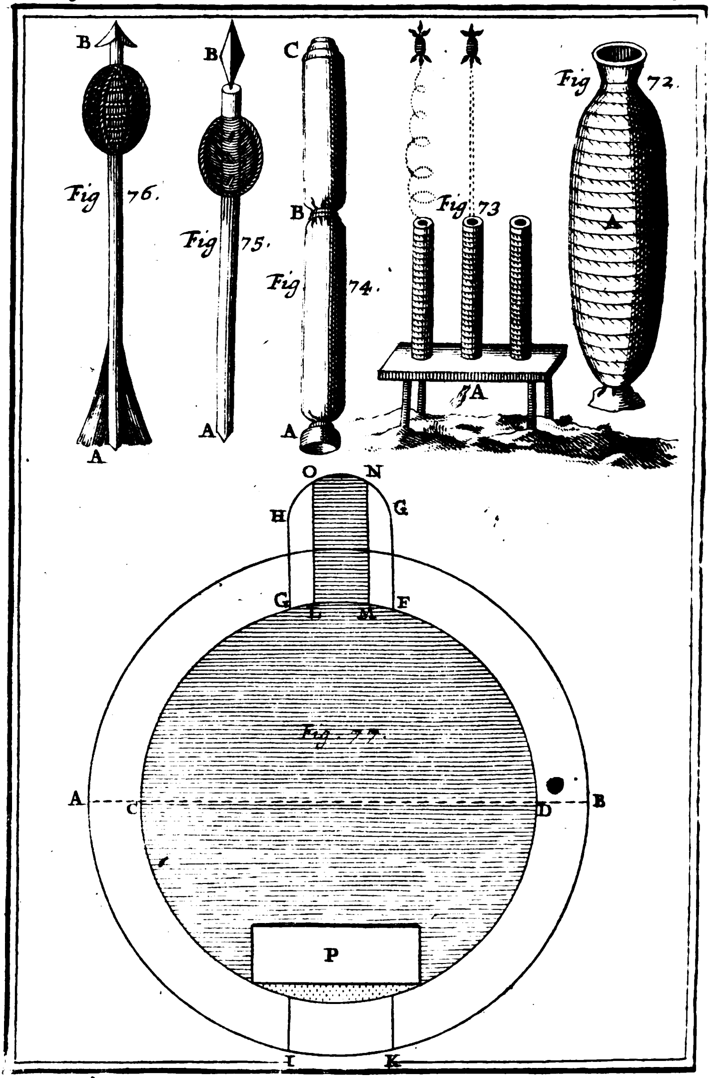


Fig 76.
Fig 75.
Fig 74.
Fig 73
Fig 72.
Fig. 77.
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with fine Powder, and choak it at the other End: Then roll it strongly with small Cord from one End to the other, as you see in A, gluing the Cord with good Glue, to keep it fast, and to strengthen the Coffin, that it may give the greater Noise when it breaks: Thus is your Fire-Link ready to be fasten'd to the end of a Rocket either with Paper, Parchment, or Cord, or otherwise; but note, that you must pierce the End of your Fire-Link, which joyns to the Rocket, and prime it with Corn-powder. Plate 24. Fig. 72.

To make *flying Fire-Links*, you must have such Coffins as for the former, only they must be a little longer, and having choak'd 'em at one End, charge them with Corn-powder, adding at last Meal-powder to the thickness of one Inch, driving all down, as in Sky-rockets, with a Mallet. Then strengthen the Coffin with Line, as in the former, after you have choak'd the other End, leaving a Hole about the bigness of a Goose-quill, to which you must put a little moistned Powder for Priming.

Or, having choak'd at one End, and charg'd your Coffin within one Inch of the other End, choak it there, leaving only a small Hole, which if quite shut up, or too small, must be open'd with a Bodkin; then fill up your empty space with Powder finely flowered, or with the Composition for Sky-rockets, which must be driven close with a Rammer and Mallet, doubling down the remaining Paper, if any, upon your Composition, which will give a fine Tail to your *Link*; and when you have made a Hole in the Middle of this last Paper, and prim'd it, your flying Link is ready to be thrown into the Air, which is done thus.

You must provide Guns or Cannons with a Vent at Bottom, where there must be a Tail somewhat long, which must pass through a Piece of Wood, such as A, that it may reach to a Fire-conveyance running along underneath, to set Fire to the Cannons one after another, which will also throw up into the Air the Links with a Noise in the same Order. Fig. 73.

PRO-

PROBLEM XVIII.

To make Serpents *for artificial Fire-works.*

SErpents are ſmall Sky-rockets, which inſtead of Mounting ſtraight upwards, riſe obliquely, and deſcend with ſeveral Turnings and Windings. The *Compoſition* for them may be much the ſame with that for Sky-rockets; or that for Ground-rockets, if you deſire their Motions to be more brisk. The Conſtruction and Proportions of their Coffin are as follows.

Plate 24. Fig. 74. The Length AC of the Coffin may be about four Inches, and it muſt be rowled on a Rowler ſomewhat bigger than a Gooſe-quill: This done you muſt choak it at one End, as at A, and filling it with Compoſition a little beyond the Middle, as to B, choak it there alſo, leaving a little Hole; the reſt you muſt fill with Corn-powder, to make a Report when it breaks, choaking it quite at the other Extremity C. The Extremity A muſt be prim'd with ſome moiſtned Powder, by which when you have fired the Compoſition in the Part AB, the Serpent will riſe into the Air, and afterwards coming down, will make ſeveral Turnings and Windings, 'till the Grain-Powder being fired, it breaks in the Air with a Bounce before it fall.

If it be made up without choaking it towards the Middle, inſtead of Turnings and Windings, it will have a waving Motion riſing and falling, till it breaks as above.

PROBLEM XIX.

To make Fire-Lances.

LAnces of Fire, are long and thick Pipes or Cannons of Wood, with Handles at the End, whereby they are made faſt to Stakes or Poſts, well fixed that may ſuſtain the force of the Fire, having ſeveral Holes to contain Rockets or Petards. They are us'd in feſtival

festival Fire-works that represent nocturnal Fights, as well for throwing Rockets, as making Vollies of Reports.

You must use 'em thus: Put a Rocket into every Hole, and fill the Bore of the Cannon with Composition, which fired will, as it consumes, fire the Rockets one after another, and throw them up into the Air. But if you would have many thrown up at once, cover the Bottom of the Lance with Composition, and thereupon place a long small Pipe fill'd with the same Composition, about which put your Rockets, 'till you have fill'd your Cannon, the prim'd End being downwards, that so firing the Composition in the Pipe, this may light that at the Bottom of the Lance, which firing the Rockets, they will mount all at once into the Air.

There may be many other ways of contriving *Fire-Lances* in imitation of this, of which I shall not speak: I shall only mention one other sort of these *Lances*. This consists of a Coffin made of strong Paper well glued, which may be of what Dimensions you think fit, according as 'tis design'd to give more or less Light; this must be fill'd with the Star Composition, (of which in *Prob.* 22.) pulveriz'd, and prim'd with Meal-powder moistned: The lower End must be stopp'd with a round piece of Wood, which must appear two Inches without the Coffin, that thereby it may be fastned at Pleasure.

