THE ENTERTAINER'S LIBRARY No 1

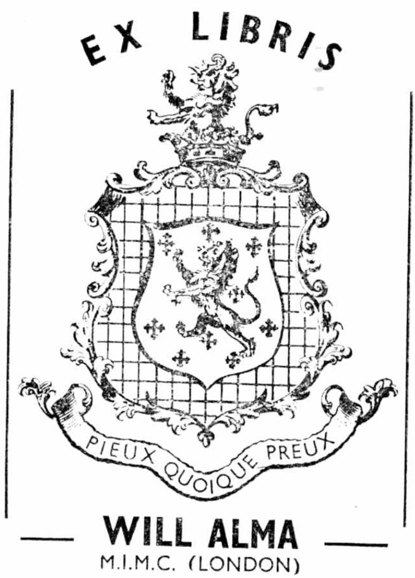
6D NET.

QUEER TRICKS IN FIGURES

TRICKS AND GAMES
AMUSING PROBLEMS
QUIBBLES AND TRIFLES

EDITED BY ARCHIBALD REITH

LONDON: FRANCIS GRIFFITHS
34 MAIDEN LANE, STRAND, W.C.



EX LIBRIS
PIEUX QUOIQUE PREUX
WILL ALMA
M.I.M.C. (LONDON)

QUEER TRICKS IN FIGURES

Edited by

ARCHIBALD REITH

LONDON: FRANCIS GRIFFITHS
34, MAIDEN LANE, STRAND, W.C.

QUEER TRICKS IN FIGURES.

I.---Tricks and Games

To Tell Any Number Thought Of.

Ask any person to think of a number, say a certain number of shillings; tell him to borrow that sum of some one in the company, and add the number borrowed to the amount thought of. It will here be proper to name the person who lends him the shillings, and to beg the one, who makes the calculation, to do it with great care, as he may readily fall into an error, especially the first time. Then say to the person,—' I do not lend you, but give you 10, add them to the former sum.' Continue in this manner:—' Give the half to the poor, and retain in your memory the other half.' Then add:—' Return to the gentleman, or lady, what you borrowed, and remember that the sum lent you, was exactly equal to the number thought of.' Ask the person if he knows exactly what remains; he will answer ' Yes.' You must then say,—' And I know also the number that remains; it is equal to what I am going to conceal in my hand.' Put into one of your hands 5 pieces of money, and desire the person to tell how many you have got. He will answer 5; upon which open your hand and shew him the 5 pieces. You may then say,—' I well knew that your result was 5; but if you had thought of a very large number, for example, two or three millions, the result would have been much greater, but my hand would not have held a number of pieces equal to the remainder.' The person then suppos-ing that the result of the calculation must be different, according to the difference of the number thought of, will imagine, that it is necessary to know the last number, in order to guess the result: but this idea is false; for, in the case which we have here supposed, whatever be the number thought of, the remainder must always be 5. The reason of this is as follows:—The sum, the half of which is given to the poor, is nothing else than twice the number thought of, plus 10; and when the poor have received their part, there remains only the number thought of, plus 5; but the number thought of is cut off when the sum borrowed is returned, and, consequently, there remain only 5.

It may be hence seen, that the result may be easily known, since it will be the half of the number given in the third part of the operation; for example, whatever be the number thought of, the remainder will be 36, or 25, according as 72 or 50 have been given. If this trick be performed several times successively, the number given in the third part of the operation must be always different; for if the result were several times the same, the deception might be discovered. When the five first parts of the calculation for obtaining a result are finished; it will be best not to name it at first, but to continue the operation, to render it more complex, by saying, for example:—'Double the remainder, deduct two, add three, take the fourth part,' &c.; and the different steps of the calculation may be kept in mind, in order to know how much the first result has been increased or diminished.—This irregular process never fails to confound those who attempt to follow it.

A SECOND METHOD.

Bid the person take 1 from the number thought of, and then double the remainder; desire him to take 1 from this double, and to add to it the number thought of; in the last place, ask him the number arising from this addition, and if you add 3 to it, the third of the sum will be the number thought of. The application of this rule is so easy, that it is needless to illustrate it by an example.

A THIRD METHOD.

Desire the person to add 1 to the triple of the number thought of, and to multiply the sum by 3: then bid him add to this product the number thought of, and the result will be a sum, from which if 3 be subtracted, the remainder will be ten times of the number required; and if the cipher on the right be cut off from the remainder, the other figure will indicate the number sought.

Example:—Let the number thought of be 6, the triple of which is 18; and if 1 be added, it makes 19; the triple of this last number is 57, and if 6 be added, it makes 63, from which if 3 be subtracted, the remainder will be 60: now, if the cipher on the right be cut off, the remaining figure, 6, will be the number required.

A FOURTH METHOD.

Bid the person multiply the number thought of by itself; then desire him to add 1 to the number thought of, and to multiply it also by itself; in the last place, ask him to tell the difference of these two products, which will certainly be an odd number, and the least half of it will be the number required.

Let the number thought of, for example, be 10; which multiplied by itself gives 100; in the next place, 10 increased by 1 is 11, which multiplied by itself, makes 121; and the difference of these two squares is 21, the least half of which, being 10, is the number thought of.

This operation might be varied by desiring the person to multiply the second number by itself, after it has been diminished by 1. In this case, the number thought of will be equal to the greater half of the difference of the two squares.

Thus, in the preceding example, the square of the number thought of is 100, and that of the same number, less 1, is 81; the difference of these is 19; the greater half of which, or 10, is the number thought of.

To Tell Two or More Numbers Thought Of.

If one or more of the numbers thought of be greater than 9, we must distinguish two cases; that in which the number of the numbers thought of is odd, and that in which it is even.

In the first case, ask the sum of the first and second; of the second and third; the third and fourth; and so on to the last; and then the sum of the first and the last. Having written down all these sums in order, add together all those, the places of which are odd, as the first, the third, the fifth, &c.; make another sum of all those, the places of which are even, as the second, the fourth, the sixth, &c.; subtract this sum from the former, and the remainder will be the double of the first number. Let us suppose for example, that the five following numbers are thought of, 3, 7, 13, 17, 20, which, when added two and two as above, give 10, 20, 30, 37, 23: the sum of the first, third, and fifth is 63, and that of the second and fourth is 57; if 57 be subtracted from 63, the remainder 6 will be the double of the first number, 3. Now if 3 be taken from 10, the first of the sums, the remainder 7 will be the second number; and by proceeding in this manner, we may find all the rest.

In the second case, that is to say, if the number of the numbers thought of be even, you must ask and write down, as above, the sum of the first and the second; that of the second and third; and so on, as before: but instead of the sum of the first and the last, you must take that of the second and last; then add together those which stand in the even places, and form them into a new sum apart; add also those in the odd places, the first excepted, and subtract this sum from the former, the remainder will be the double of the second number; and if the second number, thus found, be subtracted from the sum of the first and second, you will have the first number; if it be taken from that of the second and third, it will give the third, and so of the rest. Let the numbers thought of be, for example, 3, 7, 13, 17: the sums formed as above are 10, 20, 30, 24; the sum of the second and fourth is 44, from which if 30, the third, be subtracted, the remainder will be 14, the double of 7, the second number. The first, therefore, is 3, the third 13, and the fourth 17.

When each of the numbers thought of does not exceed 9, they may be easily found in the following manner:—Having made the person add 1 to the double of the first number thought of, desire him to multiply the whole by 5, and to add to the product the second number. If there be a third, make him double this first sum and add 1 to it: after which, desire him to multiply the new sum by 5, and to add to it the third number. If there be a fourth, proceed in the seme manner, desiring him to double the preceding sum; to add to it 1; to multiply by 5; to add the fourth number, and so on.

Then ask the number arising from the addition of the last number thought of, and if there were two numbers, subtract 5 from it; if there were three, 55; if there were four, 555; and so on; for the remainder will be composed of figures, of which the first on the left will be the first number thought of, the next the second, and so on.

Suppose the number thought of to be 3, 4, 6: by adding 1 to 6, the double of the first, we shall have 7, which being multiplied by 5, will give 35; if 4, the second number thought of, be then added, we shall have 39, which doubled gives 78, and if we add 1, and multiply 79, the sum, by 5, the result will be 395. In the last place, if we add 6, the number thought of, the sum will be 401; and if 55 be deducted from it, we shall have for remainder 346, the figures of which, 3, 4, 6, indicate in order the three numbers thought of.

The Game of the Ring.

This game is an application of one of the methods employed to tell several numbers thought of, and ought to be performed in a company not exceeding nine, in order that it may be less complex. Desire any one of the company to take a ring, and put it on any joint of whatever finger he may think proper. The question then is, to tell what person has the ring, and on what hand, what finger, and on what joint.

For this purpose you must call the first person 1, the second 2, the third 3, and so on. You must also denote the ten fingers of the two hands, by the following numbers of the natural progression, 1, 2, 3, 4, 5, &c. beginning at the thumb of the right hand, and ending at that of the left, that by this order the number of the finger may, at the same time, indicate the hand. In the last place, the joints must be denoted by 1, 2, 3, beginning at the points of the fingers.

To render the solution of this problem more explicit, let us suppose that the fourth person in the company has the ring on the sixth finger, that is to say, on the little finger of the left hand, and on the second joint of that finger.

Desire some one to double the number expressing the person which in this case will give 8; bid him add 5 to this double, and multiply the sum by 5, which will make 65; then tell him to add to this product the number denoting the finger, that is to say, 6, by which means you will have 71: and, in the last place, desire him to multiply the last number by 10, and to add to the product the number of the joint, 2; the last result will be 712; if from this number you deduct 250, the remainder will be 462; the first figure of which, on the left, will denote the person; the next, the finger, and consequently the hand; and the last, the joint.

It must here be observed, that when the last result contains a cipher, which would have happened in the present example, had the number of the finger been 10, you must privately subtract from the figure preceding the cipher, and assign the value of 10 to the cipher itself.

To tell by a Watch Dial, the Hour when a Person intends to rise.