The Name of *fiery* or *burning Lances*, and *Pikes*, is also given to a kind of Pikes, like a Javelin or Dart, with a strong Iron pointed Head, as AB, call'd by the *Latins*, *Phalarica*, and *Dardi di Fuoco* by the *Italians*, which were formerly thrown, being first fired, against the Enemies, either by the Hand, or from Engines, being cover'd between the Iron and Wood with Tow dipt in Sulphur, Rosin, *Jews* Pitch, and boiling Oyl; where they lighted they stuck, setting on fire whatever was inflammable. Remark. Plate 24. Fig. 75.

This sort of *Lances* is not now in use, but instead of them we have *Burning Arrows*, that are no less terrible, tho' not much now in Esteem: However we will here gratify the Curious with a brief Description of them. *Flaming Arrows*, are artificial Firebrands thrown amongst the Enemies Works, to reduce them

to

Plate 24. Fig. 76. to Ashes; they are made thus: Prepare a little Bag of strong course Cloth, about the bigness of a Goose's or a Swan's Egg, such as C, of a globular or sphæroidal Figure, which must be filled with a Composition made of four Pounds of beaten Powder, as much refin'd Salt-petre, two Pounds of Sulphur, and one Pound of *Græcian* Pitch: Or you may make it of two pounds of Meal-powder, eight pounds of Salt-petre refined, two pounds of Sulphur, one pound of Camphire, and one pound of Colophony: Or yet more simply thus; of three pounds of Powder, four pounds of Salt-petre, and two pounds of Sulphur. With one of these Mixtures fill the Bag, pressing it hard, and make an Hole through the Middle of it lengthwise, to receive an Arrow, like those of the ordinary Bows or Cross-bows, such as AB, the Head
Fig. 76. of it remaining without the Bag, which must be fastned so as it may not move, or slide towards the Feathers. This done, roll your Bag with strong Pack-thread as thick as possible from one End to another, and then cover it all over with Meal-powder mix'd with melted Pitch. Thus it is ready to be shot out of a Bow or Cross-bow, after it is fir'd by two little Holes made for that purpose near the Head of your Arrow.

PROBLEM XX.

To make Fire-Poles or *Perches.*

Fiery Poles or *Perches* properly speaking are what We have call'd *Fiery Lances*, of which We have spoken in the preceding Problem; which might supersede any further Labour about 'em, but that We design here to shew another way of making 'em.

You must have a Pole of some light and dry Wood ten or twelve Foot in Length, and two Inches in Thickness, in one of the Ends whereof you must make three or four Grooves or Gutters opposite to one another, two or three Foot long; In some of these put Rockets, fill'd with a Composition made of five Ounces of Powder, three Ounces of Salt-petre, one Ounce of Sulphur, and two Ounces of Coal;

Coal; in others put Petards or Crackers of Paper, which must communicate with the Rockets by Holes passing between: And last of all cover your Artifice over neatly with Paper, the better to deceive the Eyes of Spectators.

PROBLEM XXI.

To make Petards *for Fire-works of Diversion.*

PEtards or *Crackers*, for Fire-works of Pleasure, are made of Paper, or thin Pieces of Metal, as Copper, Iron, or Lead. Those of Paper have their particular Moulds, and are made as is directed in *Probl.* 11. Their Coffins are charged towards the Head, *i. e.* the upper Part, with grained Powder, which will cause the Petard to give a Report, when the Priming which is put towards the Tail is burnt: This Priming must be of a slow Composition made of Powder mix'd with one third Part of Coal, each subtilly pulveriz'd apart, that they may the more intimately incorporate. It will be convenient to keep this Composition in a moist Place, that thereby becoming wettish, it may be the more closely driven into the Coffin; and therefore if 'tis too dry, it is usual to sprinkle it a little with Oyl of Petre, or of Linseed.

When the *Petard* is of Iron, it is divided into two Partitions, by a Wheel or round Plate of Iron, fitted to its Cavity, pierc'd with a little Hole in the Middle; the Partitions are call'd *Chambers*, whereof the upper one contains the Corn-powder, and the lower, the Composition or Priming, which being fired by a small Hole at Bottom, carries the Fire to the Powder in Grains thro' the Hole in the Wheel.

A Petard may be charg'd with Grain-powder only, and strongly wadded with Paper or Tow: Or each End may be shut up with an Iron Wheel solder'd, making one Hole only in the side, by which it must be loaded and fired.

Besides these for Pleasure, there are also *Petards* made for Service in War, which are likewise of Iron or Copper, without Bottoms; they are parted into three Remark.

three equal Divisions or Chambers, the Middle of which is fill'd with Corn-powder, and the two extream ones with Lead-bullets, which are parted from the Powder with Paper, the two Ends being also stop'd by two little Paper Wheels, with a Hole in the Middle for the Priming.

PROBLEM XXII.

To make Stars *for* Sky-Rockets.

STars are little Balls, about the bigness of a Musquet-Bullet, or an Hazle-nut, made of an inflammable Composition, which gives a splendid Light, resembling that of Stars, from whence is the Name. When they are put into the Rocket, they must be cover'd with prepar'd Tow, the Manner of making which shall be taught, after that of Stars.

They are made thus: To one pound of Powder finely flowered, add four pounds of Salt-petre, and two pounds of Sulphur; and having mix'd all very well, roll up about the bigness of a Nutmeg of this Mixture in a piece of old Linnen or in Paper; then tie it well with Pack-thread, and make a Hole through the Middle, with a pretty big Bodkin, to receive some prepared Tow, which will serve for Priming: This being lighted, fires the Composition, which emitting a Flame through both Holes, gives the Resemblance of a pretty large Star.

If instead of a dry Composition, you use a moist one in form of Paste, you need only roll it into a little Ball, without wrapping it up in any thing, save, if you will, in prepared Tow, because of it self it will preserve its spherical Figure; nor needs there any Priming, because while moist you may rowl it in Meal-powder, which will stick to it, and when fired will light the Composition, and this at falling forms it self into Drops.

Remark. There are many other Ways of making Stars, too long now to be mention'd; I shall only here shew how to make *Stars of Report*, that is, Stars that give a Crack like that of a Pistol or Musquet, as follows.

Take

Take ſmall Links, made as is taught in *Probl.* 17. which you may chooſe either to roll with Line or not; tie to one End of 'em, which muſt be pierc'd, your Stars if made after the firſt manner, that is, of the dry Compoſition: Otherwiſe you need only leave a little piece of the Coffin empty beyond the Choak of the pierced End, to be fill'd with moiſt Compoſition, having firſt prim'd your Vent with Grain-Powder.

You may alſo contrive Stars, which, upon Conſumption of the Compoſition, may appear to be turn'd into Serpents, a thing eaſy to be perform'd by ſuch as underſtand what precedes; upon which account, and becauſe they are but little in uſe, I ſhall ſay no more of 'em.