The person is told to set the hands of his watch at any hour he pleases, which hour he tells you; and you add on your own mind 12 to it. You then desire him to count *privately* the number of that addition on the dial, commencing at the next hour to that at which he intends to rise, and including the hour at which he has placed the hand, which will give the answer. For example:—A intends to rise at 6 (this he conceals to himself); he places the hand at 8, which he tells B, who, in his own mind, adds 12 to 8, which makes 20. B then tells A to count 20 on the dial, beginning at the next hour at which he proposes to rise: which will be 7, and counting backwards, reckoning each hour as 1, and including in his addition the number of the hour the hand is placed at, the addition will end at 6, which is the hour proposed; thus,

The hour the hand is placed at is	8
The next hour to that which A intends to rise at is 7, which counts for	1
Count back the hours from 6, and reckon them at 1 each, there will be 11 hours, viz. 4, 3, 2, 1, 12, 11, 10, 9, 8, 7, 6, ...	11
Making	20

Any Number being chosen, by Adding a Figure to that Number to make it Divisible by 9.

If the number named be, for example, 72,857, you tell him who names it to place the number 7 between any two figures of that sum, and it will be divisible by 9. For if any number be multiplied by 9, the sum of the figures of the product will be either 9, or a number divisible by 9. But the sum of the figures named is 29, therefore 7 must be added to it to make it divisible by 9.

This trick may be diversified by specifying, before the sum is named, the particular place were the figure shall be inserted, to make the number divisible by 9.

To Find the Difference between Two Numbers, the Greater of which is Unknown.

Take as many nines as there are figures in the smaller number, and subtract that sum from the number of nines. Let another person add the difference to the larger number, and taking away the first figure of the amount, add it to the last figure, and that sum will be the difference of the two numbers. Example:—John, who is 22, tells Thomas, who is older, that he can discover the difference of their ages; he therefore privately deducts 22 from 99 (his age consisting of two figures, he of course takes two nines); the difference which is 77, he tells Thomas to add to his age, and to take away the first figure from the amount, and add it to the last figure, and that will be the difference of their ages; thus:—

The difference between John's age and 99 is	77
To which Thomas adding his age	35
The sum is	112
Then by taking away the first figure 1, and adding it to the figure 2, the sum is	13
Which added to John's age	22
Gives the age of Thomas	35

The Cancelled Figure Guessed.

To tell the figure a person has struck out of the sum of two given numbers:—Command those numbers only, that are divisible by 9; such, for instance, as 36, 63, 81, 117, 126, 162, 261, 315, 360, and 432. Then let a person choose any two of these numbers; and after adding them together in his mind, strike out from the sum any one of the figures he pleases. After he has so done, desire him to tell you the sum of the remaining figures; and it follows, that the number which you are obliged to add to this amount, in order to make it 9 or 18, is the one he struck out. Thus, suppose he chose the numbers 162 and 261, making together 423, and that he strike out the centre figure, the two other figures will, added together, make 7, which, to make 9 requires 2, the number struck out.

The Money Game.

A person having in one hand a piece of gold, and in the other a piece of silver, you may tell in which hand he has the gold, and in which the silver, by the following method:—Some value, represented by an even number, such as 8, must be assigned to the gold, and a value represented by an odd number, such as 3, must be assigned to the silver; after which, desire the person to multiply the number in the right hand by any even number whatever, such as 2; and that in the left by an odd number, as 3; then bid him add together the two products, and if the whole sum be odd, the gold will be in the right hand, and the silver in the left; if the sum be even, the contrary will be the case.

To conceal the artifice better, it will be sufficient to ask whether the sum of the two products can be halved without a remainder; for in that case the total will be even, and in the contrary case odd.

It may be readily seen that the pieces, instead of being in the two hands of the same person, may be supposed to be in the hands of two persons, one of whom has the even number, or piece of gold, and the other the odd number, or piece of silver. The same operations may then be performed in regard to these two persons, as are performed in regard to the two hands of the same person, calling the one privately the right, and the other the left.

A Person having an Equal Number of Counters or Pieces of Money, in Each Hand, to find how many he has Altogether.

Desire the person to convey any number, as 4, for example, from one hand to the other, and then ask him how many times the less number is contained in the greater. Let us suppose that he says the one is triple of the other;

and in this case multiply 4, the number of the counters conveyed from one hand into the other, by 3, and add to the product the same number, 4, which will make 16. In the last place, from the number 3 subtract unity, and if 16 be divided by 2, the remainder, the quotient 8 will be the number contained in each hand, and consequently the whole number is 16.

Let us now suppose that when four counters are conveyed from one hand to the other, the less number is contained in the greater $2\frac{1}{3}$ times; in this case we must, as before, multiply 4 by $2\frac{1}{3}$, which will give $9\frac{1}{3}$; to which if 4 be added, we shall have $13\frac{1}{3}$ or $\frac{40}{3}$; if unity be then taken from $2\frac{1}{3}$, the remainder will be $1\frac{1}{3}$ or $\frac{4}{3}$, by which if $\frac{40}{3}$ be divided, the quotient 10, will be the number of counters in each hand, as may be easily proved on trial.

Three Cards being presented to Three Persons, to Guess That which Each has Chosen.

As it is necessary that the cards presented to the three persons should be distinguished, we will call the first A, the second B, and the third C; but the three persons may be at liberty to choose any of them at pleasure. This choice, which is susceptible of six different varieties having been made, give to the first person 12 counters, to the second 24, and to the third 36: then desire the first person to add together the half of the counters of the person who has chosen the card A, the third of those of the person who has chosen B, and the fourth part of those of the person who has chosen C, and ask the sum, which must be either 23 or 24; 25 or 27; 28 or 29, as in the following table:

First.	Second.	Third.	Sums.
12	24	36	
A	B	C	23
A	C	B	24
B	A	C	25
C	A	B	27
B	C	A	28
C	B	A	29

This table shews, that if the sum be 25, for example, the first person must have chosen the card B, the second the card A, and the third the card C; and that if it be 28, the first person must have chosen the card B, the second the card C, and the third the card A, and so of the rest.

Several Cards being presented, in Succession, to Several Persons, that they may Each choose One at pleasure; to Guess that which Each has Thought of.

Shew as many cards to each person as there are persons to choose; that is to say, 3 to each if there are 3 persons. When the first has thought of one, lay aside the three cards in which he has made his choice. Present the same number to the second person, to think of one, and lay aside the three cards in the like manner. Having done the same in regard to the third person, spread out the three first cards with their faces upwards, and place above them the next three cards, and above these the last three, that all the cards may thus be disposed in three heaps, each consisting of three cards. Then ask each person in which heap the card is of which he thought, and when this is known it will be easy to tell these cards, for that of the first person will be the first in the heap to which it belongs; that of the second will be the second of the next heap, and that of the third will be the third of the last heap.

Several Numbers being disposed in a Circular Form, according to their natural series, to tell that which Anyone has Thought of.

The first ten cards of any suit disposed in a circular form, as seen in the figure below, may be employed with great convenience for performing what is announced in this problem. The ace is here represented by the letter A annexed to 1, and the ten by the letter K joined to 10.

```
              3
        2     C     4
        B           D
   1A                    E5
  10K                    F6
        I           G
        9     H     7
              8
```

Having desired the person who has thought of a number or card to touch also any other number or card, bid him add to the number of the card touched, the number of the cards employed, which in this case is 10. Then desire him to count that sum in an order contrary to that of the natural numbers, beginning at the card he touched, and assigning to that card the number of the one which he thought of; for by counting in this manner, he will end at the number or card which he thought of, and consequently you will easily know it.

Thus for example if the person has thought of the number 3, marked C, and has touched 6, marked F; if 10 be added to 6, it will make 16; if 16 be then counted from F, (the person must not count this sum aloud, but privately in his own mind) the number touched, towards E. D. C. B. A. and so on in the retrograde order, counting 3, the number thought of, on F, 4 on E, 5 on D, 6 on C, and so round to 16, the number 16 will terminate on C, and show that the person thought of 3, which corresponds to C.

Remarks:—1. A greater or less number of cards may be employed at pleasure. If there are 15 or 8 cards, 15 or 8 must be added to the number of the card touched.

2. To conceal the artifice better, you may invert the cards, so as to prevent the spots from being seen; but you must remember the natural series of the cards, and the place of the first number, or the ace, that you may know the number of the card touched, in order to find the one to which the person ought to count.

Thirty Soldiers having deserted, so to place them in a Ring, that you may save any Fifteen you please, and it shall seem the Effect of Chance.

This Recreation is usually proposed in these terms: 'It once happened that fifteen Christians and fifteen Turks being in a ship at sea, in a violent tempest, it was deemed necessary to throw *half* the number of persons overboard, in order to disburden the ship, and save the rest. To effect this, with some *appearance* of equity, it was agreed to be done by *lot*, in such a manner, that the persons being placed in a ring, every ninth man should be cast into the sea, until one half of them were thrown overboard. Now the Pilot, being a Christian, was desirous of saving those of his own *persuasion*; how ought he therefore to dispose of the crew, so that the lot might always fall upon the Turks?

This question may be resolved, by placing the men according to the *numbers* annexed to the *vowels* in the words of the following verse;

Po-pu-le-am jir-gam Ma-ter Re-gi-na fe-re-bat.
4 5 2 1 3 1 1 2 2 3 1 2 2 1

from which it appears, that you must place four of those you would save first; then five of those you would have perish. After this, *two* of those to be saved, and then one who is to perish, and so on. When this is done, you must enter the ring formed by the men, and beginning with the first of the *four* men you intend to save, count on to the ninth man, and turn him out to be punished; then counting on, in like manner, to the next *ninth* man, turn him out to be punished in like manner, and so on for the rest, until the fifteen are turned out.'

It is reported of Josephus, the author of the Jewish History, that he escaped death by practically working this problem; for being governor of Joppa, at the time it was taken by the Roman emperor Vespasian, he was obliged to secrete himself with thirty or forty of his soldiers in a cave, where they made a firm resolution to perish by famine rather than fall into the hands of the conqueror. But being at length driven to great distress, they would have destroyed each other for sustenance, had not Josephus persuaded them to die by *lot*; which he so ordered, that all of them were killed except himself and another, whom he might easily destroy, or persuade to yield to the Romans.

The Magic Century.

If the number 11 be multiplied by any one of the nine digits, the two figures of the product will always be alike, as appears from the following example:

11	11	11	11	11	11	11	11	11
1	2	3	4	5	6	7	8	9
11	22	33	44	55	66	77	88	99

Now, if another person and yourself have fifty counters a-piece, and agree never to stake more than ten at a time, you may tell him, that if he will permit you to stake first, you will always undertake to make the even century before him.

In order to do this, you must first stake one, and remember the order of the above series, constantly add to what he stakes as many as will make one more than the numbers 11, 22, 33, &c. of which it is composed, till you come to 99; after which, the other party cannot possibly make the even century himself, or prevent you from making it.