PROBLEM XXIII.

To make prepared Tow *for Priming to Fire-works.*

PRepared *Tow*, called alſo *Pyrotechnical Match*, and *Quick-match*, to diſtinguiſh it from *Common Match*, is uſed for priming all ſorts of Machins for Fire-works of Diverſion, ſuch as Rockets, Fire-Lances, Stars, and the like; and 'tis made as follows.

Take Thread of Flax, Hemp, or Cotton, and double it eight or nine times, if it is for priming your large Rockets, or Fiery Lances; but four or five Times only, if 'tis to be put through your Stars. Having made it of a Bigneſs proportion'd to your deſigned Uſe, and twiſted it, but not too hard, wet it in clean Water, which muſt be after ſqueezed out with your Hands. Then put ſome Gun-powder in a little Water, ſo as to thicken it a little; in this ſoak your Match well, turning and ſtirring it till 'tis throughly impregnated with the Powder; and then taking it out, rowl it in ſome good Powder-duſt, and hang it upon Lines to dry either in the Sun or Shade: Thus you have a *Pyrotechnical Match* ready for Uſe on all Occaſions.

Common Match, call'd alſo *Fire-cord*, is thus made: Take an unglaz'd Earthen Pot; cover its Bottom with red Sand well waſh'd and dry'd; upon this lay ſpiral-wiſe

wise plain Match of Cotton, or well clean'd Tow, half an Inch thick, the distance of half an Inch being between each Revolution, and then cover it with Sand; upon which again place a Lay of Match as before, and upon this another of Sand, and so interchangeably till the Pot is full, but finishing always with a Lay of Sand: Then cover it with an earthen Cover, and lute with Clay the Joining, so as no Air may get Entrance. This done put burning Coals round the Pot, and after it has been kept hot for some Hours, let it cool of it self; so your Match is prepar'd, which will burn without Smoke or offensive Smell.

PROBLEM XXIV.

To make Fire-Sparkles *for Sky-Rockets.*

S*Parkles* differ only from Stars in their Smallness and short Continuance, these being larger and not so soon consumed as those; which, when you have occasion to use them in Rockets, may thus be made.

Take one Ounce of beaten Powder, two Ounces of pulveris'd Salt-petre, one Ounce of liquid Salt-petre, and four Ounces of Camphire in Powder; upon these, being put into a white earthen Vessel, pour Water wherein Gum-Dragant is dissolv'd, or a Dissolution either of the last nam'd Gum, or Gum-arabick in Brandy, till you have reduc'd the Mixture unto the Consistence of a thin Pap; into which put as much Lint, made of Rags, boil'd in Brandy, Vinegar, or Salt-petre, and after dry'd, as will drink up all your Mixture; and thus have you a Matter prepar'd, which you may form into little Pills of the bigness of a Pea, to be dry'd either in the Sun or Shade, after they have been dip'd in Meal-powder, that they may easily take Fire.

PRO-

PROBLEM XXV.

To make Golden Rain *for Sky-Rockets.*

THere are ſome Sky-rockets, which in falling make little Waves in the Air, like unto Hair half curled, and are therefore call'd *Hairy Rockets*; they end in a ſort of Rain of Fire, call'd *Golden Rain.* 'Tis thus made.

Fill with the Compoſition for Sky-rockets Gooſe-quills, the Feathers being cut off; putting ſome wet Powder in the open End of each, both to keep in the Compoſition, and to ſerve for Priming: With theſe fill the Head of your Sky-rocket, and it will end in a Golden Rain very agreeable to behold.

This Golden Rain calls to my Mind a *Pyrotechnical Hail*, ſo call'd from its Reſemblance to the Natural, which is a Quantity of ſmall hard Bodies, being either pieces of Flint, round Stones, leaden Bullets, or ſquare pieces of Iron, incloſ'd in a Cartridge of Wood, Iron, or Copper, and is therefore called *Cartridge* or *Caſe-ſhot*; they are us'd in War, either in open Field to diſorder an Enemy's Army, or in a Siege to drive them away from a Breach or Gate to be ſeiz'd, being ſhot either out of a Mortar, or a Great-gun of a large Bore. Remark.

PROBLEM XXVI.

To repreſent, with Rockets, *ſeveral* Figures *in the Air.*

IF you take a Rocket of the larger Sort, and place round the Head of it many ſmall ones, fixing their Sticks all round the large Coffin upon the Head of your big Rocket, which uſes to contain the Head-works, ordering it ſo, that your ſmall Rockets take Fire whilſt the Great one is Mounting up, you will have the Reſemblance of a Tree, very delightful to the Sight; whereof the big one will repreſent the Trunk, and the little ones the Branches.

But if the ſmall Rockets take Fire when the great one is half turned in the Air, they will have the Appearance of a Comet: And when the large one is altogether turn'd, ſo that its Head points downwards to the Earth, they will exhibit the Similitude of a Fountain of Fire.

If you put on the Head of a large Rocket many Gooſe-quills, the Feathers being cut off, fill'd with Sky-rocket Compoſition, as in the preceding *Problem*; when fired, they will appear to thoſe under them as a fine ſhower of Fire; but to thoſe who view them on one ſide, like half curl'd Hair very delightful to the View.

Finally, with Serpents ty'd to a Rocket with Pack-thread, by the Ends which are not fired, leaving two or three Inches of the Thread between Each, you may repreſent at pleaſure ſeveral ſorts of Figures moſt entertaining and agreeable to the Sight.

PROBLEM XXVII.

To make Fire-Pots *for Fire-works of Diverſion.*

A *Pot of Fire,* is a large Coffin fill'd with Rockets, that take fire all together, and are diſcharg'd from the Pot without hurting it. The Bottom of the Pot muſt be cover'd with Powder-duſt, which being fired by a Match that muſt paſs through a Hole in the Middle of the Pot, will ſet fire to all the Rockets at once.

When there are many Fire-Pots, they muſt be covered with ſingle Paper, that they may not play all at once; otherways one when fired might ſet fire to another: and you muſt uſe only a ſingle Leaf of Paper, that it may not hinder the Rockets to fly out. Pots of Fire are alſo made for War-ſervice, of which in *Probl.* 35.

PRO-

PROBLEM XXVIII.

To make Fire-Balls *for Diversion, that burn swimming in the Water.*

THese Globes, or *Balls of Fire*, are made commonly of three several Figures, *viz.* either Spherical, Spheroidal, or Cylindrical. They must be made of a light Wood, that they may swim on the Water, and hollow to receive a fit Composition, which is prepared as that for Rockets; but observing the following Proportions.

To one pound of Grain-powder, put thirty two pounds of Salt-petre finely pulveris'd, eight pounds of Sulphur, one ounce of rasped Ivory, and eight pounds of Saw-dust of Wood, that hath been first boil'd in Water of Salt-petre, and after dried in the Shade, or in the Sun.

Or; to eight pounds of beaten Powder, add forty eight pounds of Salt-petre, twenty four pounds of Sulphur, one pound of Camphire, sixteen pounds of Saw-dust, one pound of yellow Amber rasped, and one pound of beaten Glass.

Or; to two pounds of beaten Powder, take twelve pounds of Salt-petre, six pounds of Sulphur, four pounds of Filings of Iron, and one pound of Greek-Pitch or Colophony.

There is no necessity your Composition should be so finely beaten as that for Rockets, 'tis sufficient if it be well mix'd and incorporated, tho' neither powder'd nor searc'd: and lest it become too dry, it will be proper to sprinkle it a little with common Oyl, or Oyl of Wall-nuts, Lin-seed, or Hemp-seed, or with Stone-oyl, or some other fat and inflammable Liquor.

In the first place to make a *Spherical Ball of Fire*, you must get a Globe or Bowl of Wood of what bigness you please, which must be hollow, and very round, as well withinside as without, so that its Thickness AC, or BD, be about one ninth part of the Plate 21, Fig 77.

Plate 24. Fig. 77. the Diameter AB: Add to the upper part of it a straight concave Cylinder, as EFGH, of which the Thickness EF, must be about one fifth part of the same Diameter AB, and the Wideness of its Cavity LM, or NO, must equalize the Thickness AC, or BD, that is one ninth part of the Diameter AB. 'Tis by this Cavity you must prime your Fire-Ball, after you have fill'd it with Composition by the lower Orifice IK, by which you shall convey into it the Petard of Metal P, which must be charg'd with good Corn-powder, and laid athwart the Orifice, as you see in the Figure.

This done, the Orifice IK, which is almost equal to the Thickness EF, or GH, of the Cylinder EFGH, must be shut up with a Bung or Stopple dip'd in melted Pitch; this Bung must be covered on the upper side with such a Weight of Lead, as may sink the Globe into the Water; so that nothing but the Part GH may appear above it, which will fall out, if the Weight of the Lead, with the Ball and Composition, be equal to that of a like Bulk of Water. If therefore thus ballanc'd it be thrown into the Water, the Weight of the Lead will keep the Orifice IK, directly down, and the Cylinder EFGH perpendicularly upright, which should be fired before the Globe is thrown in.

Plate 25. Fig. 78. In the next place, to make a *Fire-Ball* of a *Spheroidal* Figure, the Thickness AC, or BD, must be one ninth part of the shortest Diameter AB, and to the upper End of the largest Diameter, a Cylinder EFGH, must be fitted, like that of the preceding, making an Orifice, as IK, at the lower End of the same largest Diameter, and its Stopple also as before, with this Difference, that instead of covering it with Lead, and putting a Petard within, a Grenade of Lead, charg'd with good Corn-powder, must be annex'd to it without, the Neck of it entring into the Bottom of the Ball, that it may take fire when the Composition is spent.

Plate 25. Fig. 79. Lastly, a *Cylindrical Fire-ball*, such as ABCD, may be made of what Bigness you please, provided its Height AD, or BC, be the Triple of its Breadth AB, or CD, its Thickness being, as in the preceding, one

ninth

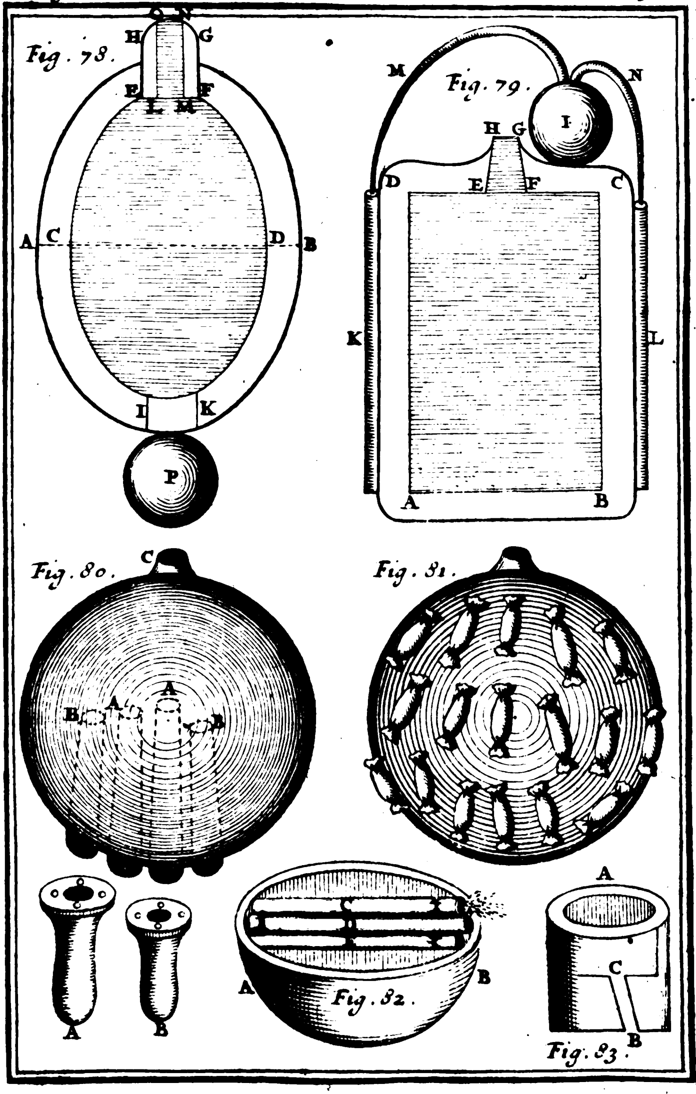
Fig. 78.
Fig. 79.
Fig. 80.
Fig. 81.
Fig. 82.
Fig. 83.

ninth part of the same Height AD, as well as the Wideness EF of the Orifice EFGH, which must be narrower by one half above than below. By this Orifice the Cylinder is to be charg'd; after which it must be fitted with a Stopple, wrapp'd round with a Cloth dip'd in melted Pitch, or Pitch and Tar, and bored Lengthwise, for holding the Priming.

This done, make fast to it, near the Priming, a little concave Globe of Metal, as I, which must first be fill'd with Water, as is done in the Æolipyles, by putting it in cold Water after it is heated pretty hot. To the sides of the Cylinder also you must fasten two small leaden Pipes, as K, L, the upper Orifice of which must be joined to the Globe I, by the two Horns M, N, made of some bending Material bor'd from one End to the other with a very small Hole, but smallest at the lower End.

Now when you have a Mind to set this Aquatick Machine a playing; first fire the Priming with a Match or otherwise, and when 'tis well lighted, throw it into the Water, so that the Bottom AB may be down; and you shall behold with Pleasure, so soon as the Fire of the Priming has heated the Globe, that the Water contain'd therein being rarefy'd, shall come out in form of Vapour impetuously by the small Holes of the Horns M, N, making a very agreeable Noise in the Orifices of the two Pipes K, L.

Remark.