If the person who is your opponent has no knowledge of numbers, you may stake any other number first, under 10, provided you afterwards take care to secure one of the last terms, 56, 67, 78, &c. or you may even let him stake first, provided you take care afterwards to secure one of these numbers.

This recreation may be performed with other numbers, but, in order to succeed, you must divide the number to be attained, by a number which is a unit greater than what you can stake each time; and the remainder will then be the number you must first stake. Suppose, for example, the number to be attained is 52 (making use of a pack of cards instead of counters) and that you are never to add more than six; then dividing 52 by 7, the remainder which is

3 will be the number you must stake first; and whatever the other stakes, you must add as much to it as will make it equal to 7, the number by which you divided; and so on.

The Game of the Bag.

To let a person select several numbers out of a bag, and to tell him the number which shall exactly divide the sum of those he has chosen:—Provide a small bag, divided into two parts, into one of which put several tickets, numbered 6, 9, 15, 36, 63, 120, 213, 309, &c.; and in the other part put as many other tickets, marked No. 3 only. Draw a handful of tickets from the first part, and after shewing them to the company, put them into the bag again, and having opened it a second time, desire any one to take out as many tickets as he thinks proper; when he has done that, you open privately the other part of the bag, and tell him to take out of it one ticket only. You may safely pronounce that the ticket shall contain the number by which the amount of the other numbers is divisible; for, as each of these numbers can be multiplied by 3, their sum total must, evidently, be divisible by that number. An ingenious mind may easily diversify this exercise, by marking the tickets in one part of the bag, with any numbers that are divisible by 9 only, the properties of both 9 and 3 being the same; and it should never be exhibited to the same company twice without being varied.

Different, yet the Same.

Write down three very different lines of figures, and announce that you can divide them among three persons without altering the amounts. You can prove, in one adding up, that each one's share will be the same, and that they will not be much the richer for it.

In writing down the figures, select only those whose digits when added together make the same sum in each case, as in the following example:—

431211	=	12
7320	=	12
129	=	12

The Mysterious Sum.

Write down four rows of dots, and under them a line of figures as in the following example:—

```
 .  .  .  .  .
 .  .  .  .  .
 .  .  .  .  .
 .  .  .  .  .
 -------------
19  9  9  9  8
```

Invite someone to insert any figures he pleases in the first and third rows, one figure for every dot.

```
 3  7  2  1  0
 .  .  .  .  .
 6  2  7  8  9
 .  .  .  .  .
 -------------
19  9  9  9  8
```

You will then immediately fill up the other two lines, and on adding the four lines together the amount will be the original sum.

Explanation.—The last line written beforehand, was only the amount of two rows of five nines.

```
 9  9  9  9  9
 9  9  9  9  9
 -------------
19  9  9  9  8
```

As the person is almost certain to write other figures than these, all you have to do is to put down those that will nine.
supply his deficiencies to make his figures each equal to

For instance if the first figure of the first row is 3, and of the third row 2, begin the second row with 6 and the fourth with 7, by which means the four lines will be equivalent to two rows of nines, and the previously declared amount be verified.

Another Mysterious Sum.

To give a person his choice out of three or four rows of figures, which shall be written down for him; to let him multiply the row he chooses, by any number; to let him suppress or rub out any of the figures; and even to alter the arrangement of the rest, to let him show the figures that remain; and to tell him then what figures he has suppressed.

Suppose the following sets of figures are given to choose from, viz.:

3 6 4 8 5 1
2 3 4 7 6 5
8 2 3 6 4 4

which are all divisible by 9 without a remainder, though the person must not be told so; and suppose he chooses the third line, and multiplies it by 6, the product will likewise be divisible by 9 without a remainder. Now supposing he suppresses the 6, then the sum of the remaining digits is 30, which contains three 9's and 3 over; which 3, wants 6 to make up 9, and therefore 6 was the figure suppressed.

When the sum of the digits, remaining after the suppression of a figure, is divisible by 9 without a remainder, then the figure suppressed must be either 9 or 0.

Two Persons selecting Two Numbers, and Multiplying Them Together, by Knowing the Last Figure of the Product, to Tell the Other Figures.

If the number 73 be multiplied by the numbers of the following arithmetical progression, 3, 6, 9, 12, 15, 18, 21, 24, and 27, their products will terminate with the nine digits in this order, 9, 8, 7, 6, 5, 4, 3, 2, 1; the numbers being as follows, 219, 438, 657, 876, 1095, 1314, 1533, 1752, 1971; therefore put into one of the divisions of divided bag, several tickets marked with the number 73, and in the other part of the bag the numbers 3, 6, 9, 12, 15, 18, 21, 24, and 27.

Then open that part of the bag which contains the numbers 73, and ask someone to take out one ticket only, then quickly change the opening, and invite another person to take a ticket, and when you have multiplied their two numbers together, by knowing the last figure of the product, you will readily tell them by the foregoing series, what the other figures are.

A Pair of Dice being Thrown, to Find the Number of Points on Each Die without Seeing Them.

Tell the person who cast the dice to double the number of points upon one of them, and add 5 to it; then, to multiply the sum produced by 5, and add to the product the number of points upon the other die. This being done, desire him to tell you the amount, and having thrown out 25, the remainder will be a number consisting of two figures, the first of which to the left, is the number of points on the first die, and the second figure to the right, the number on the other.

Thus suppose the number of points of the first die which comes up, to be 2, and that of the other 3; then, if to 4, the double of the points of the first there be added 5, and the sum produced, 9, be multiplied by 5, the product will be 45; to which if 3, the number of points on the other die, be added, 48 will be produced, from which if 25 be subtracted, 23 will remain; the first figure of which is 2, the number of on the other.
points on the first die, and the second figure 3, the number

An Arithmetical Trick with Cards.

Take a pack of cards, place the first card on the table, with the back to you; look at it; if it is a seven, put 5 more cards on the top, making 15 in number; if it is a king, knave, or queen, place 2 cards on the top as before; if an ace, 11, and so on, till the pack is laid out (always making 12 in number). If any cards remain that will not make 12, lay them separate from the rest. Now, supposing you have 6 heaps, and 3 odd cards, strike off 4 heaps, and multiply the remainder to add to the following card. It may be readily seen that it is needless to reckon the kings, which are counted 13. If any spots remain at the last card, subtract them from 13, and the remainder will indicate the spots of the card that has been drawn; if the remainder be 11, it has been a knave; if 12 it has been a queen; but if nothing remains, it has been a king. The colour of the king may be known by examining which one among the cards is wanting.

If you are desirous of employing only 32 cards, the number used at present for piquet, when the cards are added the remaining 2 heaps by 13, adding in the 3 odd cards, which would make 29; then turn up all the heaps face to you, count the pips, and there will be 29! (taking no notice of the odd cards after you have added them in). If there are 10 heaps, and no odd cards, or 12 heaps, and 2 odd cards, always cut off 4 heaps; and multiply the remaining heaps by 13, as before. Court cards count as ten.

To Guess the Number of Spots on any Card, which a Person has Drawn from a Whole Pack.

Take a whole pack, consisting of 52 cards, and desire some person in company to draw out any one at pleasure, without shewing it. Having assigned to the different cards their usual value, according to their spots, call the knave 11, and queen 12, and the king 13. Then add the spots of the first card to those of the second; the last sum to the spots of the third, and so on, always rejecting 13, and keeping as above directed, reject all the tens; then add 4 to the spots of the last card, and a sum will be obtained, which taken from 10, if it be less, or from 20 if it exceeds 10, the remainder will be the number of the card that has been drawn; so that if 2 remains, it has been a knave, if 3 a queen, if 4 a king, and so on.

If the pack be incomplete, attention must be paid to those deficient, in order that the number of the spots of all the cards wanting may be added to the last sum, after as many tens as possible have been subtracted from it; and the sum arising from this addition must, as before, be taken from 10 or 20, according as it is greater or less than 10. It is evident that by again looking at the cards, the one which has been drawn may be discovered.

The demonstration of this rule is as follows: since, in a complete pack of cards, there are 13 of each suit, the values of which are 1, 2, 3, &c., to 13, the sum of all the spots of each suit, calling the knave 11, the queen 12, and the king 13, is seven times 13 or 91, which is a multiple of 13; consequently the quadruple of this sum is a multiple of 13 also; if the spots then of all the cards be added together, always rejecting 13, we must at last find the remainder equal to nothing. It is therefore evident that if a card, the spots of which are less than 13, has been drawn from the pack, the difference between these spots and 13 will be what is wanting to complete that number; if at the end then, instead of reaching 13, we reach only 10, for example, it is evident that the card wanting is a three; and if we reach 13, it is also evident that the card wanting is one of those equivalent to 13, or a king.

If two cards have been drawn from the pack, we may tell, in like manner, the number of spots which they contain both together; that is, how much is wanting to reach 13, or that deficiency increased by 13; and to know which two, nothing is necessary but to count privately how many times 13 has been completed, for with the whole of the cards it ought to be counted 28 times; if it be counted therefore only 27 times, with a remainder, as 7 for example, the spots of the two cards drawn amount together to 6; if 13 be counted only 26 times, with the same remainder, it may be concluded that the two cards formed together 13+6, or 19.

The demonstration of the rule given when the same number of cards is used, as that employed for the game of piquet, viz., 32 cards, calling the ace 1, the knave 2, the queen 3, the king 4, and assigning to the other cards the value of their spots, is attended with as little difficulty; for in each suit there are 44 spots, making altogether 176, which, as well as 44, is a multiple of 11; we may therefore always count to 11, rejecting 11, and the number wanting to reach 11, will be the value of the card which has been drawn.

But the same number 176, if 4 were added to it, would be a multiple of 10 or of 20; and hence a demonstration also of the method which has been taught.

A Person having Drawn, from a Complete Pack of fifty-two Cards, one, two, three, four, or more Cards, to Guess the Whole Number of the Spots which they contain.

Assume any number whatever, such as 15, for example, greater than the number of the spots of the highest card, counting the knave 11, the queen 12, and the king 13, and desire the person to add as many cards from the pack, to the first card he has chosen, as will make up 15, counting the spots of that card; let him do the same thing in regard to the second, the third, the fourth, &c.; and then desire him to tell how many cards remain in the pack. When this is done, proceed as follows:—

Multiply the above number 15, or any other that may have been assumed, by the number of cards drawn from the pack, which we shall here suppose to be three; to the product, 45, add the number of these cards, which will give 48; subtract the 48 from 52, and take the remainder 4, from the cards left in the pack; the result will be the number of spots required.