There are many other ways of making these fiery Globes, for which I shall remit my Readers to Pyrotechnical Authors. I shall only add, that a Ball of Fire, like those of the first sort, may be contriv'd, which when fired in a small close Room, will emit a most acceptable Smell, the Composition of which make up as follows.

Take to eight Ounces of Salt petre, two Ounces of *Storax Calamita*, two Ounces of Frankincense, two Ounces of Mastick, one Ounce of Amber, one Ounce of Civet, four Ounces of the Saw-dust of Juniper-wood; four Ounces of the Saw-dust of Cypress-wood, and two Ounces of Oil of Spicknard. Mix and incorporate all these things together, as is said in the Composition for Rockets. Or; to four Ounces of Salt-petre, add two Ounces of Flowers of Sulphur, one Ounce of Camphire, one Ounce of yellow Amber rasp'd and well pulveris'd, two Ounces of Coal of the Lime-tree, and one Ounce of Flowers of Benjamin. All these should be pulveriz'd each apart, then mix'd and imbodied together, as in the Composition of common Rockets.

PROBLEM XXIX.

To make Fire-Balls *for Diversion, that will dance upon an Horizontal Plain.*

Plate 25. Fig. 80. MAKE a *Ball* of Wood, with a Cylinder A, like the first of the three describ'd in the preceding Problem, and charging it with a like Composition, put into it four, or more Petards or Crackers, if you please, fill'd with good Grain-powder to the Top, as AB, which must be stop'd strongly with Paper, or Tow rowl'd hard: Thus you have a Ball, which being fired by the Priming at C, will leap upon a smooth Horizontal Plain according as the Fire lays hold on the Petards.

Fig. 81. But instead of putting the Petards within, you may fasten them without to the Surface of the Globe, and they will make it to roll and dance as the Fire reaches the Petards, which, as you see in the Figure, are plac'd carelesly upon the Surface of the Ball.

Fig. 82. You may also thus contrive a like Ball, which shall roll to and fro upon an Horizontal Plain with a very swift Motion. Make two equal Hemispheres of Past-board, and fit to one of 'em, as AB, three common Rockets charged and prim'd as your ordinary Sky-rockets, without Petards, so that the Rockets, C, D, E, don't exceed in Length the Diameter of the Concavity of the Hemisphere, with the Tail of one answering the Head of the other, as in the Figure, that the Fire passing from one to another, they may burn successively: To this join the other Hemisphere, gluing them neatly together with good Paper, that they may not be separated by the Motion; there must only be made one Hole opposite to the Tail of the

the first Rocket for Priming, which being fired thereby, when spent, will fire the Second, and this in like manner the Third, which will give a continual Motion to the Ball when plac'd on an Horizontal and smooth Plain, making it to go and come with an extraordinary Swiftness.

The two Hemispheres of Paper or Past-board may be thus made: Take a large Wooden Globe, coat it all over with melted Wax, entirely covering its Surface, that you may glue to it many Fillets of strong Paper, about two or three Fingers wide, one above another to the Thickness of about two Lines. Or you may do it thus, which is in my Opinion the better and more easy Way; Dissolve in Glue-water that Mass or Past which is us'd in Paper-mills to make Paper withal, and lay it over the whole Surface of the Globe, which, when dryed by degrees at a small Fire, must be cut asunder in the Middle; so you shall have two solid Hemispheres, to be rendered concave, if you separate the Wood from the Past-board, by melting the Wax at a good Fire.

PROBLEM XXX.

To make Sky Fire-balls *for Fire-works of Diversion.*

These *Balls* are call'd *Sky* or *Air-Balls*, because they are thrown up into the Air from a *Mortar*, which is a well known Piece of Artillery, short, well-fortified, and of a large Bore, us'd in War to throw Fire-works of Service against the Enemy, and in Fire-works of Pleasure to raise into the Air Balls of Fire, and other such things, for Diversion.

Tho' these Balls are of Wood, and of a convenient Thickness, *viz.* the twelfth part of their Diameter; yet if you put too much Powder into the Mortar, they will be unable to resist its Force. Therefore it is, that you must proportion the Quantity of Powder to the Weight of the Ball to be thrown; which if it weigh four Pounds, one Ounce of Powder will serve; but if your Fire-ball weigh eight Pounds, it it will require two Ounces of Powder, and so on in the same Proportion.

It may fall out, that the Chamber of a Mortar may prove too big to contain exactly the Quantity of Powder requir'd to the Fire-ball, which should be put immediately above the Powder, that it may be thrown up and lighted at the same time; In this Case, you may make another Mortar of Wood, or of Past-board, with a Bottom of Wood, as AB, containing a Quantity of Powder proportionable to the Weight of your Ball, which may be put into the large Mortar of Brass or Iron.

Plate 25. Fig. 83.

This small Mortar must be made of light Wood, or of Paper pasted, and rowl'd in form of a Cylinder, or of an inverted Cone without a Point, save that its lower Bottom must be of Wood. The Chamber AB, where the Powder lies, must be bor'd obliquely with a small Wimble, as at BC, so as the Vent B may answer to that of the metallick Mortar, to which if you put Fire, it will light the Powder at the Bottom of the Chamber AC, immediately under the Fire-ball, which will also take Fire, and rising into the Air, will make an agreeable Noise; which otherwise would not succeed, if an empty Space were left betwixt the Fire-ball and Powder.

Plate 26. Fig. 84.

The Profil or perpendicular Section of such a Ball is represented by the Rectangle ABCD, the Breadth of which AB is almost equal to its Height AD. The Thickness of the Wood at the two Sides LM, is equal, as we have already said, to a twelfth part of the Diameter of the Ball, and the Thickness E F, of the Cover, is double that of the Sides, or equal to a sixth part of the same Diameter. The Height GK, or HI, of the Chamber GHIK, where the Priming is put, and which is bounded by the Semicircle LGHM, is one fourth part of the Breadth AB, and its Breadth GH is one sixth part of the same Breadth AB.

This Ball must be fill'd with Canes or common Reeds, of a Length fitted to the inward Height of the Ball, and charged with a slow Composition made of three Ounces of Meal-powder, one Ounce of Sulphur moistned a little with Oyl of Petre, and two Ounces