Let us suppose, for example, that the person has drawn from the pack, a 7, a 10, and a knave, which is equal to 11: to make up the number 15 with a 7, eight cards will be required; to make up the same number with a 10, will require five; and with the knave, which is equal to 11, four will be necessary. The sum of these three numbers, with the three cards, makes 20, and consequently 32 cards remain in the pack. To find the sum of the numbers, 7, 10, 11, multiply 15 by 3, which will give 45; and if the number of the cards drawn from the pack be added, the sum will be 48, which taken from 52, leaves 4. If 4 then be subtracted from 32, the remainder, 28, will be the sum of the spots contained on the three cards drawn from the pack, as may be easily proved by trial.

Another Example.—Let us suppose two cards only drawn from the pack, a 4 and a king, equal to 13; if cards be added to these to make up 15, there will remain in the pack 37 cards.

If 15 be multiplied by 2, the product will be 30, to which if 2, the number of the cards drawn from the pack, be added, we shall have 32; and if 32 be taken from 52, the remainder will be 20. In the last place, if 20 be subtracted from 37, the number of the cards left in the pack, the remainder, 17, will be the number of the spots of the 2 cards drawn from the pack.

Remarks.—I. If 4 or 5 cards are drawn from the pack, it may sometimes happen that a sufficient number will not be left to make up the number 15; but even in this case the operation may still be performed. For example, if 5 cards, the spots contained on which are 1, 2, 3, 4, 5, have been drawn; to complete with each of these cards the number 15 would require, together with the 5 cards, at least 65; but as there are only 52, there are consequently 13 too few. He who counts the pack must therefore say that 13 are wanting.

On the other hand, he who undertakes to tell the number of the spots, must multiply 15 by 5, which makes 75; and to this if 5, the number of the cards, be added, it will give 80; that is to say, 28 more than 52; if 13 then be subtracted by

28, the remainder, 15, will be the number of spots contained on these 5 cards.

But if we suppose that the cards left in the pack are, for example, 22, which would be the case if the five cards drawn were the 8, 9, 10, knave $=11$, and queen $=12$, it would be necessary to add these 22 to the excess of 5 times $15+5$, over 52, that is to say to 28, and we should have 50 for the spots of these 5 cards, which is indeed the exact number of them.

II. If the pack consists not of 52 cards, but of 40, for example, there will still be no difference in the operation; the number of the cards, which remain of these 40, must be taken from the sum produced by multiplying the made-up number by that of the cards drawn, and adding to the product the number of these cards.

Let us suppose, for example, that the cards drawn are 9, 10, 11, that the number to be made up is 12, and that the cards left in the pack are 31. Then $12 \times 3 = 36$, and 3 added for the three cards, makes 39, which subtracted from 40 leaves 1. If one then be taken from 31, the remainder 30 will be the number of the spots required.

III. Different numbers to be made up with the spots of each card chosen might be assumed; but the case would still be the same, only that it would be necessary to add these three numbers to that of the cards, instead of multiplying the same number by the number of cards drawn, and then adding the number of the cards. In this there is so little difficulty, that an example is not necessary.

IV. The demonstration of this method, which some of our readers perhaps may be desirous of seeing, is exceedingly simple, and is as follows. Let a be the number of cards in the pack, c the number to be made up by adding cards to the spots of each card drawn, and b the cards left in the pack; let x, y, z express the spots of the cards, which we shall here suppose to be 3, and we shall then have, for the number of the cards drawn, $c-x+c-y+c-z+3$; which with the cards left in the pack b, must be equal to the whole pack. Then $3c+3-x-y-z+b=a$, or $x+y+z=3c+3+b-a$, or $=b-(a-3c-3)$. But $x+y$

$+z$ is the whole number of the spots; b is the number of cards left in the pack, and $a-3c-3$ is the whole number of cards in the pack, less the product of the number to be completed by the number of the cards drawn, minus that number.

Topsy Turvy.

Six hundred and sixty so ordered may be,
That if you divide the whole number by three,
The quote will exactly in numbers express
The half of six hundred and sixty not less.

If the given figures be turned upside down, they will become 990, the third part of which, viz., 330 is half of 660.

One from Five leaves Five.

From half of five take one, then five shall still remain.
One half of FIVE = IV,
Take away I and V or 5 remains.

A Question of Halves.

Divide half of nine by half of five, the quotient shall be one.

Half of IX when cut through the middle is IV; and half of FIVE is also IV; and the quotient resulting from dividing the former by the latter is evidently 1.

Odds and Evens.

Place the nine digits, so that the sum of the odd digits may be equal to the sum of the even ones.

$$2+4+6+\cdot 8=12\cdot 8$$
$$1+3+7+\tfrac{9}{5}=12\cdot 8$$

II.---Amusing Problems

The Mule and the Ass.

A mule and an ass travelling together, the ass began to complain that her burden was too heavy. " Lazy animal " said the mule, " you have little reason to complain; for if I take one of your bags, I shall have twice as many as you, and if I give you one of mine we shall then only have an equal number." With how many bags was each loaded?

As the mule and the ass will both have equal burdens when the former gives one of his measures to the latter, it is evident that the difference between the measures which they carry is equal to 2. Now if the mule receives one from the ass, the difference will be 4; but in that case the mule will have double the number of measures that the ass has; consequently the mule will have 8, and the ass 4. If the mule then gives one to the ass, the latter will have 5, and the former 7. These were the number of the measures with which each was loaded, and which solve the problem.

The False Balance.

A Dutch cheese being placed in one of the scales of a false balance, was found to weigh 16lbs., and when placed in the other only 9lbs. What was its true weight,

The true weight was a mean proportional between the two false ones, and is found by extracting the square root of their product.

Thus $16 \times 9 = 144$ of which the square root is 12. The true weight of the cheese was therefore 12lbs.

The Climbing Snail.

A snail in climbing a mast 20 feet high, ascended 8 feet every day, and came down 4 again every night. How long would he be in getting to the top of the pole?

Since he has gained 4 feet in height every day and night, he will have advanced 12 feet in 3 days; and therefore in 4 days he will reach the top.

The Maypole.

There was a maypole which consisted of three pieces of timber, of which the first (or lowermost) was 13ft. long, the third (or uppermost) was as long as the lowermost and half the middle piece; and the middle piece was as long as the uppermost and lowermost together. How high was this maypole, and how long was each piece?

Multiply the length of the first piece 13, by 3, and the result, 39, is the length of the uppermost piece. Again multiply 13 by 4, and the result, 52, is the length of the middle piece, which is equal to the sum of the lower and the upper pieces. The height of the maypole was therefore 104ft.

A Question of Hair.

Supposing there are more persons in the world than any of them has hairs upon his head, it then necessarily follows that some two of them at least must have exactly the same number of hairs on their heads to a hair. Can you prove this?

The greatest variety that can be in the number of hairs, is equal to the greatest number that any person has; viz., one person having but one, another two, another three, and so on to the greatest number; but as, by the supposition, there are still more persons, whatever number they may have some one of the preceding must have the same. Hence the proposition is manifest.

The Schoolmaster's Trick.

A schoolmaster to amuse his scholars, showed them a number which he said was the sum of six rows, each consisting of four figures. He desired them to write down three rows of figures, to which he would add three more; and assured them that the sum of the whole should be equal to the number he showed them. How must this be done?

The number shown by the master is 29997, which is 9999×3. Now if every figure, in each row of the master's be made a complement to 9 of the scholar's row, it is evident that the sum will be equal to the number proposed.

Thus: suppose the scholar's write down three rows as follows:—

	7285 5829 3456	The scholar's rows
Then we shall have	2714 4170 6543	The master's rows
Total	29997	The number proposed.

Supposition.

Three persons were disputing about their money. Said A to B and C—" If eleven sovereigns were added to my money, I should have as much as you both." Then replied B, " If eleven sovereigns were added to my money, I should have twice as much as you both?" C answered " If eleven sovereigns were added to my stock, I should have three times as much as you both." How much had each?

A had 1; B 5; and C 7 sovereigns. Thus, A $1+11=12$; B, $5+11=16$; C, $7+11=18$.

The Unlucky Number.

Divide the number 13 into three parts, so that their squares may have equal differences, and the sum of their squares may be 75.

$1+5+7=13$

$1 \times 1 = 1$

$5 \times 5 = 25$—difference 24

$7 \times 7 = 49$—difference 24

—

75

How Old Was He?

On being asked how old he was a gentleman replied:—" The square of my age 60 years ago is double my present age." How old was he?

The gentleman's age was 72. Thus: 60 years ago he was 12 years of age. The square of 12 is 144, which divided by 2 gives 72.

A Mysterious Number.

There is a certain number which is divided into four parts. To the first part you add 2, from the second part you subtract 2, the third part you multiply by 2, and the fourth part you divide by 2, and the sum of the addition, the remainder of the subtraction, the product of the multiplication, and the quotient of the division, are all equal and precisely the same. How is this?

The number is 45, which divided into four parts, viz.: 8, 12, 5, and 20, which equal 45. To the first part you add 2: $8+2=10$; from the second you subtract 2: $12-2=10$; the third part you multiply by 2: $5 \times 2 = 10$; and the fourth part you divide by 2: $20 \div 2 = 10$. Consequently the *sum* of the addition, the *remainder* of the subtraction, the are precisely the same—10.
product of the multiplication, and the *quotient* of the division

Gripus' Will.

When Gripus died, in sterling gold was found,
Left for his family, eight thousand pound,
To be bestowed, as his last will directed,
Which did provide that none should be neglected;
For to each son (there being in number five)
Three times each daughter's portion he did give:
His daughters four, were each of them to have
Double the sum he to the mother gave.
Now that his wish may justly be fulfill'd,
What must the widow have, and what each child?

Widow's part	1
4 Daughters'	8
5 Sons'	30
	39

£8000 ÷ 39 = £205 2s. $6\frac{10}{13}$d., the widow's share.

This sum doubled = £410 5s. $1\frac{7}{13}$d., each daughter's share.

This sum trebled = £1230 15s. $4\frac{8}{13}$d., each son's share.

The Travellers.

A company of travellers spent in the Refreshment Rooms of a Railway Station the sum of six shillings and one farthing! and each of them had as many farthings to pay as there were persons in the company. How many travellers were there?

Six shillings and one farthing = 289 farthings; and this must be equal to the number of persons multiplied into the sum spent by each. In the present case the multiplicand and the multiplier are equal, and therefore we have only to find what number multiplied into itself will produce the given sum 289; or in other words, to find the square root of
= $4\frac{1}{4}$d., which is the money spent by each.
289. This = 17, the number of travellers; and 17 farthings

An Egg Problem.

Three young women went to market with eggs; the first having 50 to sell, the second 30, the third no more than ten. All three sold out, and at the same rate, and each made the same sum of money of her eggs. How were they sold?

On coming to market, eggs were selling at seven a penny, at which rate the first woman sold 49, and received sevenpence; the second sold 28, and of course received fourpence; whilst the third sold a single pennyworth; but she had three eggs remaining, whilst her companions had but one and two respectively. In the course of the day the demand increasing, she advanced her price to threepence each, for which she sold her three last eggs, and received ninepence. Her companions following her example, sold their remaining eggs for threepence each, and also realised the sum of tenpence.

First woman, for	49	eggs received	7d.
And for	1	,, ,,	3d.
	50		10d.
Second woman, for	28	eggs received	4d.
And for	2	,, ,,	6d.
	30		10d.
Third woman, for	7	eggs received	1d.
And for	3	,, ,,	9d.
	10		10d.

The Counting Match.

A in five hours a sum can count,
Which B can in eleven;
How much more then is the amount
They both can count in seven?

In 1 hour A can count $\frac{1}{5}$ of the sum.
" " B " $\frac{1}{11}$ of the sum.
" " A + B " $\frac{1}{5}+\frac{1}{11}=\frac{16}{55}$.
Therefore in 7 days they will count $\frac{16 \times 7}{55}=2\frac{2}{55}$ sums.
$2\frac{2}{55}-1=1\frac{2}{55}$ the answer required.

The Cat, the Dog, and the Leg of Mutton.

A man has a cat, a dog, and a leg of mutton, to carry over a river; but being obliged to transport them one by one on account of the smallness of the boat, in what manner is this to be done, that the cat may not be left with the dog, nor the dog with the leg of mutton.

He must first carry over the dog, and then return for the cat: when he carries over the cat, he must take back with him the dog, and leave it, in order to carry over the leg of mutton; he may then return and carry over the dog. By these means the cat will never be left with the dog, nor the dog with the leg of mutton, except when the boatman is present.

To Ascertain the Length of the Day and Night, at any Time of the Year.

Double the time of the sun's rising, which gives the length of the night, and double the time of the setting, which gives the length of the day.

The Show Pen.

At an Agricultural Show a cattle pen was formed with 50 gates of equal length. It was found necessary, later, to double the size of the pen. How many additional gates would be required to do this?

Only two. There were 48 gates on each side of the pen, one at the top, and another at the bottom. If one of the sides were moved back, and an additional gate inserted at the top and bottom, the size of the pen would be exactly doubled.

The Country Woman and the Eggs.

A country woman carrying eggs to a garrison, where she had three guards to pass, sold at the first, half the number she had and half an egg more; at the second, the half of what remained and half an egg more; and at the third, the half of the remainder and half an egg more; when she arrived at the market place, she had three dozen still to sell. How was this possible without breaking any of the eggs?

It would appear, on the first view that this problem is impossible; for how can half an egg be sold without breaking any? The possibility of it, however, will be evident, when it is considered, that by taking the greater half of an odd number, we take the exact half $+\frac{1}{2}$. It will be found therefore that the woman, before she passed the last guard, had 73 eggs remaining, for by selling 37 of them at that guard, which is the half $+\frac{1}{2}$, she would have 36 remaining. In like manner, before she came to the second guard she had 147; and before she came to the first, 295.

In a Bit of a Hole!

A ship was in a situation with a hole in one of her planks of twelve inches square, and the only piece of plank that could be had, was sixteen inches long by nine inches broad. Required to know how this said piece must be cut into four pieces, so as to repair the hole perfectly and without waste.

Cut off four inches from the narrow end of the given piece, and divide the piece so cut off into three equal pieces by cuts in the shortest direction. When arranging these three pieces lengthways on the top of the remainder, a square of twelve inches will be formed.

The Three Graces, and the Nine Muses.

The three Graces, carrying each an equal number of oranges, were met by the nine Muses, who asked for some of them; and each Grace having given to each Muse the same number, it was then found that they had all equal shares. How many had the Graces at first?

The least number which will answer this problem is 12; for if we suppose that each Grace gave one to each Muse, the latter would each have three; and there would remain 3 to each Grace. The numbers 24, 36, 48, etc., will also answer the question; and after the distribution is made, each of the Graces and Muses will have 6 or 9 or 12, etc.

Pythagoras and his School.

"Tell me, illustrious Pythagoras, how many pupils frequent thy school? One half, replied the philosopher, study mathematics, one fourth natural philosophy, one seventh observe silence, and there are three females besides."

The question here is, to find a number, the $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{7}$ of which $+3$, shall be equal to that number. It may be easily replied that this number is 28.

One Way of Putting It!

A zoological collection was being shipped to England on board a P. and O. liner. Meeting the keeper on deck one morning, one of the passengers asked him how many birds and beasts he had under his charge. The keeper made the following curious reply. "They total 72 heads and 200 feet." How many were there of each?

There were 72 animals altogether. Had they all been birds they would have had 144 feet. Had they all been beasts they would have had 288 feet. It is obvious there were some of each kind. Let us suppose the numbers equal, then: 36 birds or 72 feet + 36 beasts or 144 feet = 216 feet. But this exceeds the number of feet given by 16. Therefore we must add 8 birds and deduct 8 beasts.

44	birds	have	88 feet
28	beasts	,,	112 feet
72			200

The Blind Abbot and the Monks.

The following capital puzzle, old as it is, will be found very amusing. A convent, in which there were nine cells, was occupied by a blind abbot and twenty-four monks, the abbot lodging in the centre cell, and the monks in the side cells, three in each, forming a row of nine persons on each side of the building, as in the accompanying figure.

3	3	3
3		3
3	3	3

Fig. 1.

4	1	4
1		1
4	1	4

Fig. 2.

The abbot, suspecting the fidelity of the monks, frequently went round at night and counted them, when, if he found nine in each row, he retired to rest quite satisfied. The monks, however, taking advantage of his blindness, conspired to deceive him, and arranged themselves in the cells as in Fig. 2, so that four could go out, and still the abbot would find nine in each row.

2	5	2
5		5
2	5	2

Fig. 3.

1	7	1
7		7
1	7	1

Fig. 4.

The monks that went out returned with four visitors, and they were arranged with the monks as in fig. 3, so as to count nine each way, and consequently the abbot was again deceived.

Emboldened by success, the monks next night brought in four more visitors, and succeeded in deceiving the abbot by arranging themselves as in fig. 4.

Again four more visitors were introduced, and arranged with the monks as in fig. 5.

Finally, even when the twelve clandestine visitors had departed, carrying off six of the monks with them, the abbot, still finding nine in each row, as in fig. 6, retired to rest with full persuasion that no one had either gone out or come in.

0	9	0
9		9
0	9	0

Fig. 5.

5	0	4
0		0
4	0	5

Fig. 6.

Answer.—How it came to pass that the abbot should become confused is easily explained. The numbers in the angular cells were counted twice; these cells belonging to *two* rows, the more therefore the angular cells are filled by emptying those in the middle of each row, the double counting increases the whole sum, and the contrary is the case in proportion as the middle cells are filled by emptying the angular ones.

The Costermonger's Puzzle.

A costermonger bought 120 apples at two a penny, and 120 more of another sort at three a penny; but not liking his bargain, he mixed them together, and sold them out again at five for two pence, thinking he should recover the same sum; but on counting his money, he found that he had lost fourpence. How was this?

At first sight there appears no loss; for supposing that in selling five apples for two pence, the costermonger gave three of the sort at three a penny, and two of those at two a penny, he would receive just the same money as he bought them for; but it is evident that the latter stock would be exhausted first, and consequently the man must sell as many of the former as remained over at five for two pence (bought at two a penny or four for two pence), and would therefore lose. When all the latter sort were sold in the above manner, he would have sold only eighty of the former, for there are as many threes in one hundred and twenty, as twos in eighty; therefore the remaining forty must be sold at five for twopence, which were bought at the rate of four for two pence, thus:

If 4 : 2 :: 40 : 20, prime cost of 40 of the first sort.
5 : 2 :: 40 : 16, selling price of ditto.

4 pence loss.

The Three Jealous Husbands.

Three jealous husbands, with their wives, having to cross a river at a ferry, find a boat without a boatman; but the boat is so small that it can contain no more than two of them at once. How can these six persons cross the river, two and two, so that none of the women shall be left in company with any of the men, unless when her husband is present?

Two women cross first, and one of them rowing back the boat, carries over the third woman. One of the three women then returns with the boat, and remaining, suffers the two men, whose wives have crossed, to go over in the boat. One of the men then carries back his wife and leaving her on the bank, rows over the third man. In the last place, the woman who had crossed enters the boat, and returning twice, carries over the other two women.

This question is proposed also under the title of the three masters and the three valets. The masters agree very well, and the valets also; but none of the masters can endure the valets of the other two; so that if any one of them were left with any of the other two valets, in the absence of his master, he would infallibly cane him.

Sir Isaac Newton's Problem.

If 12 oxen will eat $3\frac{1}{3}$ acres of grass in 4 weeks, and 21 oxen will eat 10 acres of grass in 9 weeks, how many oxen will eat 24 acres in 18 weeks, *the grass being allowed to grow uniformly?*

$$\frac{3\frac{1}{3} \times 21 \times 9}{12 \times 4} = 13\tfrac{1}{8} \text{ acres.}$$

Now it appears that 21 oxen would consume $13\frac{1}{8}$ acres of grass, provided the grass did not grow during the last five weeks, but only 10 acres when it did so grow. Hence it is manifest that the quantity of grass grown upon 10 acres in five weeks is $13\frac{1}{8} - 10 = 3\frac{1}{8}$ acres.

We must now find what will be the increase of grass on 24 acres in 14 weeks, which is the difference between 4 weeks and the given 18.

$$\frac{3\frac{1}{8} \times 24 \times 14}{10 \times 5} = 21 \text{ acres.}$$

Hence the quantity of grass that will grow on 24 acres in the last 14 weeks will be equal to 21 acres; and hence the whole quantity of grass to be consumed in the 18 weeks will be equal to 45 acres.

$$\frac{12 \times 45 \times 4}{3\frac{1}{3} \times 18} = 36 \text{ oxen.}$$

To find the Number of Deals a Person may play at the Game of Whist, without holding the same Cards twice.