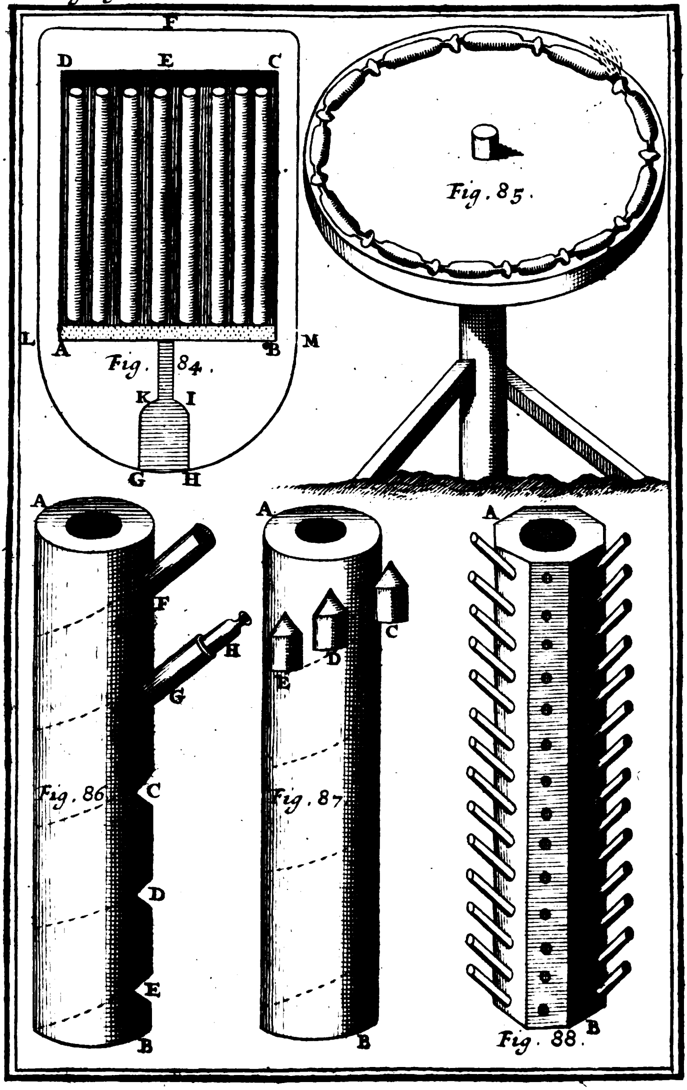
F
D
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Fig. 84.
Fig. 85.
A
F
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D
E
B
Fig. 86
A
C
D
E
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Fig. 87
A
B
Fig. 88.

Ounces of Coal: And that these Reeds or Canes may the more easily take Fire, their lower End, which rests upon the Bottom of the Ball, should be charg'd with Powder beaten and moistned in like manner with Oyl of Petre, or sprinkled with Brandy, and after dried.

This Bottom of the Ball must be covered with some Powder, half of it in Flower, and half of it in Grain, which will set fire to the lower End of the Reeds, being it self fired by the Priming put to the End of the Chamber GH, which must be fill'd with a Composition like that of the Reeds, or another slow one made of eight Ounces of Powder, four Ounces of Salt-petre, two Ounces of Sulphur, and one Ounce of Coal: Or else of four Ounces of Salt-petre, and two Ounces of Coal; all being beaten, put together, and well mixed. Plate 26. Fig. 84.

Remark. Instead of Reeds, you may charge your Ball with Ground-rockets, or with Petards of Paper, together with Stars, or Sparkles mix'd with beaten Powder and laid confusedly upon the Petards, which must be choak'd at unequal Heights, that they may not produce their Effects all at once.

There are many other ways of making these Balls, too long to be here insisted on. But you must remember to take care when they are charg'd, before they are put into the Mortar, to cover them above and all round with a Cloth dipt in Glue, and to make fast a Piece of Cloth, or Wool press'd hard into a round Form, underneath, exactly upon the Hole of the Priming, &c.

PRO-

PROBLEM XXXI.

To make Shining-Balls, *for Diversion, and for Service in War.*

FIrst, to make *Shining-balls* for Recreation; to four Pounds of Salt-petre, put six Pounds of Sulphur, two pounds of crude Antimony; four Pounds of Colophony, and four Pounds of Coal: Or, to two Pounds of Salt-petre, take one Pound of Sulphur, as much Antimony, two Pounds of Colophony, as much Coal, and one Pound of black Pitch; melt these, being well beaten, in a Kettle, or in a glaz'd earthen Pot, and thereinto throw such a Quantity of Hards of Flax, or of Hemp, as will just suffice to imbibe all the Liquor, of which as it cools make little Pellets or round Balls, to be covered over with prepared Tow, which I taught to make in *Probl.* 23. and after put into Sky-rockets, or Balls for Diversion, as is usual to be done with fiery Stars.

Next, to make *Shining* or *Flaming-balls* for Service in War, to be thrown from a Mortar against the Enemy, you must melt, in a Kettle, or glaz'd earthen Pot, as above, equal Parts of Sulphur, black Pitch, Rosin, and Turpentine, into which dip an Iron, or Stone-bullet, somewhat lower than the Bore of the Mortar, and when its Surface is cover'd with this Matter, rowl it in Corn-powder: Which done cover it over with Callico, and dip it again into the same Liquor; rolling it after in Grain-powder; this must be reiterated several times, covering, dipping, and rowling it, till it fills exactly the Bore of the Mortar or Cannon, into which you design to put it, remembring still to end your Operations with rolling it in Grain-powder, that being put into your Piece, immediately above the Charge of Powder, it may take fire as it is thrown into the Air against the Enemy, either to annoy them, or to discover their Designs, which is usually done in Sieges.

Instead of these Shining-Balls, *Red-hot-balls* are more frequently used for offending the Enemy, by burning them, their Houses, or Works. These Bullets are of Iron, and being heated red-hot in a Furnace are thus used. Your Cannon being Charg'd with Powder, freed from Corns, and pointed something upwards, you must have in readiness a Cylinder of Wood fitted exactly to its Bore, which you must put into your Gun next the Powder, and upon it you must ram down a Wad of wet Straw, Hay, or Tow of Hemp, or some such moist Materials; then putting in your Red-hot-Ball with a Ladle, immediately put Fire to your Gun. Remark.

PROBLEM XXXII.

To make a Wheel *of* Fire-works.

A *Wheel of Fire*, or Fire-works, is a Wheel of light Wood, set round with Rockets of a middle Size, the Head of one regarding the Tail of another, that when the first is spent, it may set fire to the next, which makes the Wheel turn round its fix'd Axle-tree without Intermission, till all the Rockets are consum'd. See the Figure. Plate 26. Fig. 85.

Upon this account 'tis call'd a *Fire-Wheel*, and 'tis also call'd a *Fiery Sun*, because plac'd horizontally upon a Stake somewhat large and perpendicular to the Horizon, it turns round, and represents a Sun in Night Combats, which is very diverting.

You may also make Fire-wheels which have a Situation perpendicular to the Horizon, and turn upon an Axis parallel to it, very agreeable to behold. Fire-wheels are likewise used to light other Works at a Distance, in ascending or descending upon a stretch'd Rope, like Flying-Dragons; and on many other Occasions, to the great Pleasure of the Spectators.

PRO-

PROBLEM XXXIII.

To make a Balloon, *or fiery Foot-ball.*

B*Alloons* are Coffins of a large Diameter, shot out of a Mortar whither one pleases, fill'd commonly with Serpents about the Thickness of a Ground-rocket, but not so long, with two small Fire-links of the same Length and Breadth, which being fir'd by their Priming, burst the Coffin, this having below a Fire-conveyance, at the Mouth of which there is a Priming of Cotton dipt in Powder.