The number of cards played with at *whist*, being 52, and the number dealt to each person 13; if this be taken from the whole pack, the number of cards remaining will be 39, any 13 of which may be those the person takes in; and therefore we are to find in how many ways 13 cards may be taken out of 39, which is done as follows:

Multiply 52 severally by 51, 50, 49, and *so forth*, backwards to 41, which will give 3954242643911239680000 for the product. Then divide this number, separately, by

1, 2, 3, &c. to 13, and the quotient will be 6227020800; which is the number of different ways 13 cards may be taken out of 52, and consequently the number required.

A question, somewhat similar to this, though more difficult to be solved, is, to determine the number of *fifteens* that may be made, as in the game of Cribbage, out of a common pack of 52 cards, which is found, by computation, to be no less than 17264, each counting two holes on the *board*.

Simple Division!

To distribute among three persons 21 casks of wine, 7 of them full, 7 of them empty, and 7 of them half-full; so that each of them shall have the same quantity of wine, and the same number of casks.

This problem admits of two solutions, which may be clearly comprehended by means of the two following tables:

I.

Persons	Full Casks.	Empty.	Half-Full.
1st	2	2	3
2nd	2	2	3
3rd	3	3	1

II.

Persons	Full Casks.	Empty.	Half-Full.
1st	3	3	1
2nd	3	3	1
3rd	1	1	5

The Arabians and the Stranger.

Two Arabians sat down to dinner; one had five loaves, the other three. A stranger passing by, desired permission to eat with them, to which they agreed. The stranger dined, laid down eight pieces of money and departed. The proprietor of the five loaves took up five pieces, and left three for the other, who objected, and insisted on one-half. The cause came before Ali (the magistrate) who gave the following judgment:—" Let the owner of the five loaves have seven pieces of money, and the owner of the three pieces, one." Was the sentence just?

Yes, Ali's sentence was just; for suppose the loaves to be divided into three equal parts, making twenty-four parts in all the eight loaves, and each person to eat equal or eight parts. Therefore, the stranger had seven parts of the person who contributed five loaves, or fifteen parts, and only one of him who contributed only three loaves, which made nine parts.

An Awkward Measure.

A gentleman has a bottle, containing 8 pints of choice wine, and wishes to make a present of one-half of it to a friend; but as he has nothing with which to measure it, except two other bottles, one capable of containing 5 and the other 3 pints, how must he manage, so as to put exactly 4 pints into the bottle capable of containing 5?

To enable us to resolve this problem we will call the bottle containing the 8 pints A; that of 5 pints B; and that of 3 pints C; supposing that there are 8 pints of wine in the bottle A, and that the other two are empty, as seen at D. Having filled the bottle B with wine from the bottle A, in which there will remain no more than 3 pints as seen at E, fill the bottle C from B, and consequently there will remain only 2 pints in the latter, as seen at F: then pour the wine of C into A, which will thus contain 6 pints, as seen at G, and pour the two pints of B into C, as seen at H. In the last place, having filled the bottle B from the bottle A, in which there will remain only 1 pint, as seen at I, fill up C from B in which there will remain 4 pints, as seen at K; and thus the problem is solved.

	8 A	5 B	3 C
D	8	0	0
E	3	5	0
F	3	2	3
G	6	2	0
H	6	0	2
I	1	5	2
K	1	4	3

If you are desirous of making the four pints of wine remain in the bottle A, which we have supposed to be filled with 8 pints, instead of remaining in the bottle B, fill the bottle C with wine from the bottle A, in which there will remain only 5 pints, as seen at D; and pour 3 pints of C into B, which will consequently contain 3 pints, as seen at E; having then filled C from A, in which there will remain no more than 2 pints, as seen at F; fill up B from C which will thus contain only 1 pint as seen at G. In the last place, having poured the wine of the bottle B into the bottle A, which will thus have 7 pints as seen at H; pour the pint of wine which is in C into B, consequently the latter will contain 1 pint, as seen at I; and then fill up C from A, in which there will remain only 4 pints, as was proposed, and as seen at K.

	8	5	3
	A	B	C
	8	0	0
D	5	0	3
E	5	3	0
F	2	3	3
G	2	5	1
H	7	0	1
I	7	1	0
K	4	1	3

A Capacious Body.

Mathematicians affirm that of all bodies contained under the same superficies, a sphere is the most capacious. But they have never considered the amazing capaciousness of a body, the name of which is now required, of which it may be truly affirmed, that supposing its greatest length 9 inches, greatest breadth 4 inches, and greatest depth 3 inches, yet under these dimensions it contains a solid foot?

A shoe!

III.---Problems in Progressions

Explanation of the Most Remarkable Properties of an Arithmetical Progression.

If there be a series of numbers, either increasing or decreasing, in such a manner, that the difference between the first and the second shall be equal to that between the second and third, and between the third and fourth, and so on successively; these numbers will be in arithmetical progression.

The series of numbers 1, 2, 3, 4, 5, 6, &c.; or 1, 5, 9, 13, &c.; or 20, 18, 16, 14, 12, &c.; or 15, 12, 9, 6, 3, are therefore arithmetical progressions; for in the first, the difference between each term and the following one, which exceeds it, is always 1; in the second it is 4; in like manner this difference is always 2 in the third series, which goes on decreasing; and in the fourth it is 3.

It may be readily seen, that an increasing arithmetical progression may be continued 'ad infinitum'; but this cannot be the case, in one sense, with a decreasing series; for we must always arrive at some term, from which if the common difference be taken, the remainder will be 0, or else a negative quantity. Thus, the progression 19, 15, 11, 7, 3, cannot be carried farther, at least in positive numbers; for it is impossible to take 4 from 3, or if it be taken we shall have, according to analytical expression, -1; and by continuing the subtraction we should have -5, -9, &c. (As the quantities called negative are real quantities, taken in a sense contrary to that of the quantities called positive, it is evident that, according to mathematical and analytical strictness, an arithmetical progression may be continued 'ad infinitum,' decreasing as well as increasing; but we here speak agreeably to the vulgar mode of expression.)

The chief properties of arithmetical progressions may be easily deduced from the definitions which we have here given. For a little attention will shew,

1st. That each term is nothing else than the first, plus or minus the common difference multiplied by the number of intervals between that term and the first. Thus, in the progression 2, 5, 8, 11, 14, 17, &c., the difference of which is three, there are five intervals between the sixth term and the first; and for this reason the sixth term was equal to the first plus 15, the product of the common difference 3 by 5. But as the number of intervals is always less by unity than the number of terms, it thence follows, that we may find any term, the place of which in the series is known, if we multiply the common difference by the number expressing that place less unity. According to this rule, the hundredth term of an increasing progression will be equal to the first plus 99 times the common difference. If it be decreasing, it will be equal to the first term minus that product.

In every arithmetical progression therefore, the common difference being given, to find any term the place of which is known; multiply the common difference by the number which indicates that place less unity, and add the product to the first term, if the progression be increasing, but subtract it if it be decreasing; the sum or remainder will be the term required.

2nd. In every arithmetical progression, the sum of the first and last terms is equal to that of the second and the last but one; and to that of the third and last but two, &c.; in a word, to the sum of the middle terms if the number of the terms be even, or to the double of the middle term if the number of the terms be odd.

This may be easily demonstrated from what has been said; for let us call the first term A, and let us suppose that there are twenty terms in the progression; if it be increasing, the twentieth term will be equal to A plus nineteen times the common difference; and their sum will be double the first term plus nineteen times that difference. But the second term is equal to the first plus the common difference, and the nineteenth term, or last but one, according to our supposition, is equal to the first plus eighteen times that difference. The sum therefore of the second and last but one, is twice the first term plus nineteen times the common difference, the same as before. And so of the third and last but two.

3rd. By this last property we are enabled to shew in what manner the sum of all the terms of an arithmetical progression may be readily found; for, as the first and last terms make the same sum as the second and last but one, and as the third and the last but two, &c.; in short as the two middle terms, if the number of terms be even; it thence follows, that the whole progression contains as many times the sum of the first and last terms, as there are pairs of such terms. But the number of pairs is equal to half the number of terms; consequently the sum of the whole progression is equal to the product of the sum of the first and last terms multiplied by half the number of terms, or, what amounts to the same, to half the product of the sum of the first and the last terms by the number of the terms of the progression.

If the number of the terms be odd, as 9 for example; it may be readily seen that the middle term will be equal to half the sum of the two next to it, and consequently the sum of the first and last. But the sum of all the terms, the middle term excepted, is equal to the product of the sum of the first and last terms by the number of terms less unity, for example 8 in the case here proposed, where there are 9 terms; consequently, by adding the middle term, which will complete the sum of the progression, and which is equal to half the sum of the first and last terms, we shall have, for the sum total of the progression, as many times the half sum above mentioned, as there are terms in the progression; which is the same thing as the product of half the sum of the first and last terms by the number of the terms, or the product of the whole sum by half the number of terms.

When these rules are well understood, it will be easy to resolve the following questions.

A Stone Problem.

If a hundred stones are placed in a straight line, at the distance of a yard from each other; how many yards must the person walk, who undertakes to pick them up one by one, and to put them into a basket a yard distant from the first stone.

It is evident, that to pick up the first stone, and put it into the basket, the person must walk two yards, one in going and another in returning; that for the second he must walk 4 yards; and so on, increasing by two as far as the hundredth, which will oblige him to walk two hundred yards, one hundred in going, and one hundred in returning. It may easily be perceived also, that these numbers form an arithmetical progression, in which the number of terms is 100, the first term 2, and the last 200. The sum total therefore will be the product of 202 by 50, or 10100 yards, which amount to more than five miles and a half.

The Debtor.

A merchant being considerably in debt, one of his creditors, to whom he owed £1860, offered to give him an acquittance if he would agree to pay the whole sum in 12 monthly instalments; that is to say, £100 the first month, and to increase the payment by a certain sum each succeeding month, to the twelfth inclusive, when the whole debt would be discharged; by what sum was the payment of each month increased?

In this problem the payments to be made each month ought to increase in arithmetical progression. We have given the sum of the terms, which is equal to the sum total of the debt, and also the number of these terms, which is 12; but their common difference is unknown, because it is that by which the payments ought to increase each month.

To find this difference, we must take the first payment multiplied by the number of terms, that is to say 1200 pounds, from the sum total, and the remainder will be 660; we must then multiply the number of terms less unity, or 11, by half the number of terms, or 6, and we shall have 66; by which, if the remainder 660 be divided, the quotient 10 will be the difference required. The first payment, therefore, being 100, the second payment must have been 110, the third 120, and the last 210.