The *Coffin* is made with a thick Wooden Rowler, about which is rowled strong Card-paper, glued to keep it from undoing, which being choak'd below, a Hole is made there for a Fire-conveyance, fill'd with a Composition more slow than that of Ground-rockets, being like to that of Sky-rockets: After this it may be filled with Serpents, and sometimes with Stars, and then choaked above.

PROBLEM XXXIV.

To make Pyrotechnical Maces *or* Clubs, *and other Fire-Machins, for Nocturnal Combats.*

NOcturnal Combats may be very agreeably represented in artificial Fire-works with Maces of Fire, Hangers, Scimetars, Faulchions, Swords, Cudgels, Shields, Targets, and other such Pyrotechnical Weapons; all which, besides in the Form they represent, differing but little, as to their Construction, we shall here only describe one or two for Examples, leaving the rest to the Contrivance of an ingenious Operator.

Maces or *Clubs of Fire*, being a Species of these diverting Fire balls that burn upon the Water, which we have taught to make in *Probl.* 28. it will not be needful here much to insist upon 'em. Let it suffice then

then to say, that Handles well turn'd and polish'd must be added to 'em, after you have made several Holes in them to receive your Rockets, which will be fired by the Composition at diverse times; which Composition, as is said, is the same with that of the Water-balls, or with this which follows: Take four Drahms of Sulphur, one Pound of Pitch, and two Drahms of Coal; let all be well beaten and mixed, and afterwards moistned with Brandy, or some other inflammable Liquor.

A *Fire-Hanger* is a Hanger of Wood, resembling a *Turkish* Scimetar. It is made of two Boards of dry Wood, joyning together at the Edge, and parting asunder at the Back, along which there runs as it were a triangular Groove, that must be divided into several little Partitions or Chambers by small triangular Boards; into these Partitions you may put Ground-rockets, or you may fill them with Petards, Stars, Sparkles, Shining-Balls, and other such things, which you must cover with Paper well pasted, as you must all your Hanger with Linnen Cloth. The Touch-hole must be towards the Point, by which you must set fire to its Composition contain'd in a little Canal running along the Edge, and this as it consumes will communicate the Fire to the little Chambers successively: The Composition must be of the slow Kind, made up of five Ounces of Powder, three of Salt-petre, one of Sulphur, and two of Coal.

Cimetars are crooked Hangers made of dry and light Wood, hollow also, and open in the Back, into which you must put several Rockets well glu'd and fasten'd, and so dispos'd that the Head of one may be near the Neck or Tail of another, which must be fir'd by it after its Composition is spent, as may be seen in Fire-wheels.

Targets are made of thin Boards, with a Channel running in a spiral Line, from their Circumference to the Center, for containing the Priming, which must be all covered over with a thin Covering of Wood or Past-board, bored with Holes spiral also, exactly over the Priming to receive the Ends of Rockets, which must be made fast therein.

Amongst

Amongst other Pyrotechnical Machins, we must not here forget to mention the *Fire-pipe*, which is not the least considerable among them. This may be made several Ways, of which I shall here make choice of the most simple, and most easy to be understood and performed.

Plate 26. Fig. 86. Get a wooden Pipe, as AB, of what Length and Thickness you please, about which mark out a Line winding, in Screw-fashion, from one End to the other, upon which make Holes, bored obliquely downwards in respect to the Axis of the Cylinder, as C, D, E, into which you must put Coffins or Pipes of Paper with wooden Bottoms, as F, G, to receive, the Ends of as many Ground or Sky-rockets, as you see in H, under which must be put some Powder, that must be lighted by small Pipes passing between each Hole and the Cavity of the great Pipe AB, which must be fill'd with a Composition like that of the Fire-balls that burn on the Water, the little Pipes themselves being fill'd with Powder finely pulveriz'd.

Fig. 87. Instead of Rockets fitted in Coffins obliquely ascending, you may set round the large Pipe as many Boxes of Paper, dispofed screw-wise as the Coffins, fitted with wooden Bottoms, and standing upright, that is, parallel to the Axis of the Pipe, as C, D, E, which must be glued, and well fasten'd to the Surface of the Pipe, and fill'd with a sufficient number of Ground-rockets, *&c.*

Fig. 88. For the greater Ornament, the Pipe AB, may be cut withoutside into a Prism of many Sides, and on each opposite Plain many Holes made, equidistant from one another, and bored obliquely, to receive Petards, or Rockets as before. All this will be easily apprehended by looking on the Figure.

Besides the Composition for the Aquatick Balls, you may use the following, made of six Pounds of Powder, four of Salt-petre, and one of Filings of Iron: Or this, of twelve Pounds of Powder, five of Salt-petre, three of Sulphur, two of Coal, one of Colophony, and four Pounds of Saw-dust.

PRO-

PROBLEM XXXV.

To make Fire-Pots for Service *in War.*

WE have taught, in *Probl.* 27. the Way of making Pots of Fire for diverting Fire-works, and here we are to ſhew how to make *Fire-pots* for War, which have diverſe Names according to the different Figure may be giv'n to 'em; when they are made like earthen Pots with an Handle on each Side, they are call'd *Fire-pots* or *Fire-pitchers*; when they reſemble a Bottle or a Vial, they are call'd *Fire-bottles* or *Vials*; when like a Box, *Fire-boxes*. But whatever Figure they have, they are ordinarily made in the following Manner.

Put into a Veſſel of Metal or Earth Quick-lime finely pulveris'd, or, if you can't have this, Aſhes of Oak or Aſh-wood well ſearced, till the Veſſel is fill'd to a third Part, and then fill it up to the Brims with good Corn-powder: This done cover it exactly above with ſtrong Paper, or rather with a Wheel of Wood, and wrapping it round with a Linnen Cloth pitched, tie to the Neck or Handle Ends of Match, which being lighted, and the Pot thrown amongſt the Enemies, will fire the Powder, and make a prodigious Havock among the Soldiers, the Veſſel breaking into a thouſand Pieces, which will kill all they hit: Beſides that the Quick-lime riſing up into the Air, will make a thick Duſt reſembling that of a Whirlwind, which will extremely incommode all within its Reach.

Or you may take an earthen or glaſs Veſſel with a long Neck, like a Matras or Body of an Alembick, and fill its Belly with Grain-powder, with a little Sublimate and ſome Bole-Armoniack, mixing with all theſe, if you pleaſe, ſmall Pieces of Iron, to produce as it were a Hail. Laſtly, fill the Neck of your Veſſel with a ſlow Compoſition, that after 'tis fired there may be ſufficient time to throw it where one would have it to do Execution.

These

These Fire-pots are of good Use in War: They may be thrown by the Besieged in an Attack, from the top of the Rampart, into the Moat, if the Enemy is come so far, or upon the Counterscarp, with the Hand; and out of proper Engins, they may be thrown into the Trenches and other Works of the Enemies. They may be used also against the Besieged, being thrown, out of such Machins, into a Place by the Be-siegers. They are also of great use in Naval Fights, when Vessels come to be grappled or boarded; for by throwing these Pots into the Enemy's Ship, you may either blow it up by firing their Powder, or set it on Fire, and put the Soldiers and Sailors into great Confusion.