A Well Problem.

A gentleman employed a bricklayer to sink a well, and agreed to give him at the rate of three shillings for the first yard in depth, five for the second, seven for the third, and so on increasing till the twentieth, where he expected to find water; how much was due to the bricklayer when he had completed the work?

This question may be easily answered by the rules already given; for the difference of the terms is 2, and the number of terms 20; consequently, to find the twentieth term, we must multiply 2 by 19, and add 38, the product, to the first term 3, which will give 41 for the twentieth term.

If we then add the first and last terms, that is 3 and 41, which will make 44, and multiply this sum by 10, or half the number of terms, the product 440 will be the sum of all the terms of the progression, or the number of shillings due to the bricklayer, when he had completed the work. He would, therefore, have to receive £22.

Another Well Problem.

A gentleman employed a bricklayer to sink a well to the depth of 20 yards, and agreed to give him £20 for the whole; but the bricklayer falling sick, when he had finished the eighth yard, was unable to go on with the work; how much was then due to him?

Those who might imagine that two-fifths of the whole sum were due to the workman, because 8 yards are two-fifths of the depth agreed on, would certainly be mistaken;

for it may be easily seen that, in cases of this kind, the labour increases in proportion to the depth. We shall here suppose, for it would be difficult to determine it with any accuracy, that the labqur increases arithmetically as the depth; consequently the price ought to increase in the same manner.

To determine this problem, therefore, £20 or 400 shillings must be divided into 20 terms in arithmetical progression, and the sum of the first eight of these will be what was due to the bricklayer for his labour.

But 400 shillings may be divided into twenty terms, in arithmetical proportion, a great many different ways, according to the value of the first term, which is here undetermined; if we suppose it, for example, to be 1 shilling, the progression will be 1, 3, 5, 7, &c., the last term of which will be 39; and consequently the sum of the first eight terms will be 64 shillings. On the other hand, if we suppose the first term to be $10\frac{1}{2}$, the series of terms will be $10\frac{1}{2}$, $11\frac{1}{2}$, $12\frac{1}{2}$, $13\frac{1}{2}$, $14\frac{1}{2}$, which will give 112 shillings for the sum of the first eight terms.

But to resolve the problem in a proper manner, so as to give to the bricklayer his just due for the commencement of the work, we must determine what is the fair value of a yard of work, similar to the first, and then assume that value as the first term of the progression. We shall here suppose that this value is 5 shillings; and in that case the required progression will be 5, $6\frac{11}{19}$, $8\frac{3}{19}$ $9\frac{14}{19}$, $11\frac{6}{19}$, $12\frac{17}{19}$, &c., the common difference of which is $\frac{30}{19}$, and the last term 35. Now to find the eighth term, which is necessary before we can find the sum of the first eight terms, multiply the common difference $\frac{30}{19}$ by 7, which will give $11\frac{1}{19}$, and add this product to 5 the first term, which will give the eighth term $16\frac{1}{19}$; if we then add $16\frac{1}{19}$ to the first term, and multiply the sum, $21\frac{1}{19}$, by 4, the product $84\frac{4}{19}$, will be the sum of the first eight terms, or what was due to the bricklayer, for the part of the work he had completed. The bricklayer, therefore, had to receive $84\frac{4}{19}$ shillings, or £4 4s. 2d.

The Lost Purse.

A gentleman, having lost his purse, could not tell the exact sum it contained, but recollected that when he counted the pieces two by two, or three by three, or five by five, there always remained one; and that when he counted them seven by seven, there remained nothing. What was the number of pieces in his purse?

It may be readily seen that, to solve this problem, nothing is necessary but to find a number which when divided by 7 shall leave no remainder; and which when divided by 2, 3, 5, shall always leave one. Several methods may be employed for the purpose; but the simplest is as follows:

Since nothing remains when the pieces are counted seven by seven, the number of them is evidently some multiple of 7; and since 1 remains when they are counted two by two, the number must be an odd multiple; it must therefore be some of the series 7, 21, 35, 49, 63, 77, 91, 105, &c.

This number also, when divided by 3, must leave unity; but in the above series, 7, 49, and 91, which increase arithmetically their difference being 42, are the only numbers that have the above property. It appears likewise, that if 91 be divided by 5, there will remain 1; and we may thence conclude that the first number which answers the question is 91; for it is a multiple of 7, and being divided by 2, 3, or 5, the remainder is always 1.

Several more numbers, which answer this question, may be found by the following means: continue the above progression, in this manner: 7, 49, 91, 133, 175, 217, 259, 301, until you find another term divisible by 5, that leaves unity; this term will be 301, and will also answer the conditions of the problem; but the difference between it and 91, is 210, from which it may be concluded, that if we form the progression:

91, 301, 511, 721, 931, 1141, &c.,

all these numbers will answer the conditions of the problem also.

It would, therefore, be still uncertain what money was in the purse, unless the owner could tell nearly the sum it contained. Thus, for example, if he should say that there were about 500 pieces in it, we might easily tell him that the number was 511.

Let us now suppose that the owner had said, that when he counted the pieces two by two there remained 1; that when he counted them three by three there remained 2; four by four, 3; five by five, 4; six by six, 5, and, in the last place, that when he counted them seven by seven, nothing remained.

It is here evident that the number, as before, must be an odd multiple of 7, and consequently one of the series 7, 21, 35, 49, 63, 77, 91, 105, &c. But the numbers 35 and 77, of this series, answer the conditions of leaving 2 as a remainder when divided by 3, and their difference is 42. For this reason we must form a new arithmetical progression, the difference of which is 42, viz.,

35, 77, 119, 161, 203, 245, 287, &c.

We must then seek for two numbers in it, which when divided by 4 shall leave 3 as remainder. Of this kind are the numbers 35, 119, 203, 287; and therefore we must form a new progression, the difference of the terms of which is 84,

35, 119, 203, 287, 371, 455, 623, &c.

In this new progression we must seek for two terms, which when divided by 5, shall leave 4; and it will be readily seen that these numbers are 119, and 539, the difference of which is 420. A series of terms, therefore, which answer all the conditions of the problem except 1, is

119, 539, 959, 1379, 1799, 2219, 2639, &c.

But the last condition of the problem is, that the required number, when divided by 6, leaves 5 as a remainder. This property belongs to 119, 959, 1799, &c., always adding 840; consequently the number sought is one of those in that progression. For this reason, as soon as we know nearly within what limits it is contained, we shall be able to determine it.

If the owner therefore of the purse had said, that it contained about 100 pieces, the number required would be 119; if he had said there were nearly 1000, it would be 959, &c.

IV.--Magic Squares

The name Magic Square, is given to a square divided into several other small equal squares or cells, filled up with the terms of any progression of numbers, but generally an arithmetical one, in such a manner, that those in each band, whether horizontal, or vertical, or diagonal, shall always form the same sum.

These squares have been called "Magic Squares," because the ancients ascribed to them great virtues; and because this disposition of numbers formed the basis and principle of many of their talismans.

According to this idea, a square of one cell, filled up with unity, was the symbol of the deity, on account of the unity and immutability of God; for they remarked that this square was by its nature unique and immutable; the product of unity by itself being always unity. The square of the root 2 was the symbol of imperfect matter, both on account of the four elements, and of the impossibility of arranging this square magically. A square of 9 cells was assigned or consecrated to Saturn; that of 16 to Jupiter; that of 25 to Mars; that of 36 to the Sun; that of 49 to Venus; that of 64 to Mercury; and that of 81, or nine on each side, to the Moon.

Those who can find any relation between the planets and such an arrangement of numbers, must no doubt have minds strongly tinctured with superstition; but such was the tone of the mysterious philosophy of Jamblichus, Porphyry, and their disciples. Modern mathematicians, while they amuse themselves with these arrangements, which require a fairly extensive knowledge of combination, attach to them no more importance than they really deserve.

Magic Squares are divided into even and odd. The former are those the roots of which are even numbers, as 2, 4, 6, 8, etc.; the latter of those the roots of which are odd, and which, by a nècessary consequence, have an odd number of cells; such are the squares of 3, 5, 7, 9, etc. As the arrangement of the latter is much easier than that of the former, we shall treat of them first.

I.—Odd Magic Squares.

We shall here suppose an odd square, the root of which is 5, and that it is required to fill it up with the first 25 of the natural numbers. In this case, begin by placing unity in the middle cell of the horizontal band at the top; then proceed from left to right, ascending diagonally, and when you go beyond the square, transport the next number 2 to the lowest cell of that vertical band to which it belongs; set 3 in the next cell, ascending diagonally from left to right, and as 4 would go beyond the square, transport it to the most distant cell of the horizontal band to which it belongs; set 5 in the next cell, ascending diagonally from left to right, and as the following cell, where 6 would fall, is already occupied by 1, place 6 immediately below 5; place 7 and 8 in the two next cells, ascending diagonally, as seen in the figure; and then set 9 at the bottom of the last vertical band; then 10 in the last cell on the left of the second horizontal band; then 11 below it; after which continue to fill up the diagonal with the numbers 12, 13, 14, 15; and as you can ascend no further, place the following number 16 below 15; if you then proceed in the same manner, the remaining cells of the square may be filled up without any difficulty, as seen in the above figure. The following are the squares of 3 and 7 filled up by the same method.

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

8	1	6
3	5	7
4	9	2

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	45	3	12
22	31	40	49	2	11	20

Remarks.

1.—According to this disposition, the most regular of all, the middle number of the progression occupies the centre, as 5 in the square of 9 cells, 13 in that of 25, and 25 in that of 49; but this is not necessary in the arrangement of all magic squares.

2.—In each of the diagonals, the numbers which occupy the cells equally distant from the centre, are double that in the centre; thus $30+20=47+3=28+22=24+26$ etc., are double the central number 25.

3.—The case is the same with the cells centrally opposite, that is to say, those similarly situated in regard to the centre, but in opposite directions both laterally and perpendicularly: thus 31 and 19 are cells centrally opposite, and the case is the same in regard to 48 and 2, 13 and 37, 14 and 36, 32 and 18. But it happens, that, according to this magic arrangement, those cells opposite in this manner, are always double the central number, being equal to 50, as may be easily proved.

II.—Even Magic Squares.

A.

The following is a method of constructing squares evenly even, that is to say, those, the root of which when halved is even, or can be divided by 4 without remainder. Of this kind are the squares of 4, 8, 12, etc.

Let us suppose that the annexed square is to be filled up magically, with the first 16 of the natural numbers: Fill up first the two diagonals; and for that purpose begin to count the natural numbers, in order, 1, 2, 3, 4, etc., on the cells of the first horizontal band from left to right; then proceed to the second band, and when you come to the cells belonging to the diagonals, inscribe the numbers counted as you fall upon them; by which means you will have the arrangement represented. When the diagonals have thus been filled, to fill up the cells which remain vacant, begin to count the same numbers, proceeding from the angle D in the cells of the lower band, going from right to left, and then in the next above it; and when any cells are found empty, fill them up with the numbers that belong to them: in this manner you will have the square 16 filled up magically, as seen in the annexed figure, and the sum of each band and each diagonal will be 34.

A B

1			4
	6	7	
	10	11	
13			16

C D

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

B.

The following is a method of constructing oddly even Magic Squares, that is those the root of which when halved gives an odd number; as those of 6, 10, 14, etc.

Take for example the square of the root 6. To fill it up inscribe in it the first six numbers of the arithmetical progression, 1, 2, 3, etc., according to the foregoing method; which will give the first primitive square, as in the annexed figure.

5	6	3	4	1	2
2	1	4	3	6	5
5	6	3	4	1	2
5	6	3	4	1	2
2	1	4	3	6	5
5	6	3	4	1	2

The second must be formed by filling up the cells in a vertical direction, according to the same principle, with the multiples of the root, beginning at 0, viz., 0, 6, 12, 18, 24, 30.

24	6	24	24	6	24
0	30	0	0	30	0
12	18	12	12	18	12
18	12	18	18	12	18
30	0	30	30	0	30
6	24	6	6	24	6

A

	29	12	27	28	7	26	
	2	31	4	3	36	5	
	17	24	15	16	19	14	
C	23	18	21	22	13	20	D
	32	1	34	33	6	35	
	11	30	9	10	25	8	

B

The similar cells of the two squares if then added, will form a third square, which will require only a few corrections to be magic. This third square is here annexed.

To render the square magic, leaving the corners fixed, transpose the other numbers of the upper horizontal band, and of the first vertical one on the left, by reversing all the remainder of the band; writing 7, 28, 27, 12, instead of 12, 27, etc., and in the vertical one, 32, 23, 17, and 2, from the top downwards, instead of 2, 17, etc.

It will be necessary also to exchange the numbers in the two cells of the middle of the second horizontal band at the top, of the lowest of the second vertical band on the left, and of the last on the right. The numbers in the cells A and B must also be exchanged, as well as those in C and D; by which means we shall have the square corrected and magically arranged.

29	7	28	9	12	26
32	31	3	4	36	5
23	18	15	16	19	20
14	24	21	22	13	17
2	1	34	33	6	35
11	25	10	27	30	8

V.—Quibbles and Trifles

Quite Simple.

How can you take one away from nineteen, and yet have twenty left?

XIX − I = XX.

At Sixes and Sevens.

How is it possible to take nine from six, ten from nine, and fifty from forty, and have six left,

SIX	IX	XL
IX	X	L
S	I	X

How's That?

Come, tell to me what figures three,
When multiplied by four
Make five exact, 'tis truth in fact,
This mystery explore.

In decimals $1{\cdot}25$ is $1\frac{1}{4}$ in fractions, which being multiplied by 4 makes 5.

Simple Addition.

How can five be added to six in order to make nine? Draw six straight lines thus:—

| | | | | |

Add five more and the result is nine

N I N E.

The Mysterious Century.

Write down 100 in four figures.

$99\frac{9}{9}$

An Easy Catch.

If five times four are thirty-three,
What will the fourth of twenty be?

$8\frac{1}{4}$.

Jilted!

Jack said he could eat more nuts than Jill. Jill replied that she would jolly well like to see him do it! Jack ate ninety-nine; Jill ate one hundred and won. How many more nuts did Jill eat than Jack?

Those to whom you put the question will think you said, " one hundred and *one*," and will answer accordingly.

The Truss of Hay.

A truss of hay weighing but half a hundred weight in a scale, weighed two hundred weight upon the end of a fork carried on the farmer's shoulder. How could that be?

The fork was as the steelyard; the farmer's shoulder as the fulcrum sustaining the burden between the two powers acting at both ends of the fork.

Smart Man!

It is said that a man made so many pairs of shoes in one day that it took two days to count them. But this record is surely beaten by the Irishman who built so many miles of stone wall in one day, that it took him all night and the next day to get home!

A Knotty Question.

When first the marriage knot was tied
Between my wife and me,
My age exceeded hers as much
As three times three does three.

But when ten years and half ten years
We man and wife had been,
Her age approached as near to mine
As eight is to sixteen.

How old were they when they were married?
The bride was fifteen, and the bridegroom forty-five.

Keep Your Distance!

One summer evening as I was taking a walk, I heard the voice of some one behind, calling to me; I turned back and saw it was a friend, at the distance of 400 yards, wanting to overtake me. We moved each of us 200 yards, with our faces towards each other, in a direct line, yet we were still 400 yards asunder. How can this possibly be?

The former moved 200 yards backwards with his face towards his friends', and the latter 200 yards forwards with his face towards him.

A Railway Problem.

"The 20th Century Limited" leaves New York for Chicago at 4 p.m., and travels at the rate of 75 miles an hour. Another train leaves Chicago for New York, at the same time, and travels at 60 miles an hour. When the two trains meet, which one is nearest to New York?

This sounds a genuine problem, but it is only a catch, for it is obvious that when the two trains meet, they must both be the same distance from New York.

A Talking Library.

It has been estimated that each individual averages three hours of conversation daily, at the rate of 100 words a minute, or 20 pages of an octavo volume in an hour. At this rate we talk a volume of 400 pages in a week, and 52 volumes in a year!

The Ingenious Debtor.

I owe twenty shillings to four persons, and have only nineteen shillings with which to pay them. How can I pay them all their fair demands without deduction from any?

The half, one-third, one-sixth, and one-nineteenth of nineteen shillings are:—9/6 + 6/4 + 3/2 + 1/- = 20/-. This however, is only a payment upon paper.

Simple Addition.

To a thousand add 1, twice fifty and ten,
Six-seventh's of a million's this sum I'll maintain.
M I L L I O

Two's and Twos.

It is required to place three 2's in such a manner as to form three numbers in Geometrical Progression.

$$\frac{2}{2\times2};\quad 2-\frac{2}{2};\quad \frac{2\times2}{2}$$

which denote $\frac{1}{2}$, 1 and 2 respectively, the common ratio being 2.

A Matter of Expression.

Express 12 by four figures each the same:

$$11\tfrac{1}{1}.$$

An Odd Half.

This is a truth (tho' the number's even),
That half of twelve's exactly seven.
The half of twelve will seven be,
Cut through the middle as you see. XII

A Game of Fives.

Place four 5's so that their sum shall be $6\frac{1}{2}$:

$$5{\cdot}5+\tfrac{5}{5}.$$

All in a Row.

Place in a row nine figures, each different from the other; multiply them by 8, and the product shall still consist of nine different figures.

$$\begin{array}{r} 123456789 \\ 8 \\ \hline 987654312 \end{array}$$

Three Three's.

It is required to place three 3's in such a manner, as to form three numbers in Geometrical Progression, the common ratio of which shall be 3?

$$\frac{3}{3\times3},\quad 3^{3-3},\quad \frac{3\times3}{3},$$

denoting $\frac{1}{3}$, 1 and 3 respectively.

All the Same.

Express 78 by six figures each the same: $77\frac{77}{77}$.

A Threepenny Bit.

What part of 3d. is $\frac{1}{3}$ of 2d.?

$\frac{1}{3}$ of 2d. $=\frac{2}{3}$ of 1d. $=\frac{2}{9}$ of 3.

FRANCIS GRIFFITHS' TRAVEL BOOKS.

MODERN ARGENTINA. By W. H. KOEBEL.

The El Dorado of To-Day. With Notes on Uruguay and Chile. With 123 Illustrations. .

Demy 8vo, cloth, gilt top, 12s. 6d. net; by post 13s.

TIMES:—" The book is thoroughly up-to-date, is very readable, and contains much interesting information . . . It is the pleasantly-written work of a man with observant eye and ready ear who has made the most of his time spent in the country and has succeeded in giving a vivid and intelligent account of what he has seen and heard."

SHADOWS OF OLD PARIS. By G. DUVAL.

Fully Illustrated by J. Gavin, in colour, wash, and line drawings. Crown 8vo, 12s. 6d. net; by post 13s.

" Among the crowds of idlers and pleasure seekers, of students even, who make the French capital temporarily their home, there are few lovers of tradition curious enough to turn from the dazzling modern city of pleasure, and plunge into the dingy Paris of the past. Itself a town within a town. . . . . . . The whole story of what is more passionately eloquent, what more enigmatic and mysteri-France is written on the walls of Paris; and to a student of history, ous, than those silent stones?"

BURGUNDY: THE SPLENDID DUCHY.
Studies & Sketches in South Burgundy. By PERCY ALLEN.

Fully Illustrated with eight water-colour and 86 line drawings by Miss Marjorie Nash. Foolscap quarto, cloth, 12s. 6d. net.

BURGUNDY, though quite unknown to the majority of travellers, is one of the most interesting provinces of France. This full-blooded land of fertile meadows, vine-clad slopes, and rugged hills, is the home of a people strong and vivacious—comfortable, courageous, eloquent, sonorous folk, that like a good dinner and a good story to follow, that have produced Bernard, Bossuet, Greuze, and Lamartine.

Burgundian history, too, is unrivalled in its interest for English readers. It was the scene of events vital to the making of Europe. It was one of the strongholds of Roman civilization. It saw the genesis of a religious movement that was the greatest feature of eleventh and twelfth century history.

In this, his latest book, Mr. Allen gives us all that an intelligent traveller need know concerning the southern portion of the great Duchy, a province that has not previously been adequately dealt with by an English writer. Miss Marjorie Nash's drawings, in colour and black and white, worthily complete a volume that should form a welcome addition to the library of French Travel.

WITH NAPOLEON AT WATERLOO. EDWARD BRUCE LOW, M.A.

And other unpublished documents of the Waterloo and Peninsular Campaigns, also Papers on Waterloo by the late Edward Bruce Low, M.A.

Edited, with an Introduction, by MacKenzie MacBride.

32 Illustrations. Royal 8vo. Cloth, gilt top. Price 15s. net.

LONDON: FRANCIS GRIFFITHS, 34, MAIDEN LANE, STRAND, W.C.