But when you have a Mind to use 'em for setting Ships on Fire, they must be fill'd with a Composition, that can't be extinguish'd by Water, or otherways, such as the following, which Water is so far from quenching, when once fired, that it rather encreases its Force: So that if it fall upon the Deck of any Vessel, it will burn through it in a little time, and sticking to whatever is in its Way, set all in a Flame.

Take, two Pounds of Gunpowder, two Pounds of Salt-petre, eight Ounces of Sulphur, two Drams of Camphire, four Drams of Colophony, and one Dram of Sal-Armoniack. All these put together and well mix'd, must be made into Dough or Paste with Linseed or Common Oil, which must be formed into Balls about the Bigness of a large Wall-nut, and so put into the Fire-pot, the empty Spaces being filled up with Corn and Meal-powder mixed.

PROBLEM XXXVI.

To make Fire-Crowns *for Service in War.*

Fiery Crowns, or *Fire-garlands* are little Sacks or Bags, of Linnen or Canvas, bent round in form of a Circle, being full of a Composition like that of the Fire-pots in the preceding *Problem*, or that which follows

follows in this: They are used, as Fire-pots, to throw among the Enemies for burning of Ships, and Houses. These Bags are four, five, or six Inches wide, and from three to four Foot long: And to hinder them from becoming straight when their Composition is a burning, their Ends must be well sowed together; besides you must have an Iron Circle to strengthen them, to which they are made fast by the small Cords that are to be twisted round 'em from one End to another.

Into these Bags you may put Petards of Iron loaded with good Powder and Lead-bullets, one End of 'em entring into the Bags, and their Mouth standing out, that they ma[illegible] go off, when fired by their Touch-holes that are surrounded by the Composition, which must be set on fire by two or three Holes made in this circular Bag.

Instead of Petards, you may set round the Crown Hand Grenades, about the Bigness of an Iron-bullet of one or two Pound-weight, having little Pipes three or four Inches long scr[illegible] into their Mouth, to hold them fast, and to set [illegible] on Fire, after they have been fired by the Composition of the Fire-Garland, which must be made as follows.

To four Pounds of Powder, add six Pounds of Salt-petre, two Pounds of Sulphur, and one Pound of beaten Glass: Or, put four Pounds of Powder, to six Pounds of Salt-petre, and one Pound of Colophony; all being well beaten, searced, and mixed together.

Two of these Crowns may be joyn'd together cross-wise, as the Circles of an artificial Sphere of the World: and therefore such a Machin is call'd a *Fire-sphere* or *Circle*. It must be dipt in Pitch and Tar, and have Holes made in several Places, that it may be fired on all Sides, that none may lay hands on it, nor extinguish it, when it is thrown among the Enemies, whom it will put into great Disorder, killing all in its Way. Remark.

When these Bags are not bent into a round Form, they are call'd *Fire-sacks*, as also *Fire cylinders*, from their Figure: but there is some small Difference between these two Machins, which are chiefly used in the Defence of Places besieged, as in Assaults, Scaling of the Walls, to kill and destroy in the Breaches, or in the Moats all they come near, and with their Weight to crush whatever they fall upon.

Instead of the two Crowns join'd cross-wise one within another, three or four, or more may be put together, to make up an *artificial Sphere*, the two outward and greater crossing at Right Angles, to represent the two Colures, to which others may be also added to exhibit the other Circles of the Sphere; and all of 'em well fastned together with Iron or Brass-wier.

Cylinders of Fire are Pipes of Wood, fortified at each End, and in the Middle upon the Powder-place with good Iron Hoops, and stopp'd with a Wheel or Stopple of Wood, after they have been loaded with Stones, square Pieces of Iron, and such like, which by the Violence of the Powder are driven and scattered hither and thither, to the Right and Left, and kill, break, and destroy whatever withstands.

PROBLEM XXXVII.

To make Fire-Barrels *for Defending a Breach, and Ruining the Enemies Works.*

IN the Defending of a Breach there are also used Artificial Barrels, call'd *Flaming* or *Fire-Barrels*, as also *Thundring-Barrels*, because they are employed to overwhelm and thunderstrike the Enemy, and to ruin their Works, by rolling them down from a Breach or other Eminence upon them, being bound with Iron Hoops, and containing within 'em another little Cask full of Powder, and fix'd upon an Axle-tree, in the Middle of the large one: Or Fire-pots, Petards, and Granado's wrapt up in Tow sprinkled with Oyl of Petre, and dipt in liquid Pitch, Turpentine and Colophony.

But it will be sufficient to put thereinto one large Grenade, which may be encompassed with Pieces of Stones, Flints, and square Iron or Dice-shot, and such

such like things, which being dispers'd by the Violence of the Powder, may kill, and bruise the Enemy, and destroy their Works; but you should fill up the vacuities with Quick-lime. To these Casks or Barrels, Pipes must be fitted and well fastned, for carrying Fire to the Powder, by means of a Priming to be put therein.

Remark. We forbear here to give a particular Description of some other Pyrotechnical Machins for War, which are too too common, as of *Grenades*, that are small hollow Balls or Shells, commonly of Iron, fill'd with fine Corn-Powder, which are fired by a Fuse of a slow Mixture made of equal Parts of Powder, Salt-petre, and Brimstone: Of *Bombs*, which are large hollow Balls or Shells of Iron, fill'd with Nails, Powder, and other offensive Fire-works, that are thrown into Places besieg'd, to destroy the Houses: And of *Carcasses*, which are large oval Cases made of Ribs of Iron, and fill'd with Grenades and Ends of Pistol Barrels charg'd with Powder, and wrap'd up together with the Grenades in Tow dipt in Oyl, and other Combustible Matters. They are covered over with a Course pitch'd Cloth before they are thrown from the Mortar into the Place designed, where they make a most dreadful Havock.

PROBLEM XXXVIII.

To make an Ointment *excellent for* Curing *all sorts of Burnings.*

BOil, over a gentle Fire, in common Water, Hogs Lard, or the Fat of fresh Pork, skimming it perpetually, till no further Scum arises; then expose it thus melted to cool in the clear open Air three or four Nights. After this melt the same Lard or Grease in an earthen Vessel over a slow Fire, and strain it through a Linnen Cloth into cold Water, and after wash it well in fair River or Fountain Water, to take away its Salt, which will make it become white as Snow. Finally, being thus purify'd, put it up in a glaz'd earthen Vessel, to be kept for Use upon Occasion.

If it falls out, as commonly it happens, that by a Burning Blisters arise upon the Skin, they must not be cut or broken, till the Oyntment has been us'd to it for three or four Days. You may also use the following, which you will find to be of great Efficacy, and is made of Hogs Lard melted and mix'd with two Drams of the Water of Night-shade, and one Dram of Oyl of Saturn: Or with two Ounces of Juice of Onyons, and one Ounce of Oyl of Wall-nuts.

FINIS.

